JEE Main 2021

24 FEBRUARY SHIFT I

PHYSICS

Section A : Objective Type Questions

1. The work done by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, α and β are constants.

Then, the dimensions of β will be

a. $[M^2 L T^2]$ **b.** $[M^0 L T^0]$ **c.** $[M L T^{-2}]$ **d.** $[M L^2 T^{-2}]$

2. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is

a. $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ **b.** $2\pi\sqrt{\frac{d^3}{3Gm}}$

c. $2\pi\sqrt{\frac{3Gm}{d^3}}$ **d.** $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$

3. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be

a. $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$ **b.** $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

c. $\sqrt{G(1+2\sqrt{2})}$ **d.** $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

4. Moment of inertia (MI) of four bodies, having same mass and radius, are reported as

I_1 = MI of thin circular ring about its diameter,

I_2 = MI of circular disk about an axis perpendicular to the disk and going through the centre,

I_3 = MI of solid cylinder about its axis

and I_4 = MI of solid sphere about its diameter. Then,

a. $I_1 + I_2 = I_3 + \frac{5}{2}I_4$ **b.** $I_1 + I_3 < I_2 + I_4$

c. $I_1 = I_2 = I_3 < I_4$ **d.** $I_1 = I_2 = I_3 > I_4$

5. Consider two satellites S_1 and S_2 with periods of revolution 1 h and 8 h respectively, revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is

a. 8 : 1 **b.** 1 : 8 **c.** 2 : 1 **d.** 1 : 4

6. Each side of a box made of metal sheet in cubic shape is a at room temperature T, the coefficient of linear expansion of the metal sheet is α. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

a. $4\pi a^3\alpha\Delta T$ **b.** $4a^3\alpha\Delta T$

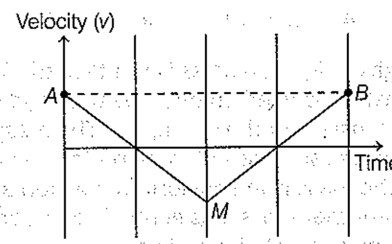
c. $\frac{4}{3}\pi a^3\alpha\Delta T$ **d.** $3a^3\alpha\Delta T$

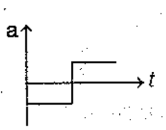
7. If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material, respectively. Choose the correct relation for these parameters.

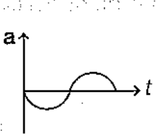
a. $Y = \frac{9K\eta}{2\eta + 3K}$ N/m^2 **b.** $Y = \frac{9K\eta}{3K - \eta}$ N/m^2

c. $K = \frac{Y\eta}{9\eta - 3Y}$ N/m^2 **d.** $\eta = \frac{3YK}{9K + Y}$ N/m^2

8. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?



a.


b.


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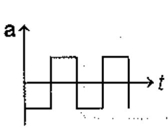

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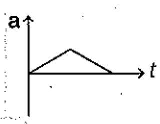


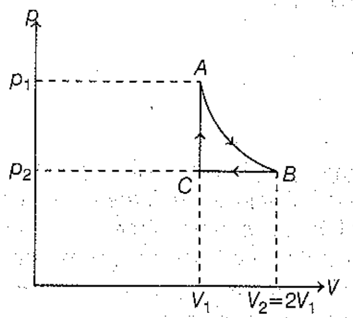
9. n mole of a perfect gas undergoes a cyclic process $ABCA$ (see figure) consisting of the following processes.

$A \to B$: Isothermal expansion at temperature T, so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from p_1 to p_2.

$B \rightarrow C$: Isobaric compression at pressure p_2 to initial volume V_1.

$C \rightarrow A$: Isochoric change leading to change of pressure from p_2 to p_1.

Total work done in the complete cycle $ABCA$ is



a. 0 b. $nRT\ln 2$

c. $nRT\left(\ln 2+\frac{1}{2}\right)$ d. $nRT\left(\ln 2-\frac{1}{2}\right)$

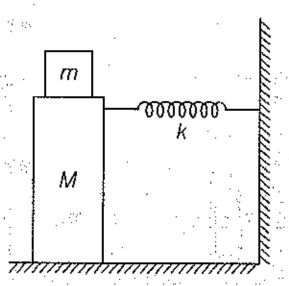
10. Match List-I with List-II.

	List-I		List-II
A.	Isothermal	1.	Pressure constant
B.	Isochoric	2.	Temperature constant
C.	Adiabatic	3.	Volume constant
D.	Isobaric	4.	Heat content is constant

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	1	3	2	4	b.	3	2	1	4
c.	2	4	3	1	d.	2	3	4	1

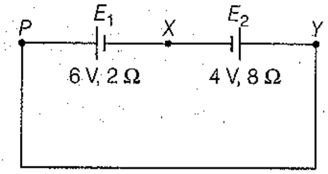
11. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be



a. $A\sqrt{\frac{M+m}{M}}$ b. $A\sqrt{\frac{M}{M+m}}$

c. $A\sqrt{\frac{M-m}{M}}$ d. $A\sqrt{\frac{M}{M-m}}$

12. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4 V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is



a. 2.0 V b. 3.6 V

c. 5.6 V d. 10.0 V

13. A current through a wire depends on time as $i=\alpha_0 t+\beta t^2$, where $\alpha_0=20$ A/s and $\beta=8$ As^{-2}. Find the charge crossed through a section of the wire in 15 s.

a. 260 C b. 2100 C

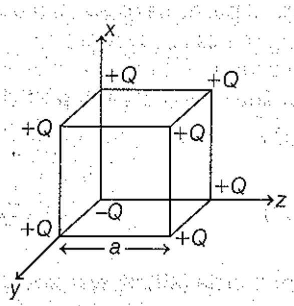
c. 11250 C d. 2250 C

14. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be

a. 1 : 2 b. 2 : 1

c. 4 : 1 d. 1 : 4

15. A cube of side a has point charges $+Q$ located at each of its vertices except at the origin, where the charge is $-Q$. The electric field at the centre of cube is



a. $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

b. $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

c. $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

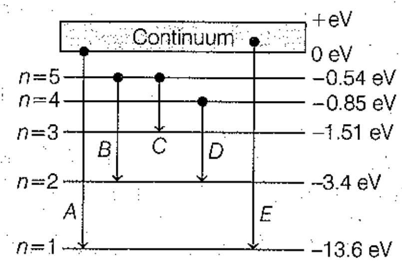
d. $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

16. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be

a. 7 b. 0.875

c. 0.5 d. 3.5

17. In the given figure, the energy levels of hydrogen atom have been shown alongwith some transitions marked *A*, *B*, *C*, *D* and *E*. The transitions *A*, *B* and *C* respectively represent



a. The first member of the Lyman series, third member of Balmer series and second member of Paschen series.
b. The ionisation potential of hydrogen, second member of Balmer series and third member of Paschen series.
c. The series limit of Lyman series, second member of Balmer series and second member of Paschen series.
d. The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

18. Given below are two statements :

Statement I Two photons having equal linear momenta have equal wavelengths.

Statement II If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

a. Both Statement I and Statement II are true.
b. Both Statement I and Statement II are false.
c. Statement I is true but Statement II is false.
d. Statement I is false but Statement II is true.

19. The focal length f is related to the radius of curvature r of the spherical convex mirror by

a. $f = r$ **b.** $f = -r$
c. $f = -\frac{r}{2}$ **d.** $f = +\frac{r}{2}$

20. In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

a. 4 : 1 **b.** 2 : 1
c. 1 : 4 **d.** 3 : 1

Section B : Numerical Type Questions

21. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be .... N. [Take, $g = 10\ \text{ms}^{-2}$]

22. An unpolarised light beam is incident on the polariser of a polarisation experiment and the intensity of light beam emerging from the analyser is measured as 100 lumens. Now, if the analyser is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be .... lumens.

23. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is ....

24. A hydraulic press can lift 100 kg when a mass m is placed on the smaller piston. It can lift ...... kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass m on the smaller piston.

25. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where, y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is .... cm.

26. A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is ....

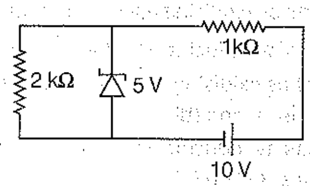
[Take, $\pi = 3.14$]

27. An audio signal $v_m = 20 \sin 2\pi(1500\, t)$ amplitude modulates a carrier

$v_c = 80 \sin 2\pi(100000\, t)$

The value of per cent modulation is ....

28. In connection with the circuit drawn below, the value of current flowing through 2kΩ resistor is .... $\times 10^{-4}$ A.



29. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is ...... $\times 10^7$ m/s.

30. A common transistor radio set requires 12 V (DC) for its operation. The DC source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (AC) on standard domestic AC supply. The number of turns of secondary coil are 24, then the number of turns of primary are ....

CHEMISTRY

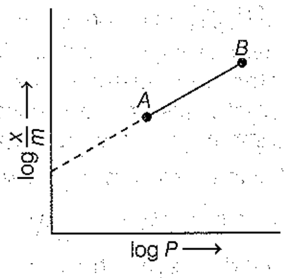
Section A : Objective Type Questions

1. Which of the following are isostructural pairs ?

A. SO_4^{2-} and CrO_4^{2-} B. $SiCl_4$ and $TiCl_4$
C. NH_3 and NO_3^- D. BCl_3 and $BrCl_3$

a. A and B only **b.** A and C only
c. B and C only **d.** C and D only

2. In Freundlich adsorption isotherm, slope of AB line is



a. n with ($n = 0.1$ to 0.5) **b.** $\log n$ with ($n > 1$)
c. $\log \frac{1}{n}$ with ($n < 1$) **d.** $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$

3. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionisation enthalpy is

a. Al < Mg < Si < S < P **b.** Mg < Al < Si < P < S
c. Mg < Al < Si < S < P **d.** Al < Mg < S < Si < P

4. Which of the following ore is concentrated using group 1 cyanide salt ?

a. Calamine **b.** Malachite **c.** Siderite **d.** Sphalerite

5. (A) $HOCl + H_2O_2 \longrightarrow H_3O^+ + Cl^- + O_2$
(B) $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$
Choose the correct option.

a. H_2O_2 acts as oxidising agent in equations (A) and (B)
b. H_2O_2 acts as reducing agent in equations (A) and (B)
c. H_2O_2 act as oxidising and reducing agent respectively in equations (A) and (B)
d. H_2O_2 acts as reducing and oxidising agent respectively in equations (A) and (B).

6. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are

a. $X = Na[Al(OH)_4]$, $Y = SO_2$, $Z = Al_2O_3$
b. $X - Al(OH)_3$, $Y - SO_2$, $Z - Al_2O_3 \cdot xH_2O$
c. $X = Al(OH)_3$, $Y = CO$, $Z = Al_2O_3$
d. $X = Na[Al(OH)_4]$, $Y = CO_2$, $Z = Al_2O_3 \cdot xH_2O$

7. The electrode potential of M^{2+}/M of $3d$-series elements shows positive value for

a. Fe **b.** Co **c.** Zn **d.** Cu

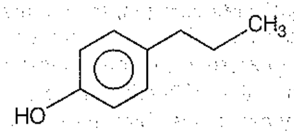
8. The major components in Gun metal are

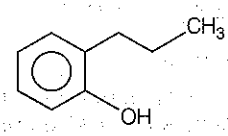
a. Cu, Sn and Zn **b.** Cu, Zn and Ni
c. Cu, Ni and Fe **d.** Al, Cu, Mg and Mn

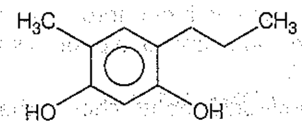
9. The gas released during anaerobic degradation of vegetation may lead to

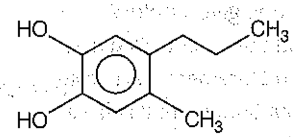
a. acid rain **b.** global warming and cancer
c. corrosion of metals **d.** ozone hole

10. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH?

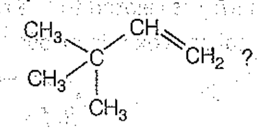
a.


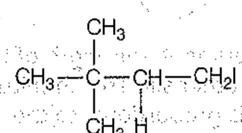
b.


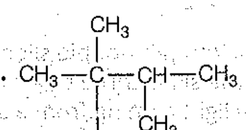
c.


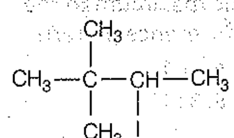
d.


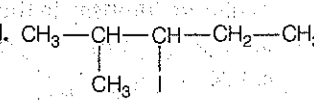
11. What is the major product formed by HI on reaction with



a.


b.


c.


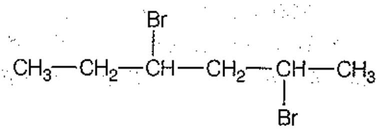
d.


12. Which of the following reagent is used for the following reaction ?

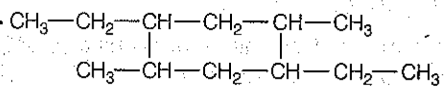
$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$

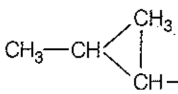
a. Copper at high temperature and pressure
b. Molybdenum oxide
c. Manganese acetate
d. Potassium permanganate

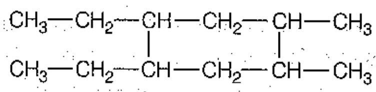
13. The product formed in the first step of the reaction of

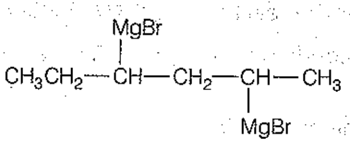
 with excess

Mg/Et_2O ($Et = C_2H_5$) is

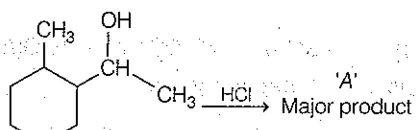
a. 

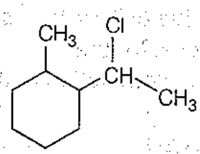
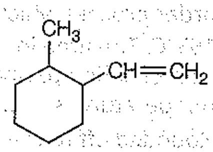
b. 

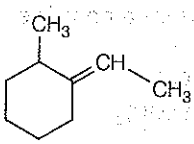
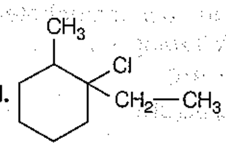
c. 

d. 

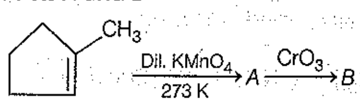
14. What is the final product (major) '*A*' in the given reaction?

