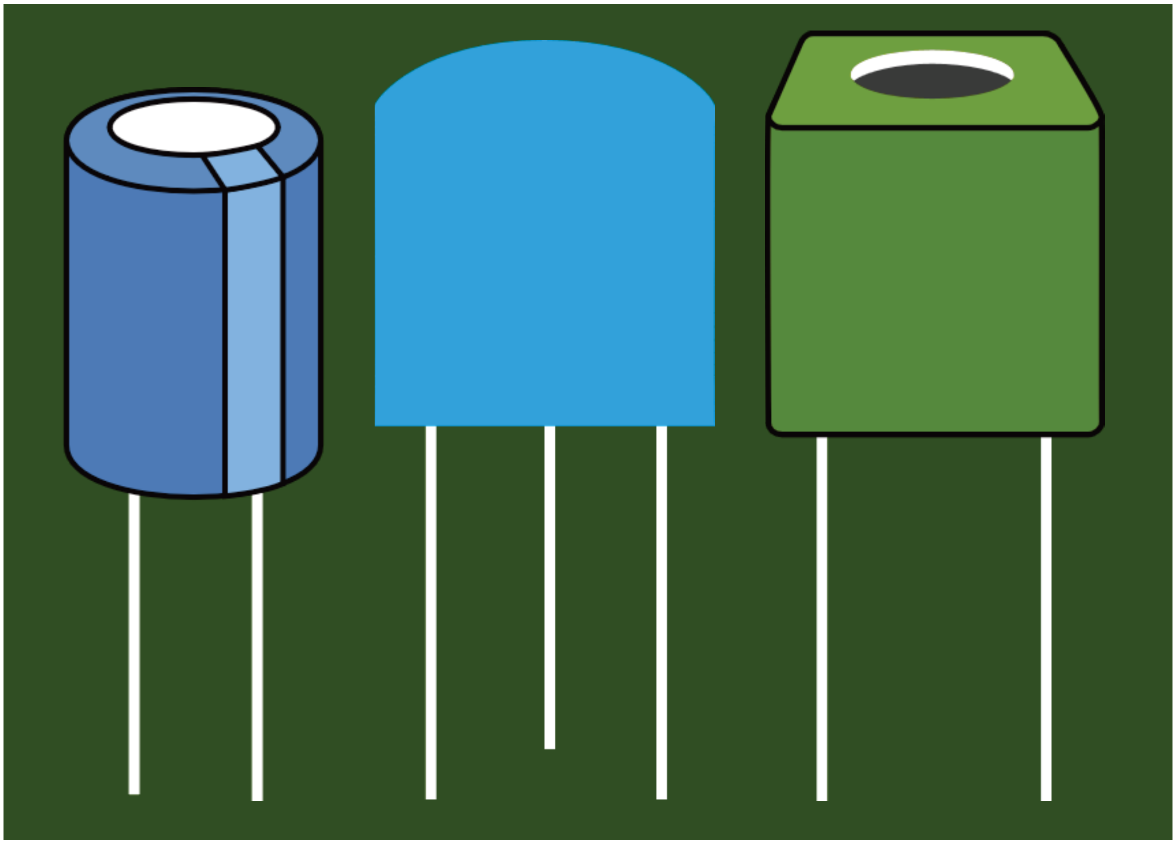
Electrostatic Potential and Capacitance



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Electrostatic Potential

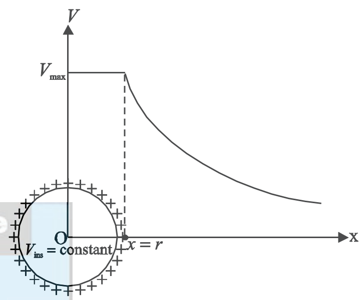
It is the potential energy possessed by a unit positive charge at the given point in an electric field and is measured by the amount of work done in moving a unit positive charge from infinity to the given point against the field. It is a scalar quantity and is measured in volt.

$$V = -\int_{\infty}^{r} dW = -\int_{\infty}^{r} \vec{E}.d\vec{r}$$

The electric potential (V) at a distance r from a charge q is given by $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}\right]$.

If ΔU is the change in potential energy of a system on a charge q move through a distance the against an electric field E, then $\Delta U = -qEd$.

Electrostatic potential at a point is $V = \frac{\Delta U}{q}$



Potential due to a charged spherical conductor

The potential energy per unit positive charge at a point in an electric field is called the electric potential at that point.

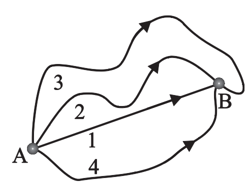
Potential due to a point charge $= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Potential due to a charged spherical conductor is $= \frac{Q}{4\pi\varepsilon_0 x}$

where x is distance of the point from the center of charged sphere.

For points on the surface or inside the conductor the potential is $= \frac{Q}{4\pi\varepsilon_0 r}$

- The work done in moving a charged particle from one point to another in an electrostatic field depends only on the initial and final points, but not on the path between the two points. In figure the work done per unit charge in moving from A to B along all the paths 1, 2, 3 and 4 are equal. i.e., $W_1 = W_2 = W_3 = W_4 = W$ (say). The work done $W = V_A \sim V_B$.



