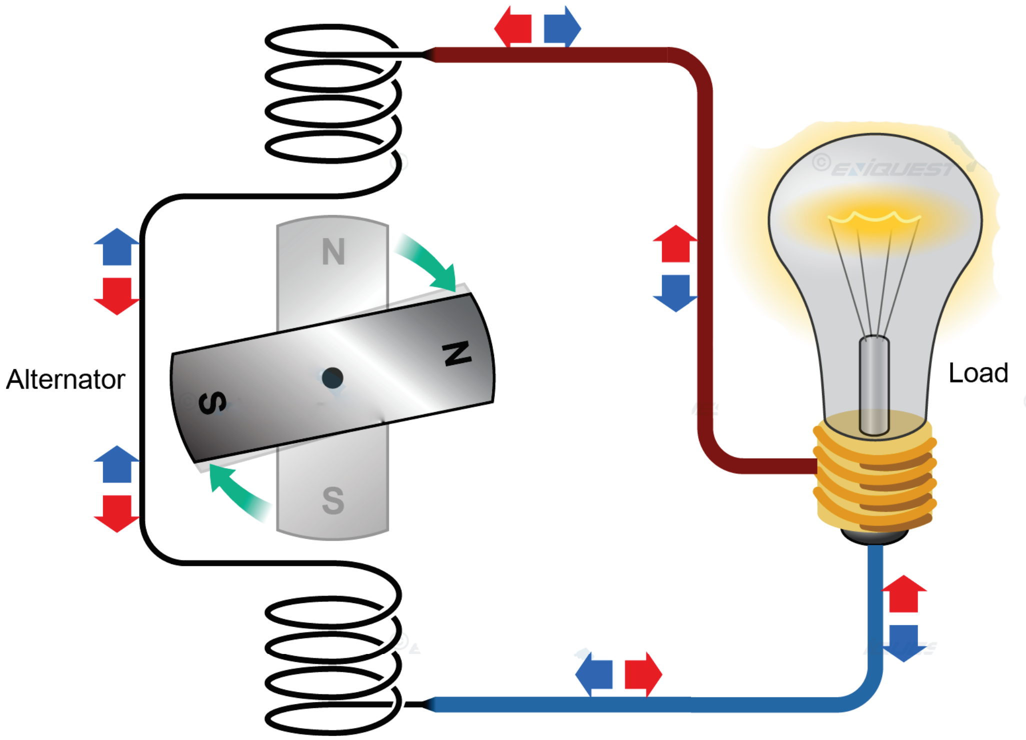
ALTERNATING CURRENT



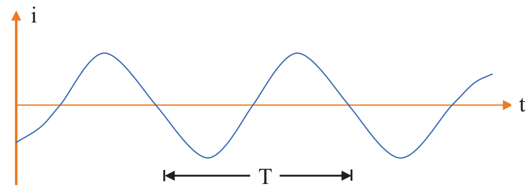
Alternating Current

AC Circuits

In the previous chapters, various circuits we have analysed were all direct current circuits. The source which drives the current in dc circuits is a battery. The main element which controls the current through the circuit is the resistance. It is a well known fact that almost all household and industrial power-distribution systems operate with alternating current (ac) and not direct currents (dc). To drive an ac through a circuit a source of alternating emf or voltage is required.

In this chapter we only study the ac that varies sinusoidally with time. Such an alternating current is given by

$i = i_o \sin(\omega t + \phi)$ ... (1)



In equation (1), 'i' gives instantaneous value of current i.e. magnitude of current at any instant of time t and i_0 the peak value or maximum value of ac. This is also called amplitude of ac. ω is the angular frequency of ac.

Further, $\omega = \frac{2\pi}{T} = 2\pi f$,

where T is the time period of ac. It is equal to the time taken by the ac to go through one complete cycle of variation (zero to maximum, maximum to zero; zero to maximum in opposite direction and finally maximum to zero). Again, f is the frequency of ac. It is equal to the number of complete cycles of variation gone through by the ac per second. A dc flows from +ve terminal to –ve through the external resistor. Magnitude of dc also remains almost constant. But ac through a resistor changes its direction alternatively.

Characteristics of ac

Instantaneous current

This is given by $i = i_0 \sin(\omega t + \phi)$ at any time t.