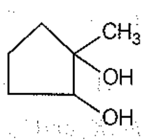
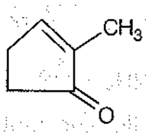


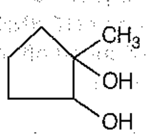
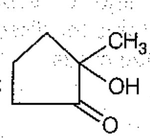
a.  **b.** 

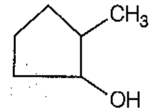
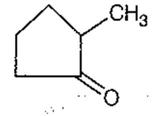
c.  **d.** 

15. Identify products *A* and *B*



a. *A* :  *B* : 

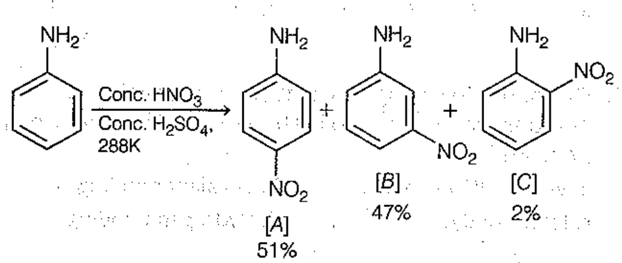
b. *A* :  *B* : 

c. *A* :  *B* : 

d. *A* : $OHC-CH_2CH_2CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

B : $HOOC-CH_2CH_2CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

16. In the following reaction the reason why *meta*-nitro product also formed is



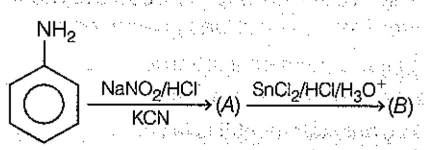
a. $-NH_2$ group is highly *meta*-directive

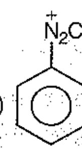
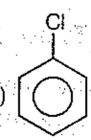
b. $-NO_2$ substitution always takes place at *meta*-position

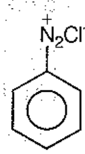
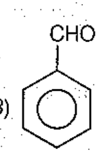
c. Formation of anilinium ion

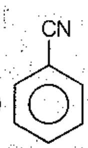
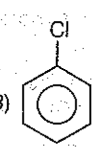
d. Low temperature

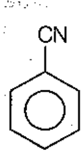
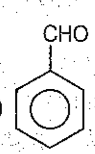
17. '*A*' and '*B*' in the following reaction are



a. (*A*)  (*B*) 

b. (*A*)  (*B*) 

c. (*A*)  (*B*) 

d. (*A*)  (*B*) 

18. Match List-I with List-II.

List-I (Monomer Unit)		**List-II** (Polymer)	
A.	Caprolactum	1.	Natural rubber
B.	2-chloro buta-1,3-diene	2.	Buna-N
C.	Isoprene	3.	Nylon-6
D.	Acrylonitrile	4.	Neoprene

Choose the correct answer from the options given below

	A	B	C	D		A	B	C	D
a.	1	2	3	4	**b.**	4	3	2	1
c.	2	1	4	3	**d.**	3	4	1	2

19. Out of the following, which type of interaction is responsible for the stabilisation of α-helix structure of proteins ?

a. van der Waals' forces **b.** Covalent bonding
c. Ionic bonding **d.** Hydrogen bonding

20. Given below are two statements.

Statement I Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

a. Both statement I and statement II are true.
b. Both statement I and statement II are false.
c. Statement I is true but statement II is false.
d. Statement I is false but statement II is true.

Section B : Numerical Type Questions

21. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is $x \times 10^{-1}$. The value of x is .... (Rounded off to the nearest integer).

22. The coordination number of an atom in a body centered cubic structure is ....

[Assume that the lattice is made up of atoms.]

23. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de-Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda Li}{\lambda_p}$ is $x \times 10^{-1}$ The value of x is ....

(Rounded off to the nearest integer)

(Mass of Li^{3+} = 8.3 mass of proton)

24. For the reaction, $A(g) \longrightarrow B(g)$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is .... (Rounded off to the nearest integer)

($R = 8.31$ J mol^{-1} K^{-1} and $\ln 10 = 2.3$)

25. When 9.45 g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is $x \times 10^{-3}$. The value of x is ....

(Rounded off to the nearest integer)

[$K_{f(H_2O)} = 1.86$ K kg mol^{-1}]

26. At 1990 K and 1 atm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of k_p for the reaction $Cl_2(g) \rightleftharpoons 2Cl(g)$ under the above conditions is $x \times 10^{-1}$ The value of x is ....

(Rounded off to the nearest integer)

27. The reaction of sulphur in alkaline medium is given below

$S_8(s) + aOH^-(aq) \longrightarrow bS^{2-}(aq) + cS_2O_3^{2-}(aq) + dH_2O(l)$

The value of 'a' is .... (Integer answer)

28. Gaseous cyclobutene isomerises to butadiene in a first order process which has a 'k' value of 3.3×10^{-4} s^{-1} at 153°C. The time in minutes it takes for the isomerisation to proceed 40 % to completion at this temperature is ....

(Rounded off to the nearest integer)

29. Number of amphoteric compounds among the following is ....

a. BeO **b.** BaO
c. $Be(OH)_2$ **d.** $Sr(OH)_2$

30. The stepwise formation of $[Cu(NH_3)_4]^{2+}$ is given below

$Cu^{2+} + NH_3 \xrightleftharpoons{K_1} [Cu(NH_3)]^{2+}$

$[Cu(NH_3)]^{2+} + NH_3 \xrightleftharpoons{K_2} [Cu(NH_3)_2]^{2+}$

$[Cu(NH_3)_2]^{2+} + NH_3 \xrightleftharpoons{K_3} [Cu(NH_3)]^{2+}$

$[Cu(NH_3)_3]^{2+} + NH_3 \xrightleftharpoons{K_4} [Cu(NH_3)_4]^{2+}$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^3, 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[Cu(NH_3)]^{2+}$] is $x \times 10^{-12}$. The value of x is .... (Rounded off to the nearest integer)

MATHEMATICS

Section A : Objective Type Questions

1. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \dfrac{x - (1/2)}{x - 1}$

Then, the composition function $f(g(x))$ is

a. one-one but not onto **b.** onto but not one-one
c. Neither one-one nor onto **d.** Both one-one and onto

2. Let p and q be two positive numbers, such that $p + q = 2$ and $p^4 + q^4 = 272$. Then, p and q are roots of the equation

a. $x^2 - 2x + 136 = 0$ **b.** $x^2 - 2x + 16 = 0$
c. $x^2 - 2x + 8 = 0$ **d.** $x^2 - 2x + 2 = 0$

3. The system of linear equations

$3x - 2y - kz = 10$
$2x - 4y - 2z = 6$
$x + 2y - z = 5m$

is inconsistent, if

a. $k \neq 3, m \neq \frac{4}{5}$ **b.** $k = 3, m = \frac{4}{5}$
c. $k = 3, m \neq \frac{4}{5}$ **d.** $k \neq 3, m \in R$

4. The value of $-{}^{15}C_1 + 2 \cdot {}^{15}C_2 - 3 \cdot {}^{15}C_3 + \dots - 15 \cdot {}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \dots + {}^{14}C_{11}$ is

a. $2^{16} - 1$ **b.** $2^{13} - 14$
c. $2^{13} - 13$ **d.** 2^{14}

5. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \dots \infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of

$\dfrac{2 \sin x}{\sin x + \sqrt{3} \cos x}, \left(0 < x < \dfrac{\pi}{2}\right)$ is

a. $\frac{1}{2}$ **b.** $\sqrt{3}$ **c.** $\frac{3}{2}$ **d.** $2\sqrt{3}$

6. $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} (\sin\sqrt{t})dt}{x^3}$ is equal to

a. 2/3 **b.** 3/2
c. 1/15 **d.** 0

7. The function

$f(x) = \dfrac{4x^3 - 3x^2}{6} - 2 \sin x + (2x - 1) \cos x$

a. increases in $\left[\frac{1}{2}, \infty\right)$ **b.** decreases in $\left[\frac{1}{2}, \infty\right)$
c. increases in $\left(-\infty, \frac{1}{2}\right]$ **d.** decreases in $\left(-\infty, \frac{1}{2}\right]$

8. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then, the number of ways, the committee can be formed, is

a. 1050 **b.** 1625
c. 560 **d.** 575

9. If $f : R \to R$ is a function defined by $f(x) = [x - 1]\cos\left(\dfrac{2x - 1}{2}\right)\pi$, where [·] denotes the greatest integer function, then f is

a. discontinuous only at $x = 1$
b. discontinuous at all integral values of x except at $x = 1$
c. continuous only at $x = 1$
d. continuous for every real x

10. If $\int \dfrac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\dfrac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to

a. (3, 1) **b.** (1, 3)
c. (– 1, 3) **d.** (1, – 3)

11. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is

a. $24\pi + 3\sqrt{3}$ **b.** $24\pi - 3\sqrt{3}$
c. $12\pi + 3\sqrt{3}$ **d.** $12\pi - 3\sqrt{3}$

12. The population $P = P(t)$ at time t of a certain species follows the differential equation $\dfrac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is

a. $\log_e 9$ **b.** $\frac{1}{2}\log_e 18$
c. $\log_e 18$ **d.** $2\log_e 18$

13. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1,1), (2, 2) and (4, 4), respectively. Then, which of these stones is / are on the path of the man ?

a. A only **b.** B only
c. C only **d.** All the three

14. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is

a. $x = a$ **b.** $x = -\frac{a}{2}$
c. $x = 0$ **d.** $x = \frac{a}{2}$

15. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is

a. 0 **b.** $2t^3$ **c.** $-t^3$ **d.** $-2t^3$

16. The equation of the plane passing through the point (1, 2, −3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is

a. $6x - 5y + 2z + 10 = 0$

b. $11x + y + 17z + 38 = 0$

c. $6x - 5y - 2z - 2 = 0$

d. $3x - 10y + 2z + 11 = 0$

17. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is

a. $2\sqrt{19}$ **b.** $19\sqrt{2}$

c. $\sqrt{38}$ **d.** 38

18. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is

a. $\frac{1}{32}$ **b.** $\frac{3}{16}$

c. $\frac{5}{16}$ **d.** $\frac{1}{2}$

19. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in metres) is

a. 25 **b.** 30

c. $20\sqrt{3}$ **d.** $25\sqrt{3}$

20. The statement among the following that is a tautology is

a. $A \wedge (A \vee B)$ **b.** $A \vee (A \wedge B)$

c. $[A \wedge (A \to B)] \to B$ **d.** $B \to [A \wedge (A \to B)]$

Section B : Numerical Type Questions

21. If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to ....

22. Let $B_i (i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let P be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)P = \alpha\beta$ and $(\beta - 3\gamma)P = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0,1)). Then, $\frac{P(B_1)}{P(B_3)}$ is equal to ....

23. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ....

24. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is ....

25. Let $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k + 2 : k \in N\}$

and $C = \{9k + l : k \in N\}$ for some $l (0 < l < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to ....

26. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is ....

27. If $\int_{-a}^{a} (|x| + |x - 2|)\,dx = 22$, $(a > 2)$ and $[x]$ denotes the greatest integer $\le x$, then $\int_{a}^{-a} (x + [x])\,dx$ is equal to ....

28. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose centre is at (2, 1), then its radius is ....

29. Let three vectors **a**, **b** and **c** be such that **c** is coplanar with **a** and **b**, **a** · **c** = 7 and **b** is perpendicular to **c**, where $\mathbf{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = 2\hat{i} + \hat{k}$, then the value of $2|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2$ is ....

30. $\lim_{n \to \infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1 + r + r^2}\right)\right\}$ is equal to ....

Answers

Physics

1. (c)	2. (b)	3. (*)	4. (d)	5. (a)	6. (d)	7. (c)	8. (a)	9. (d)	10. (d)
11. (b)	12. (c)	13. (c)	14. (d)	15. (c)	16. (a)	17. (d)	18. (c)	19. (d)	20. (a)
21. (25)	22. (75)	23. (1)	24. (25600)	25. (25)	26. (2000)	27. (25)	28. (25)	29. (15)	30. (440)

Chemistry

1. (a)	2. (d)	3. (a)	4. (d)	5. (b)	6. (d)	7. (d)	8. (a)	9. (b)	10. (b)
11. (b)	12. (b)	13. (d)	14. (d)	15. (d)	16. (c)	17. (d)	18. (d)	19. (d)	20. (b)
21. (2)	22. (8)	23. (2)	24. (1380)	25. (34.4)	26. (5)	27. (12)	28. (26)	29. (2)	30. (1.26)

Mathematics

1. (a)	2. (b)	3. (c)	4. (b)	5. (a)	6. (a)	7. (a)	8. (b)	9. (d)	10. (b)
11. (b)	12. (d)	13. (b)	14. (c)	15. (d)	16. (b)	17. (c)	18. (d)	19. (d)	20. (c)
21. (10)	22. (6)	23. (17)	24. (540)	25. (5)	26. (9)	27. (3)	28. (3)	29. (75)	30. (1)

Note *(**) None of the option is correct.*

Solutions

PHYSICS

1. *(c)* Given, work done, $W = \alpha \cdot \beta^2 e^{-\frac{x^2}{\alpha k T}}$

where, k is Boltzmann constant,

T is temperature and x is displacement.

We know that, $\frac{x^2}{\alpha k T}$ is a dimensionless quantity.

$\therefore \quad \left[\frac{x^2}{\alpha k T}\right] = [M^0 L^0 T^0] \Rightarrow [\alpha] = \frac{[x^2]}{[k]\,[T]}$

$\Rightarrow \quad [\alpha] = \frac{[L^2]}{[k]\,[T]} \quad \ldots \text{(i)}$

Since, dimensions of k are

$[k] = [M^1 L^2 T^{-2} K^{-1}] \quad \ldots \text{(ii)}$

Dimensions of temperature are

$[T] = [K] \quad \ldots \text{(iii)}$

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$[\alpha] = \frac{[L^2]}{[M^1 L^2 T^{-2} K^{-1}]\,[K]}$$

$$[\alpha] = [M^{-1} T^2]$$

According to dimensional analysis,

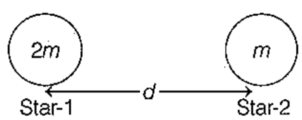
$$[W] = [\alpha \beta^2]$$

$\Rightarrow \quad [\beta^2] = \frac{[W]}{[\alpha]}$

$\Rightarrow \quad [\beta^2] = \frac{[M^1 L^2 T^{-2}]}{[M^{-1} T^2]} = [M^2 L^2 T^{-4}]$

$\Rightarrow \quad [\beta] = [MLT^{-2}]$

2. *(b)* The given situation is shown below



The gravitational force between these two stars provide the required centripetal force for rotation in a circle about their common centre.

Assuming $2m$ at origin, the centre of mass of the system lies at

$$x = \frac{2m \times 0 + m \times d}{2m + m} = \frac{d}{3}$$

Hence, $\quad F_G = F_C$

where, F_G is gravitational force between them and F_C is centripetal force.

$\Rightarrow \quad \frac{G m_1 m_2}{r^2} = 2 m \omega^2 x$

$\Rightarrow \quad \frac{G(2m)\,(m)}{d^2} = 2 m \omega^2 \times \frac{d}{3} \Rightarrow \omega^2 = \frac{3Gm}{d^3}$

$\Rightarrow \quad \omega = \sqrt{\frac{3Gm}{d^3}} \quad \ldots \text{(i)}$

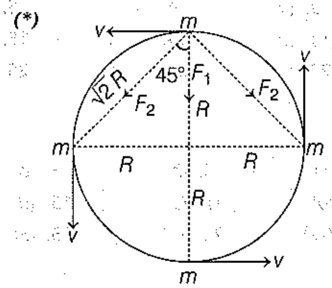
We know that, $\quad \omega = \frac{2\pi}{T}$

$\therefore \quad T = \frac{2\pi}{\omega}$

$\Rightarrow \quad T = \frac{2\pi}{\sqrt{\frac{3Gm}{d^3}}}$

$= 2\pi \sqrt{\frac{d^3}{3Gm}} \quad$ [using Eq. (i)]

3. (*)



Given, $m = 1$ kg, $R = 1$ m

We know that,

$$F = \frac{G m_1 m_2}{r^2}$$

$\because \quad F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$

and $\quad F_2 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$

Net force on one particle,

$$F_{net} = F_1 + F_2 \cos 45° + F_2 \cos 45°$$
$$= F_1 + 2F_2 \cos 45°$$
$$= \frac{Gm^2}{4R^2} + 2\left(\frac{Gm^2}{2R^2}\right) \cdot \frac{1}{\sqrt{2}}$$
$$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{\sqrt{2}R^2}$$
$$= \frac{Gm^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right]$$

As the gravitational force provides the necessary centripetal force, so

$$F_{net} = F_C = \frac{mv^2}{R}$$

Here, F_C = centripetal force.

