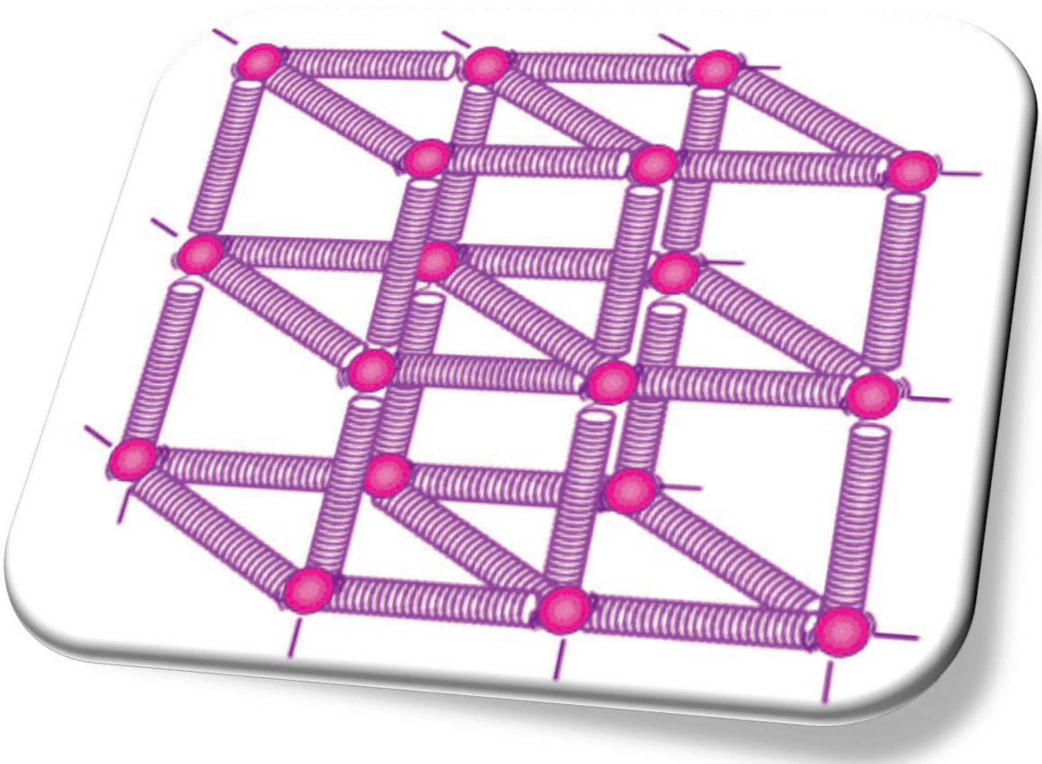
Mechanical Properties of Solids



Mechanical Properties of Solids

Elasticity is the property of the materials by virtue of which the bodies restore their natural shape and size on removal of the external (deforming) forces.

A perfect elastic body is one which completely regains its original form after the removal of the deforming forces e.g., a quartz fibre.

A perfect plastic body is one which does not regain its original form and remains in the deformed state. e.g., wet soil, wax, etc.

Stress

The restoring force developed per unit area of the body, when the body is subjected to a deforming force, is called stress.

Since the restoring force is equal and opposite to the external deforming force, the stress may be measured as the external force acting per unit area.

$$\therefore \text{ Stress} = \frac{\text{external force applied}}{\text{area}} = \frac{F}{A}$$

Strain

The ratio of change in dimension of the body to its original dimension is called strain.

There are three types of strain.

(a) longitudinal strain $= \dfrac{\text{change in length}}{\text{original length}} = \dfrac{\Delta l}{l}$

(b) volume strain $= \dfrac{\text{change in volume}}{\text{original volume}} = \dfrac{\Delta V}{V}$

(c) shearing strain is the ratio of the displacement of a layer in the direction of the tangential force and the distance of that layer from the fixed surface.

Hooke's law

Within elastic limit stress is directly proportional to strain.

i.e., stress $\propto$ strain or $\dfrac{\text{stress}}{\text{strain}} = \text{constant}$

This constant is called modulus of elasticity.

Types of modulii of elasticity.