Average current or mean current

The alternating current (ac) varies with time. Its mean value over a time interval o to t is given by

$$\bar{i} = \frac{\int_0^1 i\,dt}{\int_0^1 dt} = \frac{1}{T}\int_0^1 i\,dt$$

This, on integrating turns out to be

$$\bar{i} = -\frac{i_0}{T_\omega}[\cos(2\pi + \varphi) - \cos\varphi] = 0 \text{ for a time period } t = T \text{ and } \omega T = 2\pi$$

Mean square current

At every instant of time i^2 is +ve. The average of i^2 over a time period is given by

$$\overline{i^2} = \frac{\int_0^T i^2\,dt}{\int_0^T dt} \Rightarrow = \frac{i_0^2}{2T}\left[T - \frac{\sin(4\pi + 2\varphi) - \sin 2\varphi}{2\omega}\right] = \left(\frac{i_0^2}{\sqrt{2}}\right)$$

Root mean square current

Square root of mean square current is called root mean square current or rms current

$i_c = i_0 \sin(\omega t + \phi)$ and is given by $i_{rms} = \sqrt{\overline{i^2}} = \dfrac{i_0}{\sqrt{2}}$

The equations for mean square current and root-mean-square current are obtained for one time period. They are also valid if the average is calculated over a long period of time.

An alternating voltage which drives ac through a circuit (potential difference) may be written as

$V = V_0 \sin(\omega t + \phi)$.

This gives the instantaneous voltage. The mean value $\bar{V}$ over a complete cycle is zero, the mean square voltage over a cycle is $\dfrac{V_0^2}{2}$ and the root-mean-square voltage (rms voltage or virtual voltage) is $\dfrac{V_0}{\sqrt{2}}$.

The importance of rms current and rms voltage can be shown by considering a resistor of resistance R carrying a current i

$i = i_0 \sin(\omega t + \phi)$

The voltage across the resistor is $V = Ri = (i_0 R)\sin(\omega t + \phi)$

The thermal energy developed in the resistor during the time t to t + dt is

$i^2 R\, dt = i_0^2 R \sin^2(\omega t + \phi) dt$.

The thermal energy developed in one time period is

$$U = \int_0^T i^2 R\, dt = R\int_0^T i_0^2 \sin^2(\omega t + \varphi) dt = RT\left[\frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \varphi) dt\right] = i_{rms}^2 RT.$$

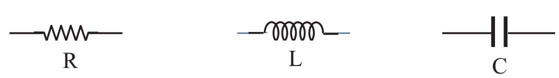
Thus, if we pass a direct current, i_{rms} passes through the resistor, it will produce the same thermal energy in a time period as that produced when the alternating current I passes through it. Similarly, a constant voltage V_{rms} applied across a resistor produces the same thermal energy as that produced by the voltage

$V = V_0 \sin(\omega t + \phi)$.

Circuit elements in ac circuit

In dc circuits except in special cases, only resistance R can control the current in the circuit, as inductor acts as short circuit while capacitor acts as open circuit.

However in case of ac circuits, resistance R, inductance L and capacitance C, or any combination of these can control the current in the circuit. Resistance R, inductance L and capacitance C are referred to as circuit elements and are shown in figure below.



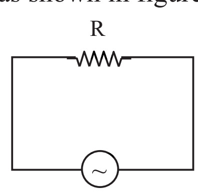
We consider the following cases to understand how R, L and C control the current in an ac circuit

(a) A Resistor in an ac circuit

If an alternating voltage $E = E_0 \sin \omega t$ is applied across a resistance as shown in figure. Kirchhoff's loop-rule at any time t, gives

$E = IR$ i.e., $I = \dfrac{E}{R}$

or $I = \dfrac{E_0}{R}\sin \omega t$ [as $E = E_0 \sin \omega t$]



$E = E_0 \sin \omega t$
circuit

From this it is evident that

1. The frequency of current in the circuit is ω and is same as that of the applied voltage.
2. In a resistance, applied voltage and the resulting current are in phase.