$\Rightarrow \quad \frac{Gm^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = \frac{mv^2}{R}$

$\Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{Gm}{R}(1 + 2\sqrt{2})}$

$\Rightarrow \quad v = \frac{1}{2}\sqrt{G(1 + 2\sqrt{2})}$

4. *(d)* Let M and R be the mass and radius of four bodies. Then, as per question, their moment of inertia are

$$I_1 = \frac{MR^2}{2}, I_2 = \frac{MR^2}{2}, I_3 = \frac{MR^2}{2},$$

$$I_4 = \frac{2}{5} MR^2$$

Clearly, $I_1 = I_2 = I_3 > I_4$

5. *(a)* Given, period of revolution of first satellite,

$$T_1 = 1\text{h}$$

Period of revolution of second satellite,

$$T_2 = 8\text{ h}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{8}$$

We know that, $\omega = \frac{2\pi}{T}$

$$\Rightarrow \quad \omega \propto \frac{1}{T}$$

$$\because \quad \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{8}{1}$$

or $\quad \omega_1 : \omega_2 = 8 : 1$

6. *(d)* We know that, $\gamma = 3\alpha$ ... (i)

where, α is the coefficient of linear expansion and γ is the coefficient of volume expansion.

We know that,

$$\frac{\Delta V}{V} = \gamma \Delta T$$

$$\Rightarrow \quad \frac{\Delta V}{V} = 3\alpha \Delta T \quad \text{[from Eq. (i)]}$$

$$\Delta V = 3a^3 \alpha \Delta T \quad [\because \text{volume of cube} = a^3]$$

7. *(c)* We know that,

$$Y = 3K(1 - 2\sigma)$$

$$\Rightarrow \quad \sigma = \frac{1}{2}\left(1 - \frac{Y}{3K}\right) \quad ...(i)$$

Also, $\quad Y = 2\eta(1 + \sigma)$

$$\Rightarrow \quad \sigma = \frac{Y}{2\eta} - 1 \quad ... (ii)$$

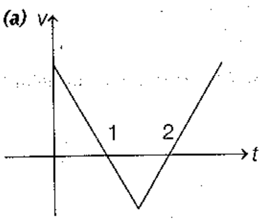
On comparing Eqs. (i) and (ii), we get

$$\left(1 - \frac{Y}{3K}\right)\frac{1}{2} = \frac{Y}{2\eta} - 1$$

On solving, we get

$$K = \frac{\eta Y}{9\eta - 3Y} \text{ N/m}^2$$

8. *(a)*



From the graph for first line, the slope is negative and intercept is positive.

So, equation of line is

$$v = -mt + c$$

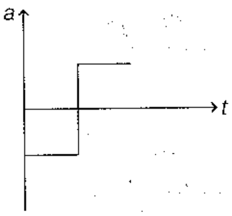
$$\Rightarrow \quad a_1 = \frac{dv}{dt} = -m$$

Similarly, for second line, the slope is positive and intercept is negative, so equation of line is

$$v = mt - c$$

$$\Rightarrow \quad a_2 = \frac{dv}{dt} = m$$

$\therefore$ The corresponding acceleration-time graph as shown below



Hence, option (a) is correct.

9. *(d)* As, AB is isothermal process, so work done by isothermal process is given by

$$W_{AB} = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$W_{AB} = nRT \ln\left(\frac{2V_1}{V_1}\right) = nRT \ln 2$$

As, BC is isobaric process, so work done by isobaric process is given by

$$W_{BC} = p\Delta V = p_2(V_1 - V_2)$$

$$= p_2\left(\frac{V_2}{2} - V_2\right) = -\frac{p_2V_2}{2}$$

$$= \frac{-nRT}{2} \quad \left[\begin{array}{l}\because pV = nRT \\ \therefore p_2V_2 = nRT\end{array}\right]$$

As, CA is isochoric process, so work done by isochoric process is given by, $W_{CA} = 0$ $\quad [\because \Delta V = 0]$

Total work done in the cycle $ABCA$,

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA}$$

$$= nRT \ln 2 + \left(\frac{-nRT}{2}\right) + 0$$

$$= nRT\left(\ln 2 - \frac{1}{2}\right)$$

10. *(d)* We know that, in isothermal process, $\Delta T = 0$

In isochoric process, $\Delta V = 0$

In adiabatic process, $\Delta Q = 0$

In isobaric process, $\Delta p = 0$

So, the correct match is,

A $\rightarrow$ 2, B $\rightarrow$ 3, C $\rightarrow$ 4, D $\rightarrow$ 1.

11. *(b)* Given, initial amplitude $= A$

Velocity at mean position, $v = A\omega$

Applying conservation of momentum at mean position, we get

$$M_1v_1 = M_2v_2$$

$$MA\omega = (M + m)v'$$

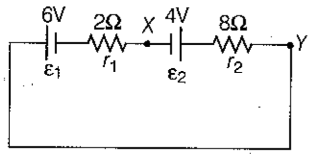
$$\Rightarrow \quad v' = \frac{MA\omega}{M + m} = \frac{MA\sqrt{\frac{k}{M}}}{M + m}$$

$$\therefore \quad v' = A'\omega' = A'\sqrt{\frac{k}{M + m}}$$

$$\Rightarrow \quad A' = \frac{MA\sqrt{\frac{k}{M}}}{M + m} \times \sqrt{\frac{M + m}{k}}$$

$$A' = \sqrt{\frac{M}{M + m}}\, A$$

12. *(c)* The circuit can be shown as below



The current through the circuit,

$$I = \frac{\varepsilon_1 - \varepsilon_2}{r_1 + r_2} = \frac{6-4}{10} = \frac{1}{5}\,\text{A}$$

$\therefore$ Potential difference across points X and Y is

$$V_{XY} = E_2 + Ir_2 = 4 + \frac{1}{5} \times 8 = 5.6\ \text{V}$$

13. *(c)* Given, $\quad i = \alpha_0 t + \beta t^2$

where, $\alpha_0 = 20\,\text{A/s}, \beta = 8\ \text{A/s}^2$

We know that, $\quad i = \frac{dq}{dt}$

$$\Rightarrow \quad \frac{dq}{dt} = i = \alpha_0 t + \beta t^2 = 20t + 8t^2$$

$$\Rightarrow \quad dq = (20t + 8t^2)dt$$

On integrating both sides, we get

$$\int_0^q dq = \int_0^{15} (20t + 8t^2)dt$$

$$q = \left[\frac{20t^2}{2} + \frac{8t^3}{3}\right]_0^{15} = 10 \times (15)^2 + \frac{8}{3} \times (15)^3$$

$$\therefore \quad q = 11250\,\text{C}$$

14. *(d)* Given, $C_1 = C_2 = C$

When both capacitors are connected in series, their equivalent capacitance will be

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$\Rightarrow \quad C_s = \frac{C}{2}$$

When both capacitors are connected in parallel, their equivalent capacitance will be

$$C_p = C + C = 2C$$

$\therefore$ The ratio of equivalent capacitance in series and parallel combination is

$$\frac{C_s}{C_p} = \frac{C/2}{2C} = \frac{1}{4}$$

$$\Rightarrow \quad C_s : C_p = 1 : 4$$

15. *(c)* We can replace $-Q$ charge at origin by $+Q$ and $-2Q$. Now, due to $+Q$ charge at every corner of cube, electric field at centre of cube is zero. So, net electric field at centre is only due to $-2Q$ charge at origin. Vector form of electric field strength,

$$\mathbf{E} = \frac{Kq\mathbf{r}}{r^3}$$

Here, position vector, $r = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$

$$\Rightarrow \quad |\mathbf{r}| = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}\,a}{2}$$

$$\Rightarrow \quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{(-2Q)\,a/2}{\left(\frac{a}{2}\sqrt{3}\right)^3} \cdot (\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{E} = \frac{-2Q\,(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})}{3\sqrt{3}\,\pi\,a^2\varepsilon_0}$$

16. *(a)* Given, emitter current, $I_E = 4$ mA

Collector current, $I_C = 3.5$ mA

Current gain in common base amplifier,

$$\alpha = \frac{I_C}{I_E}$$

$$\Rightarrow \quad \alpha = \frac{3.5}{4} = \frac{7}{8}$$

Also, current gain in common emitter amplifier,

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow \quad \beta = \frac{7/8}{1 - 7/8}$$

$$\beta = 7$$

17. *(d)* In transition A, hydrogen atom comes from higher energy state $n = \infty$ to lower energy state $n = 1$. Hence, transition A represents series limit of Lyman series. In transition B, hydrogen atom comes from higher energy state $n = 5$ to lower energy state $n = 2$. Hence, transition B represents 3rd line of Balmer series.

In transition C, hydrogen atom comes from higher energy state $n = 5$ to lower energy state $n = 3$. Hence, transition C represents 2nd line of Paschen series.

Hence, option (d) is correct.

18. *(c)* As we know, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

If linear momenta of two photons are equal, then their wavelengths is also equal.

Also, if the wavelength is decreased, then the momentum and energy of photon will increase.

Hence, option(c) is correct.

19. *(d)* For convex mirror, the focal length (f) and radius of curvature (r) are related as $f = +\frac{r}{2}$.

20. *(a)* Given, amplitude $\propto$ width of slit

$$\Rightarrow \quad A_2 = 3A_1$$

We know that,

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

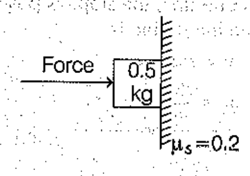
$\because$ Intensity, $I \propto A^2$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(|A_1 - A_2|)^2}$$

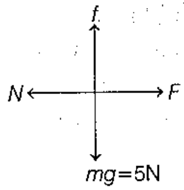
$$= \left(\frac{A_1 + 3A_1}{|A_1 - 3A_1|}\right)^2 = \left(\frac{4A_1}{2A_1}\right)^2 = \frac{4}{1}$$

$$\therefore \quad I_{\max} : I_{\min} = 4 : 1$$

21. *(25)* Given, coefficient of static friction, $\mu_s = 0.2$



Various forces acting on block are shown below



Frictional force $\leq mg$

$\Rightarrow \quad N \times 0.2 \leq 5$

$\Rightarrow \quad N \leq 25$

$\therefore$ Magnitude of horizontal force, $F = N = 25$ N

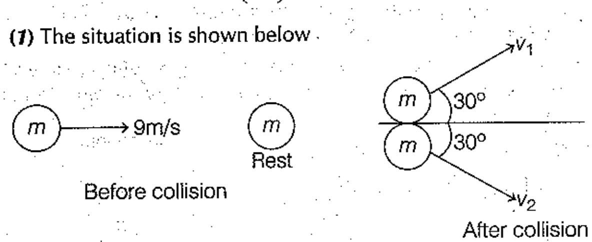
22. *(75)* Given, $I_0 = 100$ lumens

When analyser is rotated through an angle θ, the intensity of light will becomes

$$I = I_0 \cos^2\theta = 100 \times \cos^2 30^\circ$$

$$= 100 \times \left(\frac{\sqrt{3}}{2}\right)^2 = 75 \text{ lumens}$$

23. *(1)* The situation is shown below



Using conservation of linear momentum in y-direction,

$$p_i = p_f$$

As, $\quad p_i = 0$

and $\quad p_f = mv_1 \sin 30^\circ - mv_2 \sin 30^\circ$

$\Rightarrow \quad 0 = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$

$\Rightarrow \quad v_1 = v_2$ or $v_1 : v_2 = 1:1$

Since, $v_1 : v_2 = x : y$ (given)

$\therefore \quad x = 1$

24. *(25600)* According to Pascal's law,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Initially, $\quad \frac{100g}{A_1} = \frac{mg}{A_2} \quad$ ... (i)

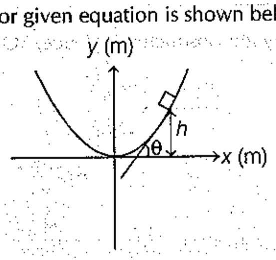
Finally, $\quad \frac{Mg}{16A_1} = \frac{mg}{\left(\frac{A_2}{16}\right)} \quad$ ... (ii)

On dividing Eqs. (i) by (ii), we get

$$\frac{100 \times 16}{M} = \frac{1}{16}$$

$\therefore \quad M = 25600$ kg

25. *(25)* The graph for given equation is shown below



At maximum height, the slope of tangent drawn,

$$\tan\theta = \frac{dy}{dx} = \frac{2x}{4} = \frac{x}{2} \quad \left[\because y = \frac{x^2}{4}\right]$$

$\Rightarrow \quad 0.5 = \frac{x}{2} \quad (\because \mu = \tan\theta)$

$\Rightarrow \quad x = 1$ m

$\therefore \quad y = \frac{x^2}{4} = \frac{1}{4} = 0.25 \text{ m} = 25 \text{ cm}$

26. *(2000)* Given, $L = 2 \times 10^{-4}$ H, $R = 6.28\,\Omega$,

$f_0 = 10$ MHz $= 10 \times 10^6$ Hz

$\therefore$ Quality factor $= \omega_0 \frac{L}{R} = 2\pi f_0 \frac{L}{R}$

$$= 2\pi \times 10 \times 10^6 \times \frac{2 \times 10^{-4}}{6.28}$$

$$= 2 \times 10^3 = 2000$$

27. *(25)* Given, audio signal,

$V_m = 20 \sin 2\pi(1500t) \quad$ ... (i)

Carrier signal, $V_c = 80 \sin 2\pi(100000t) \quad$ ... (ii)

We know that, modulation index,

$$m_f = \frac{A_m}{A_c}$$

From Eqs. (i) and (ii), we get

$$A_m = 20, A_c = 80$$

Percentage of modulation index,

$$m_f = \frac{A_m}{A_c} \times 100 = \frac{20}{80} \times 100 = 25\%$$

28. *(25)* Given, resistance, $R = 2\text{ k}\Omega = 2 \times 10^3\,\Omega$

In Zener breakdown,

$$i = \frac{V}{R} = \frac{5}{2 \times 10^3} = 2.5 \times 10^{-3}$$

$\because \quad x \times 10^{-4} = 25 \times 10^{-4}$

$\therefore \quad x = 25$

29. *(15)* Given, $\mu_r = \varepsilon_r = 2$

where, μ_r is relative permeability,

ε_r is relative permittivity.