1. Young's Modulus of Elasticity 'Y'

Within elastic limit, the ratio of longitudinal stress to longitudinal strain is called Young's modulus of elasticity.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{\ell/L} = \frac{FL}{\ell A}.$$

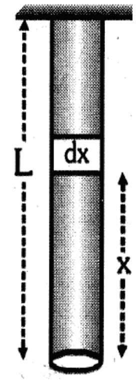
Within elastic limit, the normal force acting on a unit cross-sectional area of a wire due to which the length of the wire becomes double, is equivalent to the Young's modulus of elasticity of the material of the wire. If L is the original length of the wire, r is its radius and ℓ the increase in its length as a result of suspending a weight Mg at its lower end

then Young's modulus of elasticity of the material of the wire is $Y = \dfrac{(Mg/\pi r^2)}{(\ell/L)} = \dfrac{MgL}{\pi r^2 \ell}$

Unit of Y : N/m^2 or pascal   Dimensions of Y : $[M^1L^{-1}T^{-2}]$

- **Increment of length due to own weight**

Consider a rope of mass M and length L hanging vertically. As the tension at different points on the rope is different, stress as well as strain will be different at different points.
(i) maximum stress will be at the point of suspension
(ii) minimum stress will be at the lower end.



Consider an element of rope of length dx at x distance from the lower end,

then tension there $T = \left(\frac{M}{L}\right) x\, g$

So stress $= \frac{T}{A} = \left(\frac{M}{L}\right)\frac{xg}{A}$

Let increase in length of this element be dy then strain $= \frac{dy}{dx}$

So, Young modulus of elasticity $Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{M}{L}\frac{xg}{A}}{dy/dx} \Rightarrow \left(\frac{M}{L}\right)\frac{xg}{A}dx = Ydy$

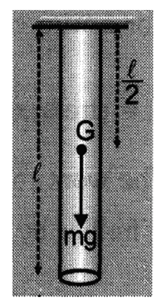
Summing up the expression for full length of the rope,

$$\frac{Mg}{LA}\int_0^L x dy = \int_0^{\Delta\ell} dy \Rightarrow \frac{Mg}{LA}\frac{L^2}{2} = Y\Delta\ell \Rightarrow \Delta\ell = \frac{MgL}{2AY}$$

[Since the stress is varying linearly we may apply the average method to evaluate strain.]

Alternate Method : Since the, weight acts at the centre of gravity, therefore

$\therefore$ The original length will be taken as $\frac{\ell}{2}$ $\therefore Y = \frac{Mg \times \frac{\ell}{2}}{A \times \Delta\ell} \Rightarrow \Delta\ell = \frac{Mg\ell}{2AY}$



But $M = (\ell A)\rho$ $\quad \therefore \Delta\ell = \frac{\ell A\rho g\ell}{2AY}$ or $\Delta\ell = \frac{\rho g\ell^2}{2Y}$

2. Bulk's modulus of elasticity 'K' or 'B'

Within elastic limit, the ratio of the volume stress (i.e., change in pressure) to the volume strain is called bulk's modulus of elasticity.

$$K \text{ or } B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{F/A}{\frac{-\Delta V}{V}} = \frac{\Delta P}{\frac{-\Delta V}{V}}$$

The minus sign indicates a decrease in volume with an increase in stress and vice-versa.

Unit of K : M/m^2 or pascal

Compressibility 'C'

The reciprocal of bulk's modulus of elasticity is defined as compressibility.

$C = \frac{1}{K}$; SI unit of C : m^2/N or pascal^{-1}

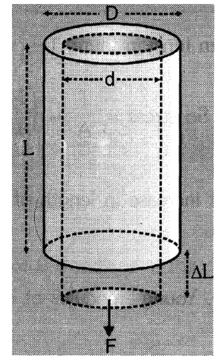
3. Modulus of Rigidity 'η'

Within elastic limit, the ration of shearing stress to shearing strain is called modulus of rigidity of a material

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \left(\frac{\frac{F_{tangential}}{A}}{\phi}\right) = \frac{F_{tangential}}{A\phi}$$

Note : Angle of shear 'φ' is always taken in radians

4. Poisson's Ratio (σ)



Within elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\beta}{\alpha}$$

$$\beta = \frac{-\Delta D}{D} = \frac{d - D}{d} \text{ and } \alpha = \frac{\Delta L}{L}$$

