Gravitation

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Kepler's laws of planetary motion

Based on the regularities in the motion of the planets, Kepler formulated a set of three laws known as Kepler's laws of planetary motion.

I Law (Law of orbits)

All planets move round the Sun in elliptical orbits with Sun at one of the foci.

II Law (Law of areas)

A line joining any planet and the Sun sweeps out equal areas in equal intervals of time.

Areal velocity: The area swept by the radius vector of a planet around the sun, per unit time is called areal velocity of the planet. Areal velocity of a planet remains constant.

III Law (Law of periods)

The square of the period of any planet about the Sun is proportional to the cube of the semi-major axis of its orbit. $T^2 \propto a^3$

where T is the period and a is the semi major axis.

If T_1 and T_2 are the periods of any two planets and r_1 and r_2 are their mean distances from the Sun, then

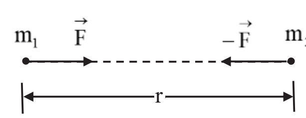
$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Nearer planets move faster. For example the orbital speed of Earth is about 30 km s^{-1}. The speed of Jupiter is about 13.2 km s^{-1} with a period of 11.86 years and that of Saturn is 9.7 km s^{-1} with a period of 29.46 years.

- Out of planets known before 18th century, Saturn is the slowest. Infact, the Sanskrit name 'shani' refers to slowly moving object. Saturn is seen for about 2½ years in each constellation $\left(\frac{T}{12} = \frac{29.46}{12} \approx 2.5 \text{ years}\right)$ and passes through 3 constellations in 7½ years, commonly known as 'saade-sath'.

Newton's Law of Gravitation: Every particle attracts every other particle with force that is proportional to the product of the masses and inversely proportional to the square of their separation and acts along the straight line joining them.

$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$



Newton's law of universal gravitation

G is a universal constant, called the constant of gravitation.

$G = 6.67 \times 10^{-11}\ N\,m^2 Kg^{-2}$

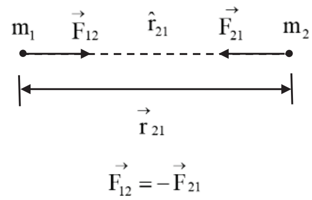
The dimensional formula for G is $[M^{-1}L^3T^{-2}]$

- The gravitational force is the weakest known force of nature.
- The value of G is the same for two particles, two celestial objects and two terrestrial objects.
- A spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its centre.
- A uniform shell of matter exerts no gravitational force on a particle located inside it. It is a gravitational shield for particles within it.
- The net gravitational force on a particle due to one or more particles is determined using the principle of superposition.

- The gravitational force on a particle would first increase slightly, eventually reach a maximum and finally decrease to zero at the centre of the earth as the particle is lowered down the centre. The reason for the initial increase is the predominance of the effect of decrease in r over that of the shell of the earth's crust that lies outside the radial position of the particle. As the centre is approached, the effect of the outer shell predominates.
- If the earth were uniformly dense, the gravitational force would decrease to zero as the particle is lowered to the centre of the earth.

Newton's law in vector form

$$\vec{F}_{21} = \left(\frac{Gm_1m_2}{r^3}\right)\vec{r}_{12} = \left(\frac{Gm_1m_2}{r^2}\right)\hat{r}_{12}$$



where $\vec{F}_{12} \rightarrow$ force exerted on particle of mass m_2 by particle of mass m_1

$\vec{r}_{12} \rightarrow$ position vector of m_2 relative to m_1

$\hat{r}_{12} \rightarrow$ unit vector in the direction of $\vec{r}_{12}$

Gravity: It is the term used to describe the force on a body near the surface of a celestial body. The earth's gravity is given by

$$F = \frac{GMm}{(R+h)^2}$$ where $M \rightarrow$ mass of the earth, $R \rightarrow$ average radius of the earth

$h \rightarrow$ height of a body of mass m above the surface of the earth.

Acceleration due to gravity (g): It is the acceleration of a body due to gravity. On the surface of the earth $g = \frac{GM}{R^2}$

- The value of g is independent of the mass of the body.
- In the absence of air resistance, heavy and light bodies released from the same height reach the ground simultaneously.
- The average density of the earth is given by $\rho = \frac{3g}{4\pi GR}$.
- Acceleration due to gravity on the surface of the moon is about one-fifth of that on the surface of the earth.

Variation of g

(i) Due to altitude: Acceleration due to gravity at a height 'h' above surface of earth is

$$g_h = \frac{GM}{(R+h)^2}$$

$$g_h = g\left[1 - \frac{2h}{R}\right] \quad \text{(for } h << R) \qquad R \rightarrow \text{radius of earth}$$

Thus g decreases with altitude.