Speed of electromagnetic wave v is given by

$$v = \frac{c}{n}$$

where, n = refractive index $= \sqrt{\mu_r \varepsilon_r} = \sqrt{4} = 2$

$\Rightarrow \quad v = \frac{3 \times 10^8}{2} = 15 \times 10^7$ m/s

$\because \quad x \times 10^7 = 15 \times 10^7$

$\Rightarrow \quad x = 15$

30. *(440)* In a transformer,

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

where, N_p = number of turns in primary circuit,

N_s = number of turns in secondary circuit = 24,

V_p = potential of primary circuit = 220V

and V_s = potential of secondary circuit = 12 V

$\Rightarrow \quad \frac{N_p}{24} = \frac{220}{12}$

$\Rightarrow \quad N_p = 440$

CHEMISTRY

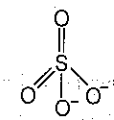
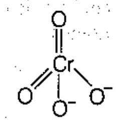
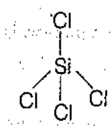
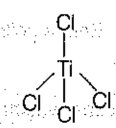
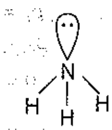
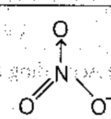
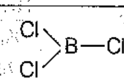
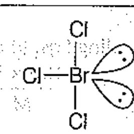
1. ***(a)*** Isostructural compounds are those compounds which have same structure as well as same hybridisation.

Formula for find hybridisation : H = (lp + σ + coordinate bond)

lp = Lone pair

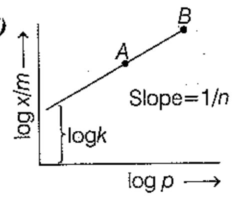
bp = Bond pair

σ = Sigma bond

S. No.	Molecule	H (Steric number)	Hybridisation	B.P.	L.P.	Shape
(A)	SO_4^{2-}	4	sp^3	4	0	 Tetrahedral
	CrO_4^{2-}	4	sp^3	4	0	 Tetrahedral
(B)	$SiCl_4$	4	sp^3	4	0	 Tetrahedral
	$TiCl_4$	4	sp^3	4	0	 Tetrahedral
(C)	NH_3	4	sp^3	3	1	 Trigonal pyramidal
	NO_3^-	4	sp^2	3	0	 Trigonal planar
(D)	BCl_3	3	sp^2	3	0	 Trigonal planar
	$BrCl_3$	5	sp^3d	3	2	 T - shape

If number of sigma bond (σ), co-ordinate bond and lone pair are same for given pairs, they are isostructural. Hence, SO_4^{2-}, CrO_4^{2-}, $SiCl_4$ and $TiCl_3$ have tetrahedral shape and are isostructural.

2. *(d)*



Given graph is Freundlich adsorption isotherm.

According to equation, $\frac{x}{m} = kp^{\frac{1}{n}}$ ...(i)

where, $\frac{x}{m}$ = amount of gas

k and n = constants depending upon nature of adsorbate and absorbent.

Now, taking log both side in Eq. (i), we get

$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$

For straight line, comparing above equation to $y = mx + c$

So, $m = \text{slope} = \frac{1}{n}$

hence, $0 \le \frac{1}{n} \le 1$

It value lies in between 0 to 1.

Hence, $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$.

3. *(a)* On moving left to right in a period of the periodic table, Ionisation energy (I.E.) increases due to increase in effective nuclear charge i.e. Z_{eff}.

But due to extra stability of fully filled and half-filled electronic configuration of Mg and P required ionisation enthalpy is more from neighbouring elements.

i.e. First ionisation enthalpy order is Al < Mg < Si < S < P.

4. *(d)* The chemical formulas of given ore in options are as follows

Sphalerite ore : ZnS

Calamine ore : $ZnCO_3$

Siderite ore : $FeCO_3$

Malachite ore : $Cu(OH)_2 \cdot CuCO_3$

Sphalerite ore containing ZnS is concentrated using group is cyanide salt (NaCN) as a depressant by froth flotation method.

5. *(b)* In equation (A), HOCl undergoes reduction in presence of H_2O_2.

$$\overbrace{\overset{(+1)}{HOCl} + H_2O_2 \longrightarrow H_3O^+ + \overset{(-1)}{Cl^-}}^{\text{Reduction}} + O_2$$

Here, oxidation state of Cl changes from +1 to −1 (i.e. reduces)

$\therefore$ H_2O_2 act as reducing agent I_2 reduces to I^- in presence of H_2O_2.

In equation (B),

$$\overbrace{\overset{0}{I_2} + \overset{+1}{H_2O_2} + 2OH^- \longrightarrow \overset{-1}{2I^-}}^{\text{Reduction}} + \overset{+2}{2H_2O} + O_2$$

Here oxidation state of iodine decreases (from 0 to −1)

$\therefore$ H_2O_2 act as reducing agent in both the equations.

6. *(d)* Al_2O_3 (aluminium oxide) was leached with alkali to get $Na[Al(OH)_4]$(X) (sodium aluminate). The solution of sodium aluminate when passed through gas Y, i.e. carbon dioxide (CO_2) forms ($Al_2O_3 \cdot xH_2O$) (Z).

Complete reactions are as follows

$$\underset{\text{Aluminium oxide}}{Al_2O_3} + \underset{\text{Sodium hydroxide}}{2\,NaOH\,(aq)} + 3H_2O(l) \longrightarrow$$

$$\underset{(X)}{2\,Na[Al(OH)_4](aq)} \xrightarrow{+\,CO_2(Y)} \underset{(Z)}{Al_2O_3 \cdot xH_2O} + \underset{\text{Sodium bicarbonate}}{2\,NaHCO_3}$$

Hence, $X = Na[Al(OH)_4]$

$Y = CO_2$

$Z = Al_2O_3 \cdot xH_2O$

7. *(d)* In the electrode potential series, only copper have positive value for electrode potential because copper has lower tendency than hydrogen to form ions. So, if standard hydrogen electrode ($E_{Cell} = 0$) is connected to copper half-cell, the copper with be relatively less negative or less number of electrons.

$E^\circ_{(Cu^{2+}/Cu)} = +\,0.34$ V; $E^\circ_{(Fe^{2+}/Fe)} = -\,0.41$ V

$E^\circ_{Co^{+2}/Co} = -\,0.28$ V; $E^\circ_{(Zn^{2+}/Zn)} = -\,0.76$ V

$\therefore$ Electrode potential of Cu $E^\circ_{Cu^{+2}/Cu}$ show positive value.

8. *(a)* 'Gun metal' also called as red brass (in USA). It is greyish in colour and originally used for making guns. The major components of 'Gun metal' are

Cu = 87%

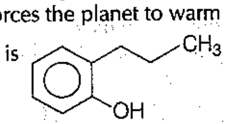
Zn = 3%

Sn = 10%

It is also used in manufacturing of gears and bearing that are to be subjected to heavy loads and low speeds.

9. *(b)* Methane (CH_4) gas is evolved due to anaerobic degradation of vegetation which causes global warming and cancer. It is a heat trapping gas that forces the planet to warm drastically and quickly.

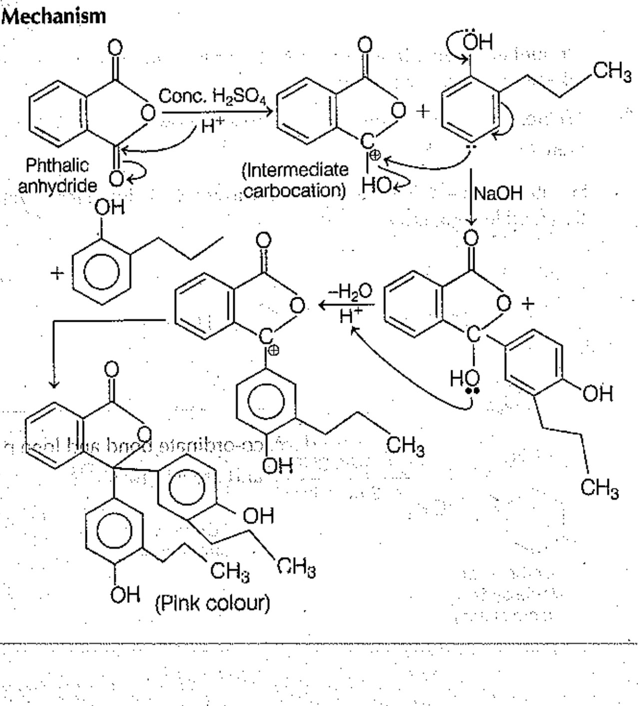
10. *(b)* Correct answer is



2-propylphenol

Firstly phthalic anhydride in presence of conc.H_2SO_4 undergoes protonation to give an intermediate carbocation. This carbocation reacts with 2-propylphenol in presence of NaOH to give pink colour compound.

Mechanism



2-propylphenol gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH.

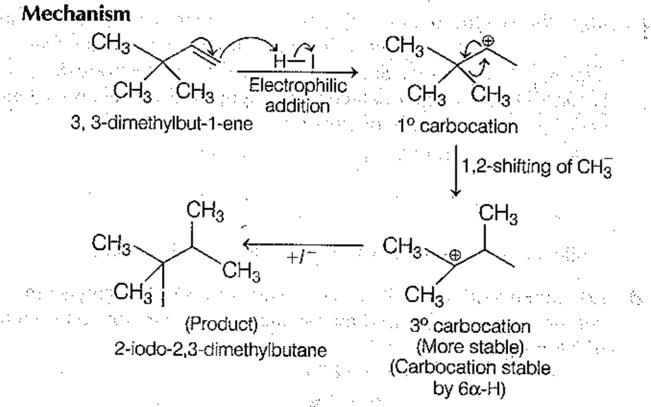
11. *(b)* The major product formed is 2-iodo-2, 3-dimethylbutane. Steps involved in the reaction are as follows :

Step 1 It is electrophilic addition reaction, π-bond of alkene attack H^+ ion of HI and form more stable carbocation.

Step 2 Formation of more stable 3° carbocation take place by 1, 2 shifting of —Me group.

Step 3 Direct addition of $I^{\ominus}$ ion .

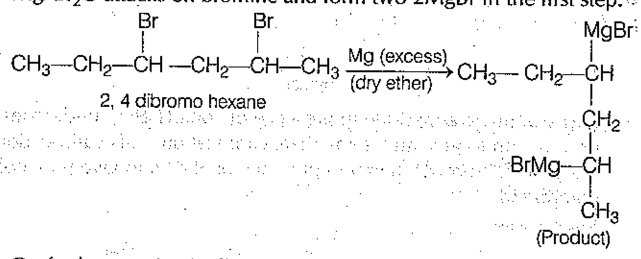
Mechanism



12. *(b)* Molybdenum oxide (Mo_2O_3) is used for oxidising alkanes to aldehyde. It used to manufacture molybdenum metal, which serves as an additive to steel and corrosive resistant alloys.

$$\underset{\text{Propane}}{CH_3—CH_2—CH_3} \xrightarrow[\text{Oxidising agent}]{Mo_2O_3} \underset{\text{Propanal}}{CH_3—CH_2—CHO}$$

13. *(d)* Here, in first step only one mole of Mg/Et_2O attacks on bromine and form two 2MgBr in the first step.

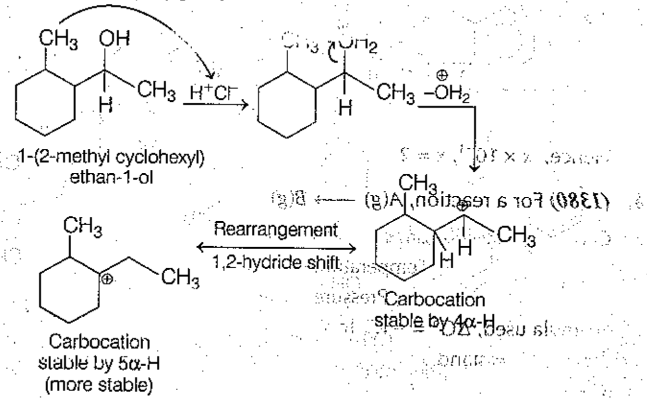


On further moving in the reaction, two MgBr are eliminated to form alkene in respective positions.

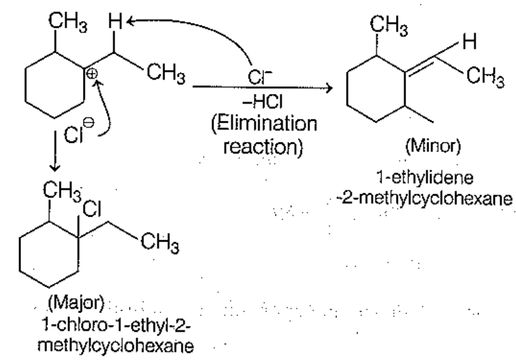
14. *(d)* Steps involved in this reaction are as follows

Step 1 $HCl \longrightarrow H^+ + Cl^-$

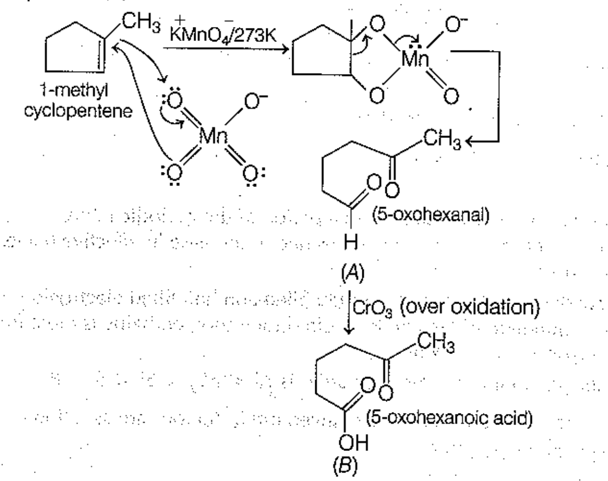
H^+ attacks on —OH (lone pair) and formed $\overset{\oplus}{O}H_2$ ion. Here, $\overset{\oplus}{O}H_2$ is the good leaving group.



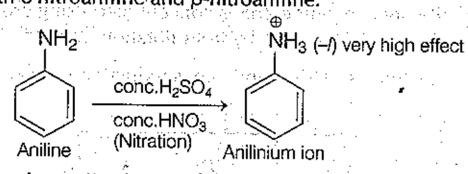
Step 2 Elimination reaction always give minor product than substitution reaction.



15. *(d)* In first step, $KMnO_4$ (dil.) help to break alkene in ketone and aldehyde and in second step, CrO_3 is used for selective over oxidation of aldehyde only. CrO_3 will form aldehyde to acid in product (*B*).



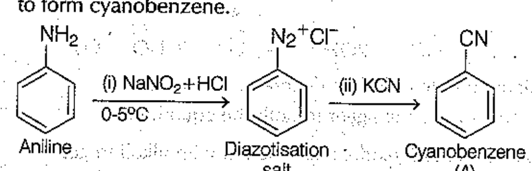
16. *(c)* Aniline on protonation forms anilinium ion, which is *meta*-directing. So, a considerable amount of *meta* product is formed alongwith *o*-nitroaniline and *p*-nitroaniline.



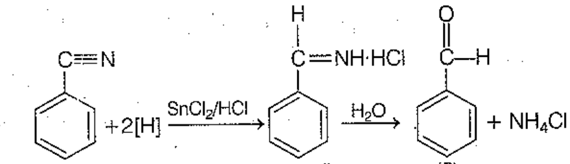
Nitrating mixture is mixture of conc. HNO_3 and a conc. H_2SO_4. When aniline is reacted with nitrating mixture *ortho*, *meta* and *para* nitroanilines are obtained.

