System of Particles and Rotational Motion

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System of Particles and Rotational Dynamics

A group of particles that can be identified and distinguished from other particles or groups is called a system. The motion of the system as a whole can be analysed by applying the laws of mechanics. This is achieved by using the concept of 'centre of mass'. Thus, identifying the centre of mass is quite important in the study of dynamics of a system.

Rigid body: It is a body whose shape and size do not change during its state of rest or motion.

A classic example of a physical system is that of a rigid body, in which the relative distance between any two particles remains unaltered during its motion. Study of rigid body motion involves physical parameters like moment of inertia, torque, angular velocity, angular momentum, translational energy and rotational energy.

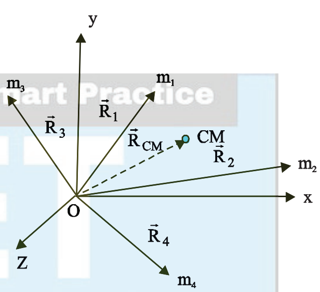
Centre of mass of a system of two particles

Centre of mass is the point at which the entire mass of a body is supposed to be concentrated.

If we have discrete system of particles as shown in the figure, then centre of mass is defined as

$$R_{cm} = \frac{m_1\vec{R}_1 + m_2\vec{R}_2 + m_3\vec{R}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$R_{cm} = \frac{1}{M}\sum m_i\vec{R}_i$$



The coordinates of centre of mass

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + m_3} = \frac{1}{M}\sum m_ix_i$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{1}{M}\sum m_iy_i$$

$$Z_{cm} = \frac{m_1z_1 + m_2z_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{1}{M}\sum m_iz_i$$

For a system having continuous distribution of the mass, the coordinates of cm are

$$X_{cm} = \frac{1}{M}\int x\,dm, \quad Y_{cm} = \frac{1}{M}\int y_{dm}, \quad Z_{cm} = \frac{1}{M}\int z\,dm$$

- Position of centre of mass is independent of coordinate system choosen.
- Centre of mass depends on the shape of the body and distribution of mass.
- Centre of mass coincides with geometric centre bodies where mass is homogeneous.
- Centre of mass remains unchanged in rotatory motion while in translatory motion position changes.
- If small position of mass m_2 is removed from a larger position of mass m_1. Then centre of mass of the remaining part is $x_{cm} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$

Motion of centre of mass

If a system of particles of masses m_1, m_2, . . move with velocities v_1, v_2, v_3 . . . respectively

- Then velocity of centre of mass is given by

$$V_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum m_i\vec{v}_i}{M}$$

($M \rightarrow$ total mass of the body)

- Momentum of the centre of mass is

  $M\vec{v}_{cm} = m_1\vec{v}_1 + m_2 + \vec{v}_2 + \ldots + m_n\vec{v}_n$

  $\vec{P}_{CM} = \vec{P}_1 + \vec{P}_2 + \ldots + \vec{P}_n$

  If $\vec{V}_{CM} = 0,\ \vec{P}_{CM} = 0$ i.e., in the frame of reference of CM, the momentum of a system is zero.

- Acceleration of CM is given by

  $$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots m_n\vec{a}_n}{m_1 + m_2 + \ldots m_n} = \frac{\sum m_i a_i}{M}$$

  Or $M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots m_n\vec{a}_n$

  $\vec{F}_{ext} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n$

  This is equation of motion of centre of mass.

  If $\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{CM} = 0 \Rightarrow \vec{v}_{CM} =$ constant.

  If $\vec{F}_{ext} = 0,$ no external force acts on a system, then the velocity of its CM remains constant.

  $\Rightarrow$ velocity of CM is not affected by internal forces.

- If $\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{CM} = 0 \Rightarrow \vec{V}_{CM} =$ constant, then $\vec{p}$ = constant.

  This leads to conservation of linear momentum.

- If a system of 2 particles of mass m_1 and m_2 separated by a distance x initially at rest, moving towards each other under the action of attractive force then the 2 particles collide at their centre of mass.

  Here $F_{12} = -F_{21}$ or $m_1a_1 = m_2a_2 \Rightarrow \frac{a_1}{a_2} = \frac{m_2}{m_1}$

  Since initial momentum = 0, centre of mass is at rest.

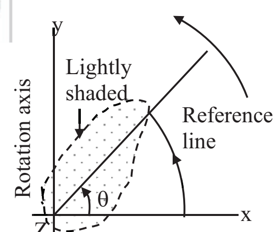
  $\vec{V}_{CM} = 0,\ m_1v_1 = m_2v_2$ or $\frac{v_1}{v_2} = \frac{m_2}{m_1}$

  $\therefore$ Ratio of distances covered by particles before collision is $\frac{x_1}{x_2} = \frac{m_2}{m_1}$

Rotational motion: A rigid body undergoes rotational motion when each of its particles travel in a circle centered on a straight line, called the axis of rotation

Rotational variables: The rotational variables are the angular equivalents of the linear quantities position, displacement, velocity and acceleration

Angular position (θ): It is the position of a fixed line perpendicular to the axis of rotation, fixed in the body, relative to a fixed axis. It is also called the angular coordinate. $\theta = \frac{s}{r}$ where



A rigid body rotating anticlockwise in the xy- phase about z axis the rotation axis

s → arc length described by a point on the reference line relative to the fixed axis.

r → radius of the arc. Its SI unit is the radian (rad) without any dimensions.