3. Current in the circuit is independent of frequency and decreases with increase in R (similar to that in dc circuits)

(b) An inductor in an ac circuit

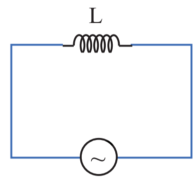
If an emf $E = E_0 \sin \omega t$ is applied across an inductor of inductance L as shown, applying Kirchhoff's loop rule we have,

$$E - L\frac{dI}{dt} = 0 \quad \text{or,} \quad L\frac{dI}{dt} = E_0 \sin \omega t$$

$$\text{or,} \quad \frac{dI}{dt} = \frac{E_0}{L}\sin \omega t$$

which on integration gives, $I = -\frac{E_0}{L\omega}\cos \omega t$

i.e., $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ with $I_0 = \frac{E_0}{\omega L}$



$E = E_0 \sin \omega t$

From this expression it is evident that

1. The frequency of current in the circuit is same as that of applied emf but current in an inductor 'lags' the applied voltage by $(\pi/2)$ [or voltage leads the current by $(\pi/2)$]
2. As we have $I_0 = (E_0/\omega L)$, the quantity ωL has the dimensions of resistance as,

$$[\omega L] = \left[\frac{\text{rad}}{\text{sec}} \times H\right] = \left[\frac{\text{rad}}{\text{sec}} \times \text{ohm} \times \text{sec}\right] = [\text{ohm}]$$

This quantity is referred to as 'inductive-reactance' and is represented by X_L and represents the opposition of a coil to ac, i.e., $X_L = \omega L = 2\pi f L$

(c) A capacitor in an ac circuit

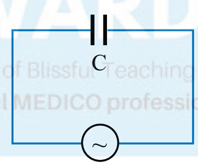
If $E = E_0 \sin \omega t$ is applied across a capacitor as shown, applying Kirchhoff's loop rule

we have, $E - \frac{q}{C} = 0$

or, $q = CE_0 \sin \omega t$ (as $E = E_0 \sin \omega t$)

or, $I = \frac{dq}{dt} = C\omega E_0 \cos \omega t$

or, $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ with $I_0 = E_0 C\omega$



$E = E_0 \sin \omega t$

From this expression it is evident that

1. Current in the circuit has same frequency as the applied voltage but leads it by $(\pi/2)$ [or voltage across a capacitor lags the current by $(\pi/2)$]
2. As $I_0 = C\omega E_0 = \frac{E_0}{X_C}$ where $X_C = \frac{1}{\omega C}$. $(1/\omega C)$ has dimensions of resistance,

as we have $\left[\frac{1}{\omega C}\right] = \left[\frac{1}{\text{rad}\, s^{-1}} \times \frac{V}{\text{coul}}\right] = \left[\frac{V}{A}\right] = \text{ohm}$

and so it represents the opposition due to capacitor to the flow of ac through it and is called 'capacitive reactance'.

Impedance

The peak current and the peak emf in all the three circuits discussed are related by

$i_0 = \frac{\varepsilon_0}{Z}$, where Z is called the impedance of circuit.

$Z = R$ for a purely resistive circuit $Z = \frac{1}{\omega C}$ for a purely capacitive circuit and $Z = \omega L$ for a purely inductive circuit.

The peak current and the peak emf are related to each other for any series circuit (one-loop circuit) having an ac source. The general name for Z is impedance.

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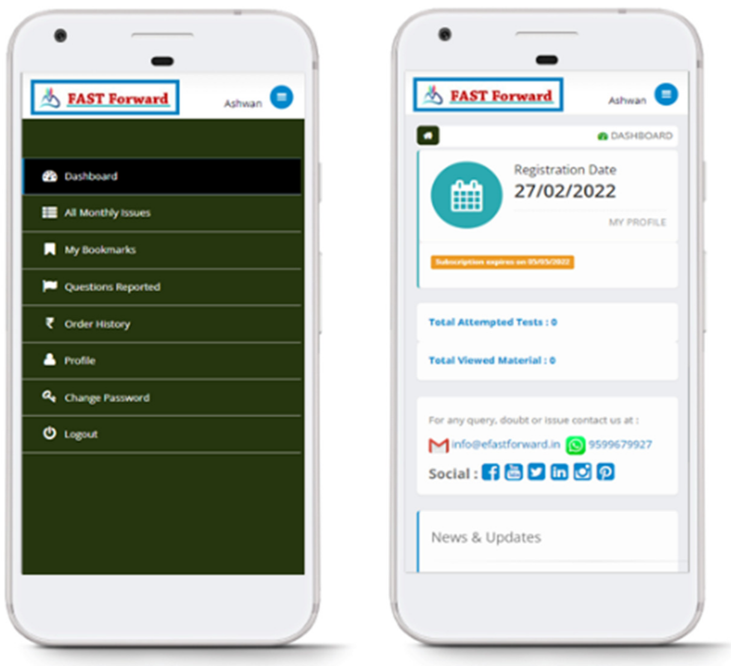

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