Aniline being basic in nature, reacts with acids to form anilinium ion which is *meta* directing.

17. *(d)* **Step 1** In step 1, $NaNO_2$ + HCl, 0-5°C used for diazotisation reaction. It will form diazonium salt. Further, it will react with KCN to form cyanobenzene.

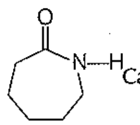


Step 2 In step 2, $SnCl_2$ and HCl is a Stephen's reduction reagent. Cyanobenzene reduced to benzaldehyde by $SnCl_2$/HCl.

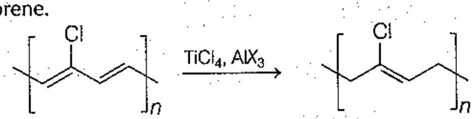


Cyanobenzene Intermediate (B) Benzaldehyde

18. *(d)*

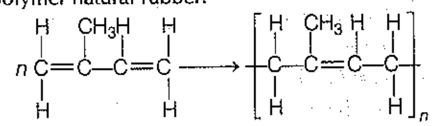
(a)  Caprolactum is the monomeric unit of polymer nylon-6 $\left[HN-(CH_2)_5-\overset{O}{\overset{\|}{C}} \right]_n$

(b) 2-chlorobuta -1, 3-diene is the monomeric unit of polymer neoprene.



2-chlorobuta-1,3-diene Neoprene

(c) 2-methylbuta-1, 3-diene (isoprene) is the monomeric unit of polymer natural rubber.



2-methyl buta-1,3-diene (Isoprene) Natural rubber

Value of n is about 10,000.

(d) $CH_2{=}CH{-}CN$ (acrylonitrile) is one of the monomeric unit of polymer buna-N.

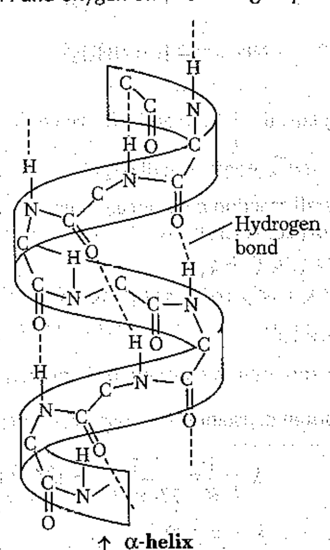
$$n\,CH_2{=}CH{-}CH{=}CH_2 + n\,CH_2{=}\overset{CN}{\overset{|}{CH}}$$

1, 3-butadiene Acrylonitrile

$$\xrightarrow{\text{Polymerisation}} \left[CH_2{-}CH{=}CH{-}CH_2{-}\overset{CN}{\overset{|}{CH}}{-}CH_2 \right]_n$$

Buna–N

19. *(d)* α-helix structure of proteins is stabilised by H-bonds between hydrogen of —NH and oxygen of —C= O group is amino acids.



α-helix

Secondary structure

20. *(b)* (i) In presence of luminous flame, blue cupric metaborate is reduced to colourless cuprous metaborate.

$$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow[\text{flame}]{\text{Luminous}} 2CuBO_2 + Na_2B_4O_7 + CO$$

If non-luminous flame is present in reaction, cupric metaborate is obtained by heating boric anhydride with copper sulphate.

$$CuSO_4 + B_2O_3 \xrightarrow[\text{flame}]{\text{Non-luminous}} \underset{\text{Cupric metaborate}}{Cu(BO)_2} + SO_3$$

So, both statements are false.

21. *(2)* Given, weight of compound $A = 4.5$ g

Molecular weight of compound $A = 90$ g/mol

Volume of solution (in mL) = 250 mL

Now, molarity is defined as number of moles of solute or compound A divided by volume of solution (in L).

$$M = \frac{\text{Number of moles of solute } (n)}{\text{Volume of solution}}$$

$$= \frac{\frac{4.5}{90}}{\frac{250}{1000}} = 0.2 \text{ or} = 2\times10^{-1}\text{ M}$$

$$\therefore \quad n = \frac{\text{Weight of solute (compound } A)}{\text{Molecular weight of solute (compound } A)}$$

Hence, $x \times 10^{-1}$ μ

$x = 2$

22. *(8)* Coordination number is the number of nearest neighbours of a central atom in the structure.

bcc has a coordination number of 8 and contains 2 atoms per unit cell.

This is because each atom touches four atoms in the layer above it, four in the layer below it and none in its own layer.

23. *(2)* Given, mass of Li^{3+} = 8.3 times of mass of proton formula,

De-Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2mqV}}$

Here, h = Planck's constant = 6.624×10^{-34} J-s

m = Mass of atom

q = Charge (or number of electrons)

$$\lambda_{Li} = \frac{h}{\sqrt{2m_{Li}(3e)V}} \quad \text{...(i)}$$

$$\lambda_P = \frac{h}{\sqrt{2m_P(e)V}} \quad \text{...(ii)}$$

Now, Eq. (i) divided by Eq. (ii), we get,

$$\frac{\lambda_{Li}}{\lambda_P} = \sqrt{\frac{m_P(e)\,V}{m_{Li}\,(3e)\,V}}$$

We know that $m_{Li} = 8.3\,m_p$

$$\frac{\lambda_{Li}}{\lambda_P} = \sqrt{\frac{m_P\times e\times V}{8.3\,m_P\times 3e\times V}} = \sqrt{\frac{1}{8.3\times3}} = \frac{1}{5} = 0.2$$

$$2\times10^{-1}$$

Hence, $x\times10^{-1}$, $x = 2$

24. *(1380)* For a reaction, $A(g) \longrightarrow B(g)$

Given, K_p (equilibrium constant) = 100

Temperature = 300 K

Pressure = 1 atm

Formula used, $\Delta G° = -RT \ln K_p$ ...(i)

Here, $\Delta G°$ = standard Gibb's free energy

R = gas constant = 8.31 J mol^{-1} K^{-1}

Put value in Eq (i), we get

$$\Delta G^\circ = -R(300)\ln 100$$
$$\Delta G^\circ = -R(300)(2)\ln(10)$$

$\because \ln(10) = 2.3$

$$\Delta G^\circ = -R(300)(2)(2.3)$$
$$\Delta G^\circ = -1380R$$

Hence, $\Delta G^\circ = -xR$

$$x = 1380$$

25. *(34.4)* $ClCH_2COOH \rightleftharpoons ClCH_2COO^- + H^+$

$t=0$	1	0	0
at $t=t$	$1-\alpha$	α	α

Here, $i = \dfrac{\text{Final moles}}{\text{Initial moles}}$

$$i = 1-\alpha+\alpha+\alpha \Rightarrow i = 1+\alpha$$

Formula used for freezing point; $\Delta T_f = iK_f m$

Here, K_f = freezing constant of H_2O

$K_f(H_2O) = 1.86\ \text{K kg mol}^{-1}$

m = molarity

$$\Delta T_f = iK_f m$$
$$0.5 = (1+\alpha)(1.86)\frac{(9.45/94.5)}{(500/1000)}$$
$$\frac{5}{3.72} = 1+\alpha \Rightarrow \frac{5}{3.72} - 1 = \alpha$$
$$\Rightarrow \quad \alpha = \frac{1.28}{3.72} = \frac{32}{93} = 0.344$$

Percentage of dissociation = 34.4%

26. *(5)* $Cl_2(g) \rightleftharpoons 2Cl(g)$

Let, moles of both of Cl_2 and Cl molecule be x.

Partial pressure of Cl is, $p_{Cl} = \dfrac{x}{2x} \times 1 = \dfrac{1}{2}$

Partial pressure of Cl_2 is

$$p_{Cl_2} = \frac{x}{2x} \times 1 = \frac{1}{2}$$

Now, $$K_p = \frac{(p_{Cl})^2}{p_{Cl_2}} \Rightarrow K_p = \frac{(1/2)^2}{1/2} = \frac{1}{2} = 0.5$$

or 5×10^{-1}

Hence, $x \times 10^{-1}$

$$x = 5$$

27. *(12)* The two half reaction, one separately are as follows

$S_8 + 16e^- \longrightarrow 8S^{2-}$ (Reduction)

$S_8 + 12H_2O \longrightarrow 4S_2O_3^{2-} + 24H^+ + 16e^-$

$2S_8 + 12H_2O \longrightarrow 8S^{2-} + 4S_2O_3^{2-} + 24H^+$ (Oxidation)

For balancing in basic medium,

Add an equal number of OH^- that of H^+,

we get

$$2S_8 + 12H_2O + 24OH^- \longrightarrow 8S^{2-} + 4S_2O_3^{2-} + 24H_2O$$
$$2S_8 + 24OH^- \longrightarrow 8S^{2-} + 4S_2O_3^{2-} + 12H_2O$$

or $S_8 + 12OH^- \longrightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$ ...(i)

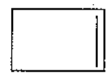
On comparing (i) with

$S_8 + aOH^-(aq) \longrightarrow bS^{2-}(aq) + cS_2O_3^{2-} + dH_2O$

We get, $a = 12$

$b = 4;\ c = 2;\ d = 6$

28. *(26)*



$\longrightarrow H_2C{=}CH{-}CH{=}CH_2$

Cyclobutene 1, 3-butadiene

It is a first order isomerisation reaction. Integrated rate law for 1st order reaction is

$$kt = \ln\frac{[A]_0}{[A]_t} \quad ...(i)$$

Here, k = rate constant

$[A]_0$ = initial concentration

$[A]_t$ = concentration at time 't'

Given, $k = 3.3 \times 10^{-4}\ s^{-1}$

$[A]_0 = 100 \Rightarrow [A]_t = 100 - 40 = 60$

Put values in Eq. (i), we get

$$3.3 \times 10^{-4}s^{-1} \times t = \ln\frac{100}{60}$$
$$t = 1547.95\ s = 25.79\ \text{min} \quad (1\ \text{min} = 60\ s \text{ or } 1\ s = \frac{1}{60}\ \text{min})$$
$$= 26 \text{ minutes}$$

29. *(2)* An amphoteric compound is a molecule or ion that can react with both as an acid or as a base.

BeO = Amphoteric

BaO = Basic

$Be(OH)_2$ = Amphoteric

$Sr(OH)_2$ = Basic

Both beryllium compound BeO and $Be(OH)_2$ are amphoteric in nature while compound BaO and $Sr(OH)_2$ are basic in nature, they form alkaline solution in H_2O.

30. *(1.26)* Given, stability constant value, $k_1 = 10^4$

$$k_2 = 1.58 \times 10^3$$
$$k_3 = 5 \times 10^2$$
$$k_4 = 10^2$$

$$Cu^{2+} + NH_3 \xrightleftharpoons{k_1} [Cu(NH_3)]^{2+} \quad ...(i)$$
$$[Cu(NH_3)]^{2+} + NH_3 \xrightleftharpoons{k_2} [Cu(NH_3)_2]^{2+} \quad ...(ii)$$
$$[Cu(NH_3)_2]^{2+} + NH_3 \xrightleftharpoons{k_3} [Cu(NH_3)_3]^{2+} \quad ...(iii)$$
$$[Cu(NH_3)_3]^{2+} + NH_3 \xrightleftharpoons{k_4} [Cu(NH_3)_4]^{2+} \quad ...(iv)$$

$\vdots \quad \vdots \quad \vdots$

On adding Eqs. (i), (ii), (iii) and (iv), we get

$$Cu^{2+} + 4NH_3 \xrightleftharpoons{k} [Cu(NH_3)_4]^{2+}$$

$\therefore$ The overall reaction constant (k) or equilibrium constant for formation of $[Cu(NH_3)_4]^{2+}$ is

$$k = k_1 \times k_2 \times k_3 \times k_4$$
$$k = 10^4 \times 1.58 \times 10^3 \times 5 \times 10^2 \times 10^2$$
$$k = 7.9 \times 10^{11}$$

where, k = equilibrium constant for formation of $[Cu(NH_3)_4]^{2+}$

So, equilibrium constant k' for dissociation of $[Cu(NH_3)_4]^{2+}$ is $\dfrac{1}{k}$.

$$k' = \frac{1}{k} = \frac{1}{7.9 \times 10^{11}} = 1.26 \times 10^{-12}$$

Hence, $$k' = x \times 10^{-12}$$
$$x = 1.26$$

MATHEMATICS

1. *(a)* Given, $f(x) = 2x - 1;\ f : R \to R$

$$g(x) = \frac{x - 1/2}{x - 1};\ g : R - \{1\} \to R$$

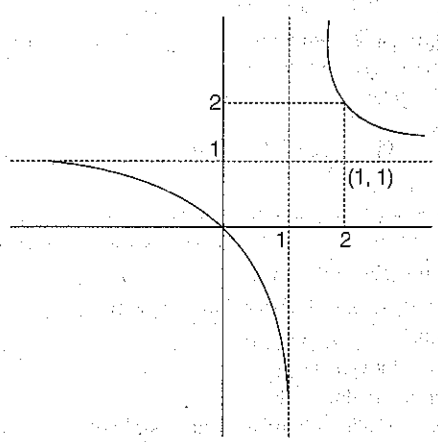
$$f[g(x)] = 2g(x) - 1$$

$$= 2 \times \left(\frac{x - \frac{1}{2}}{x - 1}\right) - 1 = 2 \times \left(\frac{2x - 1}{2(x - 1)}\right) - 1$$

$$= \frac{2x - 1}{x - 1} - 1 = \frac{2x - 1 - x + 1}{x - 1} = \frac{x}{x - 1}$$

$$\therefore \quad f[g(x)] = 1 + \frac{1}{x - 1}$$

Now, draw the graph of $1 + \dfrac{1}{x - 1}$,



$\because$ Any horizontal line does not cut the graph at more than one points, so it is one-one and here, co-domain and range are not equal, so it is into.

Hence, the required function is one-one into.

2. *(b)* Given, p and q are positive numbers.

$$p + q = 2 \qquad \text{...(i)}$$

$$p^4 + q^4 = 272$$

$$\Rightarrow \quad (p^2 + q^2)^2 - 2p^2q^2 = 272$$

$$\Rightarrow \quad [(p + q)^2 - 2pq]^2 - 2p^2q^2 = 272$$

$$\Rightarrow \quad [(2)^2 - 2pq]^2 - 2p^2q^2 = 272 \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad (4 - 2pq)^2 - 2p^2q^2 = 272$$

$$\Rightarrow 16 + 4p^2q^2 - 16pq - 2p^2q^2 = 272$$

$$\Rightarrow \quad 2p^2q^2 - 16pq - 256 = 0$$

$$\Rightarrow \quad p^2q^2 - 8pq - 128 = 0$$

$$\therefore \quad pq = \frac{8 \pm \sqrt{64 + 4 \times 128}}{2 \times 1}$$

$$= \frac{8 \pm \sqrt{576}}{2}$$

$$= \frac{8 \pm 24}{2}$$

$$\therefore \quad pq = 16, -8$$

Here, $pq = -8$ is not possible as p and q are positive.

$$\therefore \quad pq = 16$$

Now, the equation whose roots are p and q is

$$x^2 - (p + q)x + pq = 0$$

$$\Rightarrow \quad x^2 - 2x + 16 = 0$$

3. *(c)* Given, $3x - 2y - kz = 10$

$$2x - 4y - 2z = 6$$

$$x + 2y - z = 5m$$

Condition for inconsistency $\Rightarrow \Delta = 0$

and atleast one of the $\Delta_x, \Delta_y, \Delta_z$ is non-zero.

$$\text{Now, } \Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 3(4 + 4) + 2(-2 + 2) - k(4 + 4)$$

$$= 24 - 8k$$

Now, $\Delta = 0$

$$\therefore \quad 24 - 8k = 0$$

$$\Rightarrow \quad 8k = 24 \Rightarrow k = \frac{24}{8} = 3$$

$$\Delta_x = \begin{vmatrix} 10 & -2 & -k \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix}$$

$$= 10(4 + 4) + 2(-6 + 10m) - 3(12 + 20m)$$

$$= 80 - 12 + 20m - 36 - 60m$$

$$= 32 - 40m = 8(4 - 5m)$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -k \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix}$$

$$= 3(-6 + 10m) - 10(-2 + 2) - 3(10m - 6)$$

$$= -18 + 30m - 30m + 18$$

$$= 0$$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix}$$

$$= 3(-20m - 12) + 2(10m - 6) + 10(4 + 4)$$

$$= -60m - 36 + 20m - 12 + 80$$

$$= -40m + 32 = 8(4 - 5m)$$

Here, either Δ_x or $\Delta_z \neq 0$

$$\Rightarrow \quad 8(4 - 5m) \neq 0$$

$$\Rightarrow \quad m \neq 4/5$$

Hence, the required values are $k = 3;\ m \neq \dfrac{4}{5}$.