Angular displacement: It is the difference in the angular coordinates of rotating body at times t_1 and

$t_2 = t_1 + \Delta t$

$\Delta\theta = \theta_2 - \theta_1$

Angular velocity

(i) **Average angular velocity (ω_{av}):** It is the ratio of the angular displacement to the elapsed time

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

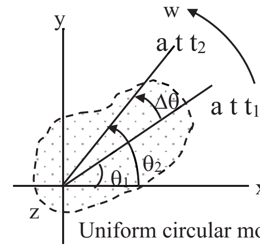
(ii) **Angular velocity(ω):** It is the rate of change of angular displacement

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

It is the limiting value of the average angular velocity

The SI unit of angular velocity is rad.s^{-1}.

Its dimensional formula is $[M^{\circ}\ L^{\circ}\ T^{-1}]$



Uniform circular motion

Angular acceleration

(i) **Average angular acceleration(α_{av}):** It is the ratio of the change in angular velocity to the elapsed time

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

(ii) **Angular acceleration(α):** It is the rate of change of angular velocity

$$\alpha = \lim_{\Delta t \to o} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

It is the limiting value of the average angular acceleration.

Its SI unit is rad s^{-2}

Its dimensional formula in $[M^{\circ}\ L^{\circ}\ T^{-2}]$

- Finite angular displacements are not vectors since they do not obey commutative law
- Infinitesimal (differential) angular displacements are, however, treated as vectors.
- The direction of rotational variables specifies the axis of rotation, and is given by the right hand rule
- Nothing moves in the direction of angular variables, i.e., along the axis of rotation.

Relations connecting linear and angular variables:

(a) $s = r\theta$; (b) $v = \omega r$; (c) $a = \alpha r$

The following table contains linear as well as angular equations of motion for constant acceleration

Angular			Linear		
Sl.no.	Equation	Missing quantity	Sl.no.	Equation	Missing quantity
1	$\omega = \omega_o + \alpha t$	θ	1	$v = u + at$	s
2	$\theta = \omega_o t + \frac{\alpha t^2}{2}$	ω	2	$s = ut + \frac{at^2}{2}$	v
3	$\omega^2 = \omega_o^2 + 2\alpha\theta$	t	3	$\omega^2 = u^2 + 2as$	t
4	$\theta = \omega_{av} t = \frac{(\omega + \omega_o)t}{2}$	α	4	$s = u_{av}\ t = \frac{(u+v)t}{2}$	a
5	$\theta_n = \omega_o + \frac{\alpha(2n-1)}{2}$	ω	5	$s_n = u + \frac{a(2n-1)}{2}$	v
6	$\alpha = \frac{\theta_2 - \theta_1}{t^2}$	ω_o, ω	6	$a = \frac{s_2 - s_1}{t^2}$	u, v

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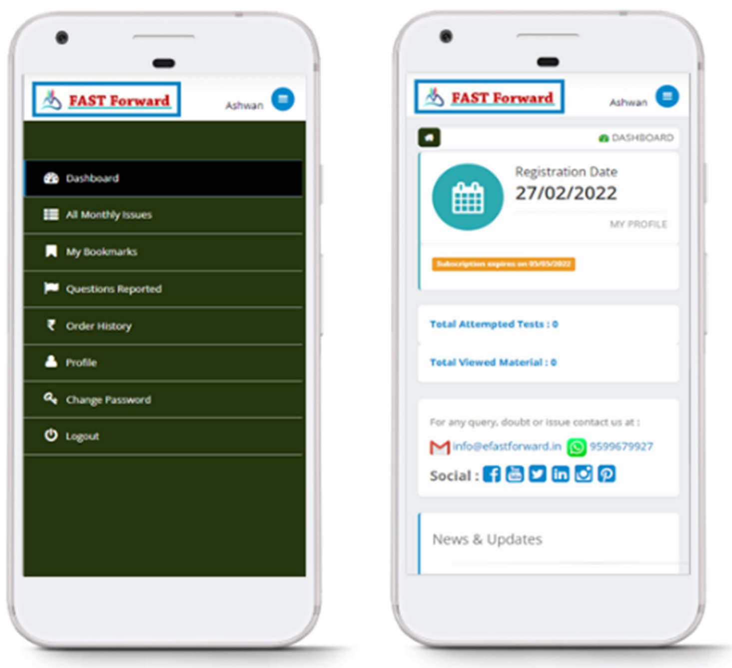

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