(ii) Due to depth: Acceleration due to gravity at a depth 'd' below the surface of earth is

$$g_d = g\left[1 - \frac{d}{R}\right] \qquad \text{for } (d << R)$$

at $d = R$, i.e., centre of earth

$g_d = 0$

$\therefore$ g decreases with depth

(iii) Due to rotation of earth: Acceleration due to gravity at a latitude λ is given by

$g_\lambda = g - R\omega^2 \cos^2 \lambda \qquad \omega \rightarrow$ angular velocity of earth

(a) At poles: $\lambda = \frac{\pi}{2}$

$$\therefore g_p = g - R\omega^2 \cos\frac{\pi}{2} = g_{max}$$

Thus g is maximum at poles.

(b) At equator: $\lambda = 0$

$$\therefore\ g_{eq} = g - R\omega^2 \cos 0$$
$$= g - R\omega^2$$
$$= g_{min}$$

(c) If earth stops rotating about its axis, the value of g at the equator will increase by 0.38 %, but at poles it remains constant.

(d) If angular speed (ω) increases by 17 times present value, there will be weightlessness on the equator. But g at the poles do not change. Earth's duration of day reduces to 84 minutes.

(iv) Due to nonspherical shape of earth

Polar radius (R_p) > equatorial radius and $g \propto \frac{1}{R^2}$

Value of g increases from poles to radius.

g is maximum at poles and minimum at non spherical shape of earth.

- Latitude is the angle made by the line joining the centre of the earth and a point on the surface of the earth with the equator.
- If h is not small enough compared to R, $g = g\left[1+\frac{h}{R}\right]^{-2}$.
- The weight of an object decreases with altitude as well as depth. On the other hand, it increases with latitude.
- The free-fall acceleration g measured on the equator of the real, rotating earth is slightly, less than the gravitational acceleration (a_g) due strictly to the gravitational force $a_g - g = 0.03\,\text{m s}^{-2}$.

Gravitational field and potential

Gravitational field

The strength of a **gravitational field at any point is defined as the gravitational force experienced by unit mass at that point.**

If $\vec{f}$ is the force acting on a mass m at a point, the gravitational field at that point is

$$\vec{F}_G = \frac{\vec{f}}{m}. \qquad \ldots (1)$$

Gravitational field due to a point mass at a distance 'r' is given by

$$\vec{F}_G = \frac{\vec{f}}{m} = -\frac{GM}{r^2}\hat{r} \qquad \ldots (2)$$

The gravitational field at a point due to an object is inversely proportional to its distance from the object and it is a vector directed towards the object. SI unit of gravitational field is N kg^{-1}.

Gravitational potential energy

The concept of potential energy is already introduced. The potential energy of a system corresponding to a conservative force is defined as $U_f - U_i = -\int_i^f \vec{F}.d\vec{r}$

The change in the potential energy of a system is equal to the negative of the work done by the internal conservative forces.

For the small displacements of a body near the earth's surface, we have used the equation $U_f - U_i = mgh$

But the idea of gravitational potential energy is not confined to earth-particle system. In general, for a two particle system, we can write

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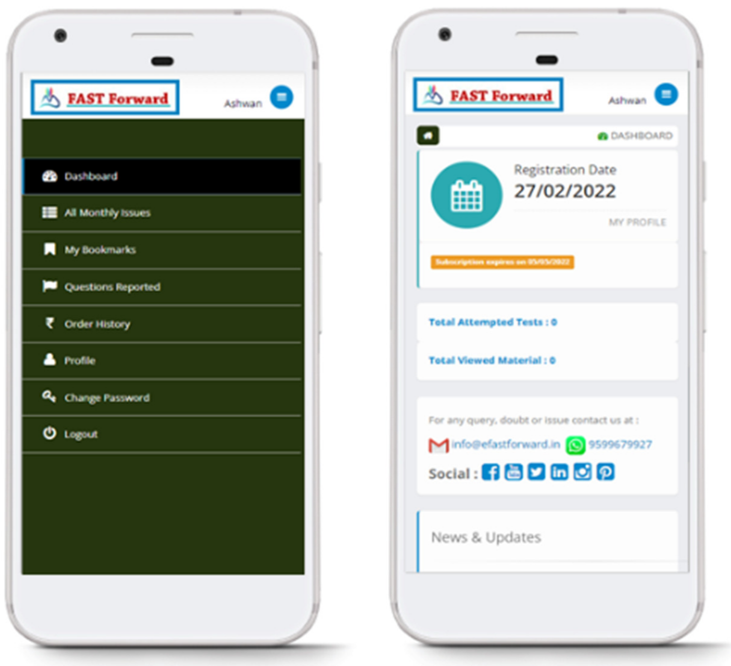

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