4. *(b)* Given, $(-{}^{15}C_1 + 2\cdot{}^{15}C_2 - 3\cdot{}^{15}C_3 + \ldots - 15\cdot{}^{15}C_{15})$
$+ ({}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11})$

Let $S_1 = -{}^{15}C_1 + 2\cdot{}^{15}C_2 - 3\cdot{}^{15}C_3 + \ldots - 15\cdot{}^{15}C_{15}$

$$= \sum_{r=1}^{15}(-1)^r \cdot r \cdot {}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\, {}^{14}C_{r-1}$$

$$= 15(-{}^{14}C_0 + {}^{14}C_1 - {}^{14}C_2 + \ldots - {}^{14}C_{14})$$

$$= 15(0) = 0$$

$$S_2 = {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$$

$$= ({}^{14}C_1 + {}^{14}C_3 + \ldots + {}^{14}C_{13}) - {}^{14}C_{13}$$

$$= 2^{13} - 14$$

Now, the required value is

$(-{}^{15}C_1 + 2\cdot{}^{15}C_2 - 3\cdot{}^{15}C_3 + \ldots - 15\cdot{}^{15}C_{15})$
$+ ({}^{14}C_1 + {}^{14}C_3 + \ldots + {}^{14}C_{11})$

$$= S_1 + S_2 = 0 + 2^{13} - 14 = 2^{13} - 14$$

5. *(a)* Given, $t^2 - 9t + 8 = 0$, is satisfied by

$$e^{(\cos^2 x + \cos^4 x + \cos^6 x + \ldots \infty)\log_e 2}$$

Now, $a^{\log_c b} = b^{\log_c a}$

$\therefore$ $e^{(\cos^2 x + \cos^4 x + \ldots \infty)\log_e 2}$

$$= 2^{(\cos^2 x + \cos^4 x + \infty)\log_e e} = 2^{\cos^2 x + \cos^4 x + \ldots\ldots \infty}$$

Here, $\cos^2 x + \cos^4 x + \ldots \infty$ are in GP,

where $a = \cos^2 x$

$r = \cos^2 x < 1$ $\quad\left(\because 0 < x < \frac{\pi}{2}\right)$

$\therefore$ $S_\infty = \dfrac{a}{1-r}$

$\therefore$ $S_\infty = \dfrac{\cos^2 x}{1-\cos^2 x} = \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$

$\therefore$ $\cos^2 x + \cos^4 x + \ldots \infty = \cot^2 x$

Now, $2^{\cos^2 x + \cos^4 x + \ldots \infty} = 2^{\cot^2 x}$

Now, roots of equation $t^2 - 9t + 8 = 0$, are

$(t-1)(t-8) = 0$

$t = 1, 8$

$\therefore$ $2^{\cot^2 x} = 1$ or 8

$\Rightarrow$ $2^{\cot^2 x} = 1 = 2^0$ or $2^{\cot^2 x} = 8 = 2^3$

$\Rightarrow$ $\cot^2 x = 0$ or $\cot^2 x = 3 \Rightarrow \cot x = 0$ or $\cot x = \sqrt{3}$

But here, $0 < x < \dfrac{\pi}{2}$.

$\therefore$ $\cot x = 0$ not possible.

Hence, $\cot x = \sqrt{3}$ is only possible value.

Now, $\dfrac{2\sin x}{\sin x + \sqrt{3}\cos x}$

Dividing numerator and denominator by $\sin x$, we get

$$\frac{2}{1+\sqrt{3}\cot x}$$

$$= \frac{2}{1+\sqrt{3}\times\sqrt{3}} \quad [\because \cot x = \sqrt{3}]$$

$$= \frac{2}{1+3} = \frac{2}{4} = \frac{1}{2}$$

6. *(a)* Given, $\lim_{x\to 0}\dfrac{\int_0^{x^2}(\sin\sqrt{t})dt}{x^3}$

$\because$ It is of the form $\dfrac{0}{0}$.

By differentiating numerator and denominator,

$$\lim_{x\to 0}\frac{\sin\sqrt{x^2}\cdot 2x}{3x^2} = \lim_{x\to 0}\frac{\sin x\cdot 2x}{3x^2}$$

$$= \frac{2}{3}\lim_{x\to 0}\frac{\sin x}{x} = \frac{2}{3}(1) = \frac{2}{3}$$

7. *(a)* Given, $f(x) = \dfrac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$

$$f'(x) = \frac{12x^2 - 6x}{6} - 2\cos x + (2x-1)(-\sin x) + \cos x(2)$$

$$= (2x^2 - x) - 2\cos x - 2x\sin x + \sin x + 2\cos x$$

$$= 2x^2 - x - 2x\sin x + \sin x$$

$$= 2x(x - \sin x) - 1(x - \sin x)$$

$f'(x) = (2x-1)(x - \sin x)$

for $x > 0$, $x - \sin x > 0$

$x < 0$, $x - \sin x < 0$

for $x \in (-\infty, 0] \cup \left[\dfrac{1}{2}, \infty\right)$, $f'(x) \geq 0$

for $x \in \left[0, \dfrac{1}{2}\right]$, $f'(x) \leq 0$

Hence, $f(x)$ increases in $\left[\dfrac{1}{2}, \infty\right)$.

8. *(b)* Given,

Number of Indians = 6

Number of foreigners = 8

Committee of atleast 2 Indians and double number of foreigners is to be formed. Hence, the required cases are

$$(2I, 4F) + (3I, 6F) + (4I, 8F)$$

$$= {}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$$

$$= (15\times 70) + (20\times 28) + (15\times 1)$$

$$= 1050 + 560 + 15 = 1625$$

9. *(d)* Given, $f(x) = [x-1]\cos\left(\dfrac{2x-1}{2}\right)\pi$ where $[\cdot]$ is greatest integer function and $f: R \to R$

$\because$ It is a greatest integer function then we need to check its continuity at $x \in I$ except these it is continuous.

Let, $x = n$ where $n \in I$

Then LHL $= \lim_{x\to n^-}[x-1]\cos\left(\dfrac{2x-1}{2}\right)\pi$

$$= (n-2)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

RHL $= \lim_{x\to n^+}[x-1]\cos\left(\dfrac{2x-1}{2}\right)\pi$

$$= (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

and $f(n) = 0$.

Here, $\lim_{x\to n^-} f(x) = \lim_{x\to n^+} f(x) = f(n)$

$\therefore$ It is continuous at every integers.

Therefore, the given function is continuous for all real x.

10. *(b)* Given, $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx$

$$= a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$$

Put, $\sin x + \cos x = t$

Also, $\sin^2 x + \cos^2 x + 2\sin x \cos x = t^2$

$\Rightarrow \quad 2\sin x \cos x = t^2 - 1$

$\Rightarrow \quad \sin 2x = (t^2 - 1)$

and $(\cos x - \sin x)\, dx = dt$

Now, $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = \int \frac{dx}{\sqrt{8 - (t^2 - 1)}}$

$$= \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + c$$

Now, according to question,

$$\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$$

$\Rightarrow \quad \sin^{-1}\left(\frac{t}{3}\right) + c = a \sin^{-1}\frac{(\sin x + \cos x)}{b} + c$

$\Rightarrow 1 \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$

$(\because t = \sin x + \cos x)$

$\therefore \quad a = 1$

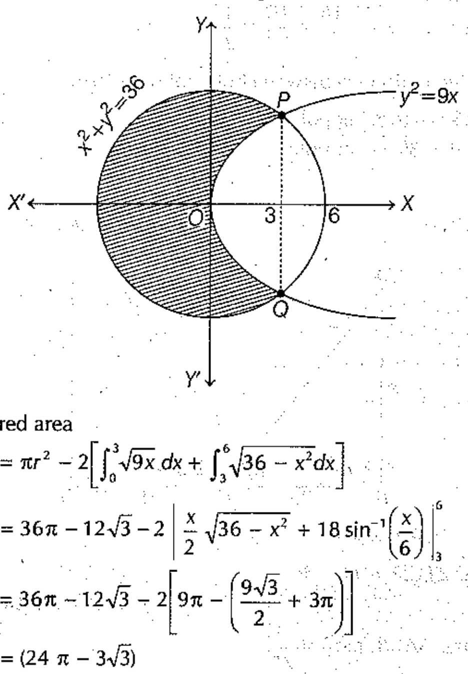
$b = 3$

Hence, $(a, b) = (1, 3)$

11. *(b)* Given, equation of circle $\Rightarrow x^2 + y^2 = 36$

Equation of parabola $\Rightarrow y^2 = 9x$

We have to find area of shaded region.



Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\, dx + \int_3^6 \sqrt{36 - x^2} dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left|\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right|_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left[9\pi - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right]$$

$$= (24\pi - 3\sqrt{3})$$

12. *(d)* Given, $\frac{dP}{dt} = 0.5P - 450$

and $\quad P(0) = 850$

$\because \quad \frac{dP}{dt} = \frac{1}{2}P - 450 = \frac{P - 900}{2}$

$\Rightarrow \quad \int_0^t \frac{dP}{P - 900} = \int_0^t \frac{dt}{2}$

$\Rightarrow \quad [\log_e |P - 900|]_0^t = \left\{\frac{t}{2}\right\}_0^t$

$\Rightarrow \log_e |P(t) - 900| - \log_e |P(0) - 900| = \frac{t}{2}$

$\Rightarrow \quad \log_e |P(t) - 900| - \log_e 50 = \frac{t}{2}$

Let at $t = t_1$, $P(t) = 0$

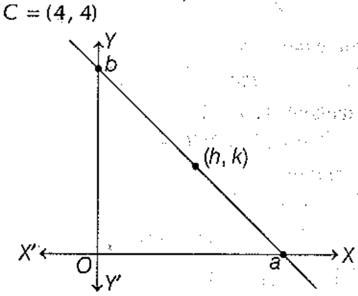
Hence, $\log_e |P(t) - 900| - \log_e 50 = t_1/2$

$\therefore \quad t_1 = 2\log_e 18$

13. *(b)* Given, position of $A = (1, 1)$

Position of $B = (2, 2)$

Position of $C = (4, 4)$



Let x-intercept be a and y-intercept be b.

Equation of line traced is

$$\frac{x}{a} + \frac{y}{b} = 1$$

This is the equation of path, let a point (h, k) lie on this path.

Then $\quad \frac{h}{a} + \frac{k}{b} = 1 \quad$ ...(i)

Also, AM of reciprocal of a and $b = \frac{1}{4}$

$\therefore \quad \frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$

$\frac{1}{a} + \frac{1}{b} = \frac{1}{2} \quad$ ... (ii)

On comparing Eqs. (i) and (ii), we get

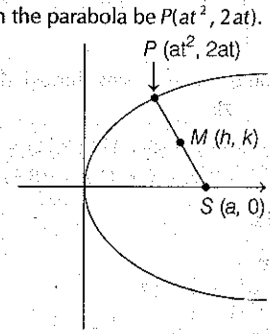
$(h, k) = (2, 2)$

Hence, the required stone is $B(2, 2)$.

14. *(c)* Given, equation of parabola $\Rightarrow y^2 = 4ax$

Focus $= S(a, 0)$

Let any point on the parabola be $P(at^2, 2at)$.



and let the mid-point of PS be $M(h, k)$.

$\therefore \quad h = \dfrac{at^2 + a}{2}; k = \dfrac{2at + 0}{2}$

$\Rightarrow \quad t^2 = \dfrac{2h - a}{a}; t = \dfrac{k}{a}$

$\Rightarrow \quad t^2 = \dfrac{k^2}{a^2}$

Now, $\dfrac{2h - a}{a} = \dfrac{k^2}{a^2}$

$\Rightarrow \quad 2h - a = \dfrac{k^2}{a} \Rightarrow k^2 = a(2h - a)$

$\therefore$ Locus of (h, k) is $y^2 = a(2x - a)$

$$y^2 = 2a\left(x - \frac{a}{2}\right)$$

$\therefore$ The directrix of this parabola is

$$x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$$

15. *(d)* Given, curve $\Rightarrow y = x^3$ ... (i)

$$P(t, t^3)$$

Equation of tangent at $P(t, t^3)$

$$(y - t^3) = 3t^2(x - t) \quad \text{... (ii)}$$

From Eqs. (i) and (ii),

$$x^3 - t^3 = 3t^2(x - t)$$

$\Rightarrow (x - t)(x^2 + t^2 + xt) = 3t^2(x - t)$

$\Rightarrow \quad x^2 + xt - 2t^2 = 0$

$\Rightarrow \quad (x - t)(x + 2t) = 0$

$\Rightarrow \quad x = t$ or $x = -2t$

This is not possible.

Now, the coordinate of $Q = (x, y) = (-2t, (-2t)^3)$ $\quad [\because y = x^3]$

$\therefore \quad Q = (-2t, -8t^3)$

$\therefore$ Ordinate of the point dividing PQ in the ratio 1 : 2 is

$$\frac{2t^3 + (-8t^3)}{1 + 2} = -2t^3$$

16. *(b)* Given, equation of planes are

$$3x + y - 2z = 5$$
$$2x - 5y - z = 7$$

and point $(1, 2, -3)$.

Normal vector of required plane $= \mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix}$

$= \hat{\mathbf{i}}(-1 - 10) - \hat{\mathbf{j}}(-3 + 4) + \hat{\mathbf{k}}(-15 - 2)$

$= -11\hat{\mathbf{i}} - \hat{\mathbf{j}} - 17\hat{\mathbf{k}}$

Now, the equation of plane passing through $(1, 2, -3)$ having normal vector $(-11\hat{\mathbf{i}} - \hat{\mathbf{j}} - 17\hat{\mathbf{k}})$ is

$$-[11(x - 1) + (y - 2) + 17(z + 3)] = 0$$

$\Rightarrow \quad 11x + y + 17z + 38 = 0$

17. *(c)* Given, $P(1, 1, 9)$.