Path independence of work done

- Work done in moving a charge over a closed path in an electric field is zero. Electric field is a conservative field.
- The electric potential at a point due to positive charges is positive and due to negative charges is negative. The net potential at a point is the algebraic sum of the potentials due to different charges. Electric potential is scalar additive. Only the potential difference between two points $V_A \sim V_B$ has a definite value. But, the absolute values of V_A and V_B are arbitrary and they have no physical significance. Only potential difference between two points is physically meaningful.
- The potential difference between two points is the work done in moving one coulomb of positive charge from one point to the other. It is measured in volt.
- Electric potential at a point due to a short dipole is given by $V = \frac{q(2a)\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}$,

where p = electric dipole moment.

- Potential at a point due to a dipole on the axial line is inversely proportional to the square of the distance from the dipole. At large distances, $V \propto \frac{1}{r^2}$, whereas, for a point charge, $V \propto \frac{1}{r}$.
- Potential is maximum ($\theta = 0$ or π and $\cos\theta = \pm 1$), along the axis of the dipole.
- At any point on the equatorial line, potential is zero (Thus, equatorial line of a dipole is equipotential).
- Electric potential due to a system of charges is given by $V = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$.
- For a continuous distribution of charge, $V = \int \frac{dQ}{4\pi\varepsilon_0 r}$
- $dQ = \lambda\, dl$, for linear distribution of charge.
- $dQ = \sigma dS$, for surface distribution of charge.
- $dQ = \rho dv$, for volume distribution of charge.

Equipotential surface

An equipotential surface is a surface passing through points at the same electric potential.

- Equipotential surfaces around a point charge are concentric spherical surfaces.
- Equipotential surfaces around a charged cylinder or line of charge are concentric cylindrical surfaces.
- Equipotential surfaces due to a large charged plane are planes parallel to the surface.
- For a uniform spherical distribution of charges (like a spherical shell or a sphere), the equipotential outside the spheres are concentric spheres around the centre of distribution of charges.
- All the lines in a plane passing through the centre of a dipole and perpendicular to the axis of the dipole lie on an equipotential surface of zero potential. If the dipole lies along x-axis, the y-z plane ($x = 0$) is an equipotential surface with $V = 0$.
- Electric field lines are perpendicular to an equipotential surface and hence work done in moving a charge on an equipotential surface is zero.

Potential energy

- Potential energy of a system of two charges is given by $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$
- Potential energy of a system of three charges is given by $U = \left(\frac{1}{4\pi\varepsilon_0}\right)\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]$
- Energy of a system of several point charges is given by $U = \sum_{i \neq j} U_{ij} = \sum_{i \neq j}\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q_i q_j}{r_{ij}}\right)$

Capacitors

When a conductor is charged, the charge spreads on its surface. If the conductor is smooth, it retains the charge for considerable time. Thus, a conductor can be used to store charge. The ability of a conductor to store charge is called its capacitance.

The capacitance of a conductor is defined as its ability to store charge and is measured by the ratio of the charge added to the conductor to the rise in its potential.

i e., $C = \dfrac{Q}{V}$

The unit of capacitance in SI is farad (*F*). One farad is the capacitance of a conductor if its potential rises by 1 volt when a charge of 1 coulomb is added to it.

Capacitance of a Spherical Conductor

Consider a spherical conductor of radius *r*. Its capacitance, $C = 4\pi\varepsilon_0 r$.

If the capacitor is filled with a dielectric of relative permittivity $\varepsilon_r, C = 4\pi\varepsilon_0\varepsilon_r r$.

	When $C = 1$ F, $r = 9 \times 10^9$, m $= 9 \times 10^6$ km, i.e. the size of conductor needed to have a conductor of capacitance of 1 C is 9×10^6 km.

Principle of a capacitor

An earthed conductor kept close to a charged conductor decreases the potential of the charged conductor. Hence for a given Q, the corresponding V is small; hence the ratio $C = \dfrac{Q}{V}$ is large. i.e., the *capacitance of the* conductor *increases.*

Capacitor

It is an arrangement of two conductors separated by a dielectric. One important use of a capacitor is to store charge.

Capacitance of some simple capacitors

(a) Spherical capacitor

$R = n^{1/3}r$, where ε_r is the relative permittivity of the medium between the outer sphere of radius *b* and inner sphere of radius *a*.

	C can be increased by ▪ decreasing (b – a) i.e., by bringing the spheres as close as possible, introducing a medium of higher ε_r.

(b) Parallel plate capacitor

$C = \dfrac{\varepsilon_0\varepsilon_r A}{d}$, where ε_r is the relative permittivity of the medium between two parallel plates of area *A* and separated by a distance *d*.

(c)Cylindrical capacitor: $C = \dfrac{2\pi\varepsilon_0\varepsilon_r L}{2.303\log_{10}\dfrac{b}{a}}$, where *L* is the length, *b* is the radius of the outer cylinder and *a* is the radius of the inner cylinder.

	• Suppose there are *n* charged drops, each of capacitance *C*, charged to potential *V* with charge *q*, surface density σ and potential energy *U* coalesce to form a single drop. For such a drop, • Total charge = nq • Total capacitance = $n^{1/3}C$ • Potential = $n^{2/3}V$ • Surface density of charge = $n^{1/3}\sigma$, and • Total potential energy = $n^{2/3}U$.