MOTION IN A PLANE

For Quick Revision and Smart Practice

NEET FAST FORWARD

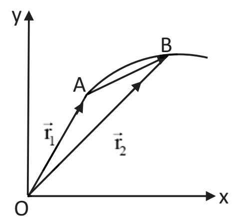
An Innovation of **NEET Experts** with **30+ years** of Blissful Teaching Experience and Inspiration of **Lakh+ successful MEDICO professionals**

India's First e - Magazine with Live Testing

MOTION IN A PLANE

Motion with uniform acceleration in a plane

Consider a point object moving in XY plane with an uniform acceleration $\vec{a}$. Let us suppose 'O' be the origin for measuring time and position of the object. Let the object be at positions A and B at times t_1 and t_2 respectively, where $\overline{OA} = \vec{r}_1$ and $\overline{OB} = \vec{r}_2$.



Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of object at instants t_1 and t_2 respectively,

then constant acceleration is given by

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

$$\vec{v}_2 - \vec{v}_1 = \vec{a}(t_2 - t_1)$$

$$\vec{v}_2 = \vec{v}_1 + \vec{a}(t_2 - t_1)$$

If $t_1 = 0$, $t_2 = t$, $\vec{v}_1 = \vec{u}$ and $\vec{v}_2 = v$ then

$$\boxed{\vec{v} = \vec{u} + \vec{a}t}$$

The above equation can be expressed in terms of rectangular components in XY plane as follows.

$\vec{u} = u_x\hat{i} + u_y\hat{j}$ where $u = \sqrt{u_x^2 + u_y^2}$

$\vec{v} = v_x\hat{i} + v_y\hat{j}$ where $v = \sqrt{v_x^2 + v_y^2}$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$ where $a = \sqrt{a_x^2 + a_y^2}$

Also, $v_x = u_x + a_x t$ and $v_y = u_y + a_y t$

Displacement in x-direction is given by $x - x_0 = u_x t + \frac{1}{2}a_x t^2$

And displacement in y-direction is given by $y - y_0 = u_y t + \frac{1}{2}a_y t^2$

- ## Projectile

Projectile is the name given to a body thrown with some initial velocity making an angle θ [≠ 90°] with the horizontal direction, and then allowed to move in two dimensions under the action of gravity alone, without being propelled by any engine or fuel.

The path followed by a projectile is called its 'trajectory'. Examples of projectile are:

(i) A ball hit by a bat

(ii) A bullet fired from a gun or pistol

(iii) A javelin thrown by an athlete

(iv) A shot-put sphere thrown by an athlete

(v) A body dropped from an aeroplane in flight / bus / train

In the above examples, we find that a projectile moves under the combined effect of two velocities:

1. A uniform velocity in the horizontal direction, which would not change provided there is no air resistance.

2. A uniformly changing velocity in the vertical direction due to gravity.

To study the motion of a projectile, the following assumptions are made.

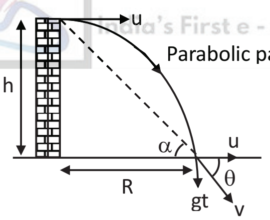
(i) There is no resistance of air.

(ii) The effect due to rotation of earth and curvature of the earth is neglected.

(iii) The acceleration due to gravity (g) is constant in magnitude and in direction at all points of the motion of projectile.

▪ Horizontal Projectile

When a body is projected horizontally with a velocity from a point above the ground level, it is called a "horizontal projectile". When a stone is projected horizontally with a velocity 'u' from the top of a tower of height 'h' it describes a parabolic path as shown in figure.



(i) Time of descent, $t = \sqrt{\frac{2h}{g}}$ (independent of 'u')

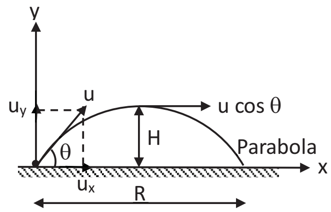
(ii) Horizontal displacement (or) range is $R = u\sqrt{\frac{2h}{g}}$

(iii) The speed with which it hits the ground is $v = \sqrt{u^2 + 2gh} = \sqrt{u^2 + g^2t^2}$

(iv) The angle at which it strikes the ground is $\theta = \tan^{-1}\left(\frac{gt}{u}\right) = \tan^{-1}\left(\frac{\sqrt{2gh}}{u}\right)$

(v) If α is angle of elevation of point of projection from the point where the body hits the ground, then $\tan\alpha = \frac{h}{R} = \frac{\frac{gt^2}{2}}{ut} = \frac{gt}{2u} \Rightarrow \tan\alpha = \frac{\tan\theta}{2}$

▪ Oblique Projectile



Any body projected into air with some velocity at an angle θ [≠ 90° and 0°] with the horizontal is called an "oblique projectile".

(i) Horizontal component of velocity is $u_x = u\cos\theta$, remains constant throughout the journey.

(ii) Vertical component of velocity $u_x = u\sin\theta$, varies at the rate of 'g'.

(iii) After a time 't':

(a) Horizontal component of velocity is $v_x = u\cos\theta$ (= u_x)

(b) Vertical component of velocity is $v_y = u_y - gt = u\sin\theta - gt$

(c) Resultant velocity is $v = \sqrt{v_x^2 + v_y^2}$

(d) Direction of velocity is given by $\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

where α is the angle that $\vec{v}$ makes with horizontal.

(e) Horizontal displacement in a time 't' is $x = u_x t = (u\cos\theta)\,t$

(f) Vertical displacement in a time 't' is $y = u_y t - \frac{1}{2}gt^2 = (u\sin\theta)t - \frac{1}{2}gt^2$

(g) Net displacement of the body is $S = \sqrt{x^2 + y^2}$

(h) Equation of trajectory (which is a parabola) of an oblique projectile is

$$y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2 = Ax - Bx^2$$

From the above equation,

- $\theta = \tan^{-1}(A)$
- Range of projectile is $R = \frac{A}{B}$
- Maximum height is $H = \frac{A^2}{4B}$