Equation of plane

$$x + y + z = 17$$

Equation of line $\Rightarrow \dfrac{x - 3}{1} = \dfrac{y - 4}{2} = \dfrac{z - 5}{2}$

$\Rightarrow \quad \dfrac{x - 3}{1} = \dfrac{y - 4}{2} = \dfrac{z - 5}{2} = \lambda$ (let)

$\Rightarrow \quad x = \lambda + 3; y = 2\lambda + 4; z = 2\lambda + 5$

$\therefore$ The point we have is $(\lambda + 3, 2\lambda + 4, 2\lambda + 5)$.

$\because$ This point lies on the plane $x + y + z = 17$.

$\therefore \quad \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \quad \lambda = 1$

$\therefore$ The coordinate of point is $(4, 6, 7)$

$\therefore$ Required distance between $(1, 1, 9)$ and $(4, 6, 7)$ is

$$= \sqrt{(4 - 1)^2 + (6 - 1)^2 + (7 - 9)^2}$$
$$= \sqrt{9 + 25 + 4} = \sqrt{38}$$

18. *(d)* Given, P(odd number 2 times) $= P$(even number 3 times)

$\Rightarrow \quad {}^nC_2\left(\dfrac{1}{2}\right)^n = {}^nC_3\left(\dfrac{1}{2}\right)^n$

where, n = Number of times the die is thrown.

$\Rightarrow \quad \dfrac{n!}{2!(n-2)!} \times \left(\dfrac{1}{2}\right)^n = \dfrac{n!}{3!(n-3)!} \times \left(\dfrac{1}{2}\right)^n$

$\Rightarrow \quad \dfrac{3! \times (n-3)!}{2! \times (n-2)!} = 1$

$\Rightarrow \quad \dfrac{3 \times 2! \times (n-3)!}{2! \times (n-2)(n-3)!} = 1 \Rightarrow 3 = (n - 2)$

$\Rightarrow \quad n = 5$

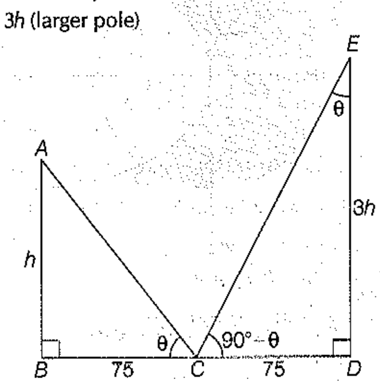
$\therefore$ Probability of getting an odd number, odd number of times

$$= P(1) + P(3) + P(5)$$
$$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$$
$$= \frac{16}{2^5} = \frac{16}{32} = \frac{1}{2}$$

19. *(d)* Given, distance between both poles = 150 m

Let $AB = h$ (smaller pole)

and $DE = 3h$ (larger pole)



$\angle ACB = \theta$

Then, $\angle ECD = 90° - \theta$

[$\because$ angles are complementary]

In ΔACB, $\tan\theta = \dfrac{h}{75}$

Also, in ΔECD, $\tan\theta = \dfrac{75}{3h}$

$\Rightarrow \quad \dfrac{h}{75} = \dfrac{75}{3h}$

$$\Rightarrow \quad h^2 = \frac{(75)^2}{3}$$

$$\therefore \quad h = \sqrt{\frac{75 \times 75}{3}} = 25\sqrt{3} \text{ m}$$

$\therefore$ Required height $= 25\sqrt{3}$ m

20. *(c)* Given, $[A \wedge (A \to B)] \to B$

$$= A \wedge (\sim A \vee B) \to B$$
$$= [(A \wedge \sim A) \vee (A \wedge B)] \to B$$
$$= (A \wedge B) \to B$$
$$= \sim A \vee \sim B \vee B$$
$$= t$$

Hence, $[A \wedge (A \to B)] \to B$ is a tautology.

21. *(10)* Given, $\alpha_{\text{least}} = p$

$$\alpha_{\max} = q$$

Equation given is $z + \alpha|z - 1| + 2i = 0$; $z \in c$ and $i = \sqrt{-1}$

Let $z = x + iy$

Then, $z + \alpha|z - 1| + 2i = 0$

$\Rightarrow x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$

$\Rightarrow (x + \alpha\sqrt{(x-1)^2 + y^2}) + i(y + 2) = 0$

$\therefore \quad y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$y = -2$ and $x^2 = \alpha^2(x^2 + 1 - 2x + y^2)$

$x^2 = \alpha^2(x^2 - 2x + 5) \qquad (\because y = -2)$

$$\Rightarrow \quad \alpha^2 = \frac{x^2}{x^2 - 2x + 5}$$

$$\therefore \quad \alpha^2 \in \left[0, \frac{5}{4}\right]$$

$$\therefore \quad \alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$$

Now, $4(p^2 + q^2) = 4[(\alpha_{\text{least}})^2 + (\alpha_{\max})^2]$

$$= 4\left[\left(-\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2\right]$$

$$= 4 \times \left[\frac{5}{4} + \frac{5}{4}\right] = 10$$

22. *(6)* Given, B_1, B_2 and B_3 are three independent events.

Let x, y, z be the probability of B_1, B_2, B_3, respectively.

$P(\text{only } B_1 \text{ occur}) = \alpha$

$P(B_1) \cdot P(\overline{B}_2) \cdot P(\overline{B}_3) = \alpha$

$\Rightarrow \quad x \cdot (1 - y) \cdot (1 - z) = \alpha$

$P(\text{only } B_2 \text{ occur}) = \beta$

$P(\overline{B}_1) \cdot P(B_2) \cdot P(\overline{B}_3) = \beta$

$\Rightarrow \quad (1 - x) \cdot y \cdot (1 - z) = \beta$

$P(\text{only } B_3 \text{ occur}) = \gamma$

$\Rightarrow \quad P(\overline{B}_1) \cdot P(\overline{B}_2) \cdot P(B_3) = \gamma$

$\Rightarrow \quad (1 - x) \cdot (1 - y) \cdot z = \gamma$

$P(\text{none occur}) = P$

$P(\overline{B}_1) \cdot P(\overline{B}_2) \cdot P(\overline{B}_3) = P$

$\Rightarrow \quad (1 - x) \cdot (1 - y) \cdot (1 - z) = P$

Now, we have given relations $(\alpha - 2\beta)P = \alpha\beta$

$\Rightarrow [x(1-y)(1-z) - 2y(1-x)(1-z)](1-x)(1-y)(1-z)$

$= x \cdot (1-y)(1-z) \cdot y(1-x)(1-z)$

[putting the value of α, β, P]

$\Rightarrow \quad (1 - z)[x(1-y) - 2y(1-x)] = x \cdot y \cdot (1 - z)$

$\Rightarrow \quad x - xy - 2y + 2xy = xy$

$\Rightarrow \quad x = 2y \qquad \text{... (i)}$

Similarly, on solving the second relation, $(\beta - 3\gamma)P = 2\beta\gamma$ by putting β, γ and P,

we get $y = 3z \qquad \text{... (ii)}$

From Eqs. (i) and (ii), we get

$$x = 2 \times 3z$$

$$\Rightarrow \quad x = 6z \Rightarrow \frac{x}{z} = 6$$

Now, $\dfrac{P(B_1)}{P(B_3)} = \dfrac{x}{z} = 6$

$\therefore$ Required answer is 6.

23. *(17)* Given, $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$

$$\Rightarrow \quad |P| = \begin{vmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{vmatrix} = (20 + 12\alpha)$$

According to the question,

$$PQ = kI_3 \Rightarrow Q = kP^{-1}I_3$$

Now, $Q = \dfrac{k}{|P|}(\text{adj } P)\, I_3$

$$= \frac{k}{(20 + 12\alpha)}\begin{bmatrix} 5\alpha & -10 & -\alpha \\ 3\alpha & 6 & -3\alpha - 4 \\ -10 & 12 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\because \quad q_{23} = -\frac{k}{8}$$

$$\therefore \quad \frac{k}{(20 + 12\alpha)}(-3\alpha - 4) = -\frac{k}{8}$$

$$\Rightarrow \quad 2(3\alpha + 4) = 5 + 3\alpha \Rightarrow 3\alpha = -3$$

$$\Rightarrow \quad \alpha = -1$$

Also, $|Q| = \dfrac{k^3\,|I|}{|P|}$

$$\Rightarrow \quad \frac{k^2}{2} = \frac{k^3}{20 + 12\alpha} \Rightarrow 20 + 12\alpha = 2k$$

$\Rightarrow \quad 2k = 20 - 12 \qquad [\because \alpha = -1]$

$\Rightarrow \quad 2k = 8$

$k = 4$

$\therefore$ Required value of $k^2 + \alpha^2 = 4^2 + (-1)^2 = 17$

24. *(540)* Given, M is a 3×3 matrix.

Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, then $M^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

Now, $M^TM = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \times \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Sum of diagonal elements

$= a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$ ...(i)

According to the question, the entries are $\{0, 1, 2\}$. [$\because M^TM = 7$]

i.e. $\{a, b, c, \dots, h, i\} = \{0, 1, 2\}$

So, for Eq. (i) to be true, there are two cases.

Case I When 7 – 1's are there and 2 – 0's are there.

$\Rightarrow \quad {}^9C_7 \times {}^2C_2 = 36$ ways of arrangements.

Case II When 1 – 2 is there, 3 – 1's and 5 – 0's are there.

$${}^9C_1 \times {}^8C_3 \times {}^5C_5 = 9 \times \frac{8!}{3!5!} \times 1$$

$= 504$ ways of arrangements.

$\therefore$ Total possible arrangements $= 36 + 504 = 540$

25. *(5)* Given, $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k + 2 : k \in N\}$

$c = \{9k + l : k \in N\}$

$\because$ 3 digit number of the form $3k + 2$ are

$\{101, 109, \dots 992\}$

$$\Rightarrow \quad \text{Sum} = \frac{100}{2}[101 + 992] = \frac{100 \times 1093}{2}$$

Similarly, 3-digit number of the form $9k + 5$ is

$$\frac{100}{2}[104 + 995] = \frac{100 \times 1099}{2}$$

[$\because$ numbers are 104, 113, ..., 995]

$$\text{Their sum} = \frac{100 \times 1093}{2} + \frac{100 \times 1099}{2}$$

$= 100 \times 1096 = 400 \times 274$

Hence, we can say the value of $l = 5$

as the second series of numbers obtained by set C is of the form $9k + 5$.

$\therefore$ Required value of $l = 5$

26. *(9)* Given, equation $\dfrac{4}{\sin x} + \dfrac{1}{1 - \sin x} = \alpha$

Let $f(x) = \dfrac{4}{\sin x} + \dfrac{1}{1 - \sin x}$

$$\Rightarrow \quad y = \frac{4 - 3\sin x}{\sin x(1 - \sin x)}$$

Let $\sin x = t$ when $t \in (0, 1)$ as $x \in \left(0, \dfrac{\pi}{2}\right)$. $\left[\begin{array}{l} \because t_1 = \sin 0 = 0 \\ t_2 = \sin\dfrac{\pi}{2} = 1 \end{array}\right]$

Now, $y = \dfrac{4 - 3t}{t - t^2}$

$$\frac{dy}{dt} = \frac{-3(t - t^2) - (1 - 2t)(4 - 3t)}{(t - t^2)^2} = 0$$

$\Rightarrow \; 3t^2 - 3t - (4 - 11t + 6t^2) = 0$

$\Rightarrow \quad 3t^2 - 8t + 4 = 0$

$\Rightarrow \quad 3t^2 - 6t - 2t + 4 = 0$

$\Rightarrow \quad 3t(t - 2) - 2(t - 2) = 0$

$\Rightarrow \quad (t - 2)(3t - 2) = 0$

$\therefore \quad t - 2 = 0$ or $3t - 2 = 0$

$t = 2$ or $t = 2/3$

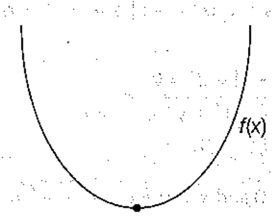
$\sin x = 2$ or $\sin x = \dfrac{2}{3}$

$x = \sin^{-1}\left(\dfrac{2}{3}\right)$ [$\because \sin\theta \le 1$]

Also, $y = \dfrac{4 - 3t}{t - t^2} = \dfrac{4 - 3 \times 2/3}{\dfrac{2}{3} - \left(\dfrac{2}{3}\right)^2}$ [$\because t = 2/3$]

$y = 9$

Now, required graph is



$\left[\sin^{-1}\left(\dfrac{2}{3}\right), 9\right]$

From the graph, minimum value of

$f(x) \ge 9$

$\alpha \ge 9$

$\therefore \alpha$ has minimum value 9.

27. *(3)* Given, $\int_{-a}^{a} (|x| + |x - 2|)\, dx = 22$

$\Rightarrow \int_{-a}^{0}(-2x + 2)\, dx + \int_0^2 (x + 2 - x)\, dx + \int_2^a (2x - 2)\, dx = 22$

$\Rightarrow (x^2 - 2x)\Big|_0^{-a} + (2x)\Big|_0^2 + (x^2 - 2x)\Big|_2^a = 22$

$\Rightarrow \quad a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$\Rightarrow \quad 2a^2 = 18$

$\Rightarrow \quad a^2 = 9$

$\Rightarrow \quad a = 3$

$\therefore \quad \int_a^{-a} x + [x]\, dx = \int_3^{-3} (x + [x])\, dx$

$= -\int_{-3}^{3} (x + [x])\, dx$

$= -[-3 - 2 - 1 + 1 + 2]$

$= -[-3] = 3$

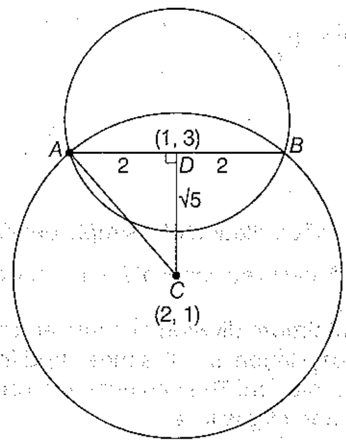
$\therefore$ Required value is 3.