$-1 \le \sigma \le 0.5$ (theoretical limit)

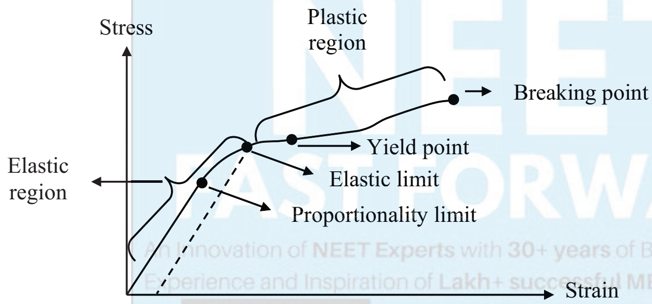
$\sigma \approx 0.2 - 0.4$ (experimental limit)

- **Relation between Y, K, η and σ :** (To be remembered)

$$Y = 3K(1 - 2\sigma), \qquad Y = 2\eta(1 + \sigma), \quad \frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}.$$

☞	Reciprocal of bulk modulus is called compressibility.

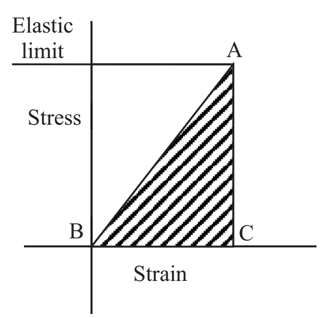
The stress v/s strain graph for a wire under the action of deforming forces is as shown below:



Elastic potential energy in a stretched wire

The elastic potential energy stored in a wire stretched by a length l is given by $U = \frac{AY}{2L} l^2$

where L is the original length of the wire
A is the cross-section of the wire
Y is the Young's modulus of the wire.



The above expression may be expressed as

$U = \frac{1}{2} \times$ maximum stretching force × extension

or $U = \frac{1}{2} \times$ stress × strain × volume

or $U = \frac{1}{2} \times Y \times (\text{strain})^2 \times$ volume

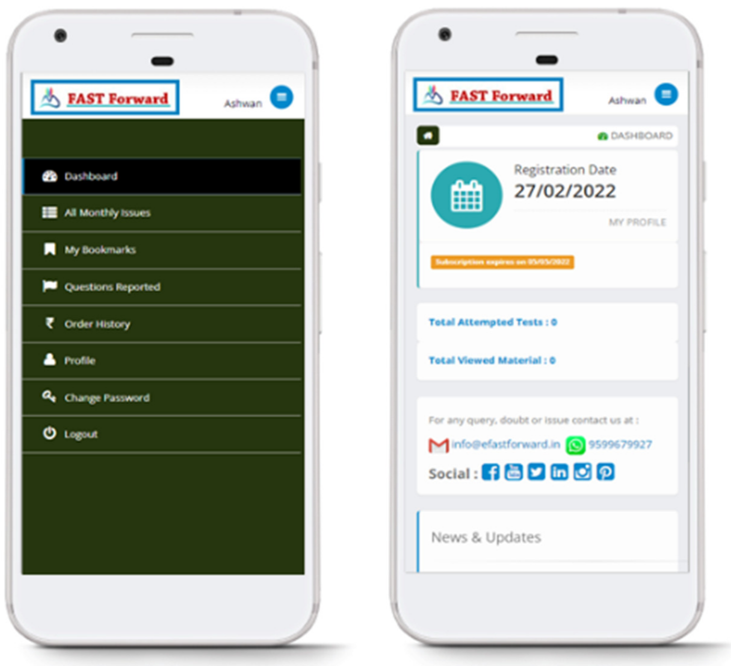
The energy stored by a member within elastic limit is called elastic potential energy. The Area under the stress shown give within elastic limit will be elastic potential energy.

$= \text{Area of triangle ABC} = \frac{1}{2}$

Factor Affecting Elasticity

- **Effect of Temperature**

TO DOWNLOAD/VIEW FULL FILE



Download Android App

Fast Forward a work of Adhipati Creations that provides the best app for NEET, JEE, BITSAT, CUET and CBSE exam preparation.