28. *(3)* Given, circle $\Rightarrow x^2 + y^2 - 2x - 6y + 6 = 0$

Coordinate of $D = (1, 3)$

Radius $= r = \sqrt{1^2 + 3^2 - 6}$

$= \sqrt{4} = 2$ units



$CD = \sqrt{(2-1)^2 + (1-3)^2}$

$= \sqrt{1+4} = \sqrt{5}$

Now, by using Pythagoras theorem in ΔADC,

$AC^2 = AD^2 + CD^2$

$= (2)^2 + (\sqrt{5})^2$

$= 4 + 5$

$AC^2 = 9$

$AC = \sqrt{9} = 3$

$\therefore$ Required radius $= 3$

29. *(75)* Given, $\mathbf{c}$ is co-planar with $\mathbf{a}$ and $\mathbf{b}$

$\mathbf{a} \cdot \mathbf{c} = 7$

$\mathbf{b} \perp \mathbf{c} \Rightarrow \mathbf{b} \cdot \mathbf{c} = 0$

$\mathbf{a} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}$

Now, $\mathbf{c} = \lambda[\mathbf{b} \times (\mathbf{a} \times \mathbf{b})]$ [$\because$ $\mathbf{c}$ is coplanar with $\mathbf{a}$ and $\mathbf{b}$]

$= \lambda[(\mathbf{b} \cdot \mathbf{b})\, \mathbf{a} - (\mathbf{b} \cdot \mathbf{a})\, \mathbf{b}]$

$= \lambda[(\sqrt{5})^2(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - (-2+1)(2\hat{\mathbf{i}} + \hat{\mathbf{k}})]$

$= \lambda[5(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + 2\hat{\mathbf{i}} + \hat{\mathbf{k}}]$

$= \lambda(-3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

Now, $\mathbf{c} \cdot \mathbf{a} = 7$

$\Rightarrow \quad \lambda(-3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) \cdot (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 7$

$\Rightarrow \quad 3\lambda + 5\lambda + 6\lambda = 7$

$\Rightarrow \quad \lambda = \frac{7}{14} = \frac{1}{2}$

$\therefore 2\,|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2$

$= 2\left|\left(\frac{-3}{2} - 1 + 2\right)\hat{\mathbf{i}} + \left(\frac{5}{2} + 1\right)\hat{\mathbf{j}} + (3 + 1 + 1)\hat{\mathbf{k}}\right|^2$ [by putting $\mathbf{a}, \mathbf{b}, \mathbf{c}$]

$= 2\left(\frac{1}{4} + \frac{49}{4} + 25\right) = 25 + 50 = 75$

$\therefore$ Required value is 75.

30. *(1)* Given, $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$

$= \tan\left(\lim_{n\to\infty} \sum_{r=1}^{n} [\tan^{-1}(r+1) - \tan^{-1} r]\right)$

$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1) - \frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$

$= \tan\frac{\pi}{4} = 1$

Hence, the required value is 1.

JEE Main 2021

24 FEBRUARY SHIFT II

PHYSICS

Section A : Objective Type Questions

1. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is

a. circular **b.** elliptical
c. parabolic **d.** straight line

2. Two electrons each are fixed at a distance $2d$. A third charge proton placed at the mid-point is displaced slightly by a distance $x(x << d)$ perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency?
(m = mass of charged particle)

a. $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ **b.** $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$ **c.** $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ **d.** $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$

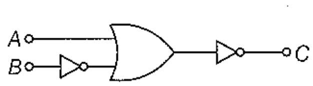
3. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules

a. continuously lose their energy till it reaches wall
b. are attracted by the walls of container
c. continuously stick to the walls of container
d. suffer change in momentum when impinge on the walls of container

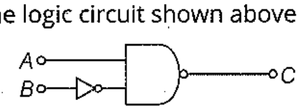
4. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains

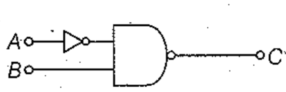
a. increase in size but no change in orientation
b. have no relation with external magnetic field
c. decrease in size and changes orientation
d. may increase or decrease in size and change its orientation

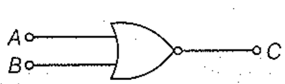
5.

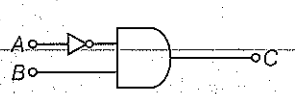


The logic circuit shown above is equivalent to

a.


b.


c.


d.


6. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 1.0 m from metre scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of g will be

a. 1.13% **b.** 1.03% **c.** 1.33% **d.** 1.30%

7. Given below are two statements:

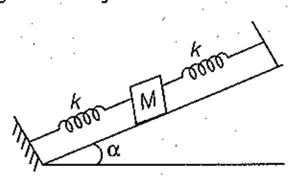
Statement I p-n junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

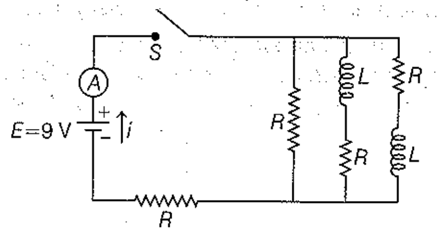
a. Statement I is false but Statement II is true.
b. Both Statement I and Statement II are true.
c. Both Statement I and Statement II are false.
d. Statement I is true but Statement II is false.

8. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, then the frequency of oscillation of given body is



a. $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ **b.** $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$
c. $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ **d.** $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

9. Figure shows a circuit that contains four identical resistors with resistance $R = 2.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH and an ideal battery with electromotive force $E = 9$ V. The current i just after the switch S is closed will be

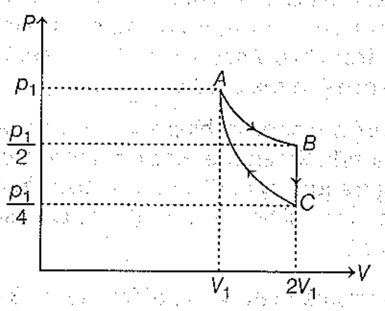


a. 2.25 A **b.** 3.0 A **c.** 3.37 A **d.** 9 A

10. The de-Broglie wavelength of a proton and α-particle are equal. The ratio of their velocities is

a. 4 : 3 **b.** 4 : 1 **c.** 4 : 2 **d.** 1 : 4

11. If one mole of an ideal gas at (p_1, V_1) is allowed to expand reversibly and isothermally (A to B), its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value ($B \to C$). Then, it is restored to its initial state by a reversible adiabatic compression (C to A). The net work done by the gas is equal to



a. $RT\left(\ln 2 - \frac{1}{2(\gamma-1)}\right)$ **b.** $-\frac{RT}{2(\gamma-1)}$

c. 0 **d.** $RT\ln 2$

12. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be

a. 10^{-3} nm **b.** 10^{-1} nm **c.** 10^{-2} nm **d.** 10^{-4} nm

13. Which of the following equations represents a travelling wave?

a. $y = A\sin(15x - 2t)$ **b.** $y = Ae^{-x^2}(vt + \theta)$

c. $y = Ae^{x}\cos(\omega t - \theta)$ **d.** $y = A\sin x \cos \omega t$

14. According to Bohr atom model, in which of the following transitions will the frequency be maximum ?

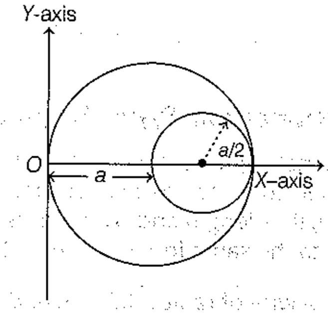
a. $n = 4$ to $n = 3$ **b.** $n = 2$ to $n = 1$

c. $n = 5$ to $n = 4$ **d.** $n = 3$ to $n = 2$

15. If the source of light used in a Young's double slit experiment is changed from red to violet, then

a. the consecutive fringe lines will come closer
b. the central bright fringe will become a dark fringe
c. the fringes will become brighter
d. the intensity of minima will increase

16. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius a as shown in figure. The centroid of the remaining circular portion with respect to point O will be



a. $\frac{1}{6}a$ **b.** $\frac{10}{11}a$

c. $\frac{5}{6}a$ **d.** $\frac{2}{3}a$

17. Zener breakdown occurs in a p-n junction having p and n both

a. lightly doped and have wide depletion layer
b. heavily doped and have narrow depletion layer
c. lightly doped and have narrow depletion layer
d. heavily doped and have wide depletion layer

18. Match List-I with List-II.

List-I		List-II
A. Source of microwave frequency	1.	Radioactive decay of nucleus
B. Source of infrared frequency	2.	Magnetron
C. Source of gamma rays	3.	Inner shell electrons
D. Source of X-rays	4.	Vibration of atoms and molecules
	5.	LASER
	6.	R-C circuit

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	6	4	1	5	**b.**	6	5	1	4
c.	2	4	6	3	**d.**	2	4	1	3

19. A particle is projected with velocity v_0 along X-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin, i.e. $ma = -\alpha x^2$. The distance at which the particle stops is

a. $\left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{2}}$ **b.** $\left(\frac{2mv_0}{3\alpha}\right)^{\frac{1}{3}}$

c. $\left(\frac{2mv_0^2}{3\alpha}\right)^{\frac{1}{2}}$ **d.** $\left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}$

20. A body weights 49 N on a spring balance at the North pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?
(Use, $g = \frac{GM}{R^2} = 9.8\ ms^{-2}$ and radius of earth, $R = 6400$ km)

a. 49 N   b. 48.83 N
c. 49.83 N   d. 49.17 N

Section B : Numerical Type Questions

21. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be .... cm.

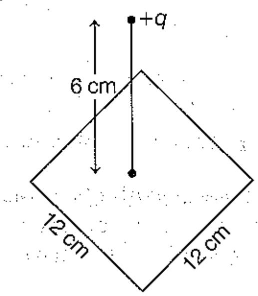
22. A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is ....

23. A uniform thin bar of mass 6 kg and length 2.4 m is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is .... $\times 10^{-1}$ kg-m^2.

24. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(KE)_A : (KE)_B$ will be $\frac{A}{1}$, so the value of A will be ....

25. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms^{-1}. The value of x will be ....

26. A point charge of + 12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be .... $\times 10^3$ N-m^2/C.



27. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is – 5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W. The value of x is ....
[Gain in dB $= 10 \log_{10}\left(\frac{P_0}{P_i}\right)$]

28. A series L-C-R circuit is designed to resonate at an angular frequency $\omega_0 = 10^5$ rad/s. The circuit draws 16 W power from 120 V source at resonance. The value of resistance R in the circuit is .... Ω.

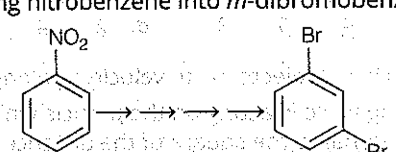
29. Two cars are approaching each other at an equal speed of 7.2 km/h. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be .... Hz. [Velocity of sound in air is 340 m/s.]

30. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be .... $\times 10^{-2}$ cm.

CHEMISTRY

Section A : Objective Type Questions

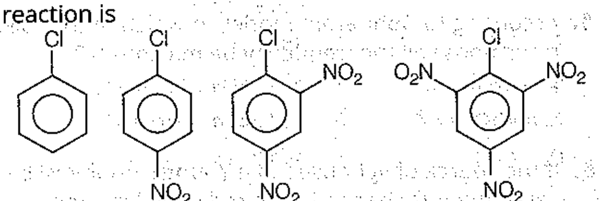
1. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene?



a. $\xrightarrow{NaNO_2}$ / $\xrightarrow{HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{H^+}$
b. $\xrightarrow{Br_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$
c. $\xrightarrow{Sn/HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{H^+}$
d. $\xrightarrow{Sn/HCl}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{NaNO_2}$ / $\xrightarrow{NaBr}$

2. Most suitable salt which can be used for efficient clotting of blood will be
a. $NaHCO_3$ b. $FeSO_4$ c. $Mg(HCO_3)_2$ d. $FeCl_3$

3. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is



(i) (ii) (iii) (iv)

a. (iv) < (iii) < (ii) < (i) b. (iv) < (i) < (ii) < (iii)
c. (iv) < (i) < (iii) < (ii) d. (i) < (ii) < (iii) < (iv)

4. According to Bohr's atomic theory,

I. kinetic energy of electron is $\propto \frac{Z^2}{n^2}$

II. the product of velocity (v) of electron and principal quantum number (n), $vn \propto Z^2$.

III. frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$

IV. coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$

Choose the most appropriate answer from the options given below.

a. Only III b. Only I c. I, III and IV d. I and IV

5. Match List-I with List-II.

	List-I	List-II
A.	$R-\overset{O}{\overset{\|}{C}}-Cl \longrightarrow R-CHO$	1. Br_2/ NaOH
B.	$R-CH_2-COOH \longrightarrow R-\underset{Cl}{\underset{\|}{C}H}-COOH$	2. H_2/ Pd-$BaSO_4$
C.	$R-\overset{O}{\overset{\|}{C}}-NH_2 \longrightarrow R-NH_2$	3. Zn(Hg) / Conc.HCl
D.	$R-\overset{O}{\overset{\|}{C}}-CH_3 \longrightarrow R-CH_2-CH_3$	4. Cl_2 / Red P, H_2O

Choose the correct answer from the options given below.

	A	B	C	D
a.	2	1	4	3
b.	3	4	1	2
c.	2	4	1	3
d.	3	1	4	2

6. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are

a. 5.82, 0 and 0 BM b. 4.90, 0 and 1.73 BM
c. 5.92, 4.90 and 0 BM d. 4.90, 0 and 2.83 BM

7. Match List-I with List-II.

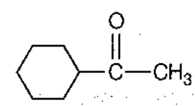
List-I (Salt)	List-II (Flame colour wavelength)
A. LiCl	1. 455.5 nm
B. NaCl	2. 670.8 nm
C. RbCl	3. 780.0 nm
D. CsCl	4. 589.2 nm

Choose the correct answer from the options given below.

	A	B	C	D
a.	4	2	3	1
b.	2	1	4	3
c.	1	4	2	3
d.	2	4	3	1

8. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ and H_2SO_4?

a. $CH_3-\overset{O}{\overset{\|}{C}}-H$

b. 

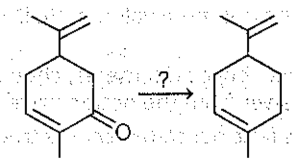
c. $CH_3-CH_2-\overset{O}{\overset{\|}{C}}-H$

d. $CH_3-\overset{O}{\overset{\|}{C}}-CH_2CH_3$

9. In polymer buna-S: 'S' stands for

a. sulphonation b. strength
c. sulphur d. styrene

10.



Which of the following reagent is suitable for the preparation of the product in the above reaction?

a. $NaBH_4$ b. NH_2-NH_2 / $C_2H_5\overset{\ominus}{O}\overset{\oplus}{Na}$
c. Ni / H_2 d. Red P + Cl_2

11. Match List-I and List-II.

List-I	List-II
A. Valium	1. Antifertility drug
B. Morphine	2. Pernicious anaemia
C. Norethindrone	3. Analgesic
D. Vitamin B_{12}	4. Tranquiliser

Choose the correct answer from the option given below.

	A	B	C	D
a.	4	3	2	1
b.	4	3	1	2
c.	2	4	3	1
d.	1	3	4	2

12. Match List-I with List-II.

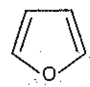
List I (Metal)	List II (Ores)
A. Aluminium	1. Siderite
B. Iron	2. Calamine
C. Copper	3. Kaolinite
D. Zinc	4. Malachite

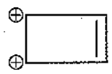
Choose the correct answer from the options given below.

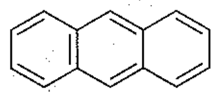
	A	B	C	D		A	B	C	D
a.	4	3	2	1	**b.**	2	4	1	3
c.	1	2	3	4	**d.**	3	1	4	2

13. Which one of the following compounds is non-aromatic ?

a. 

b. 

c. 

d. 

14. What is the correct order of the following elements with respect to their density ?

a. Cr < Zn < Co < Cu < Fe
b. Zn < Cu < Co < Fe < Cr
c. Zn < Cr < Fe < Co < Cu
d. Cr < Fe < Co < Cu < Zn

15. Given below are two statements.

Statement I The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below.

a. Statement I is false but statement II is true.
b. Both statements are true.
c. Statement I is true but statement II is false.
d. Both statements are false.

16. The incorrect statement among the following is

a. $VOSO_4$ is a reducing agent.
b. Cr_2O_3 is an amphoteric oxide.
c. RuO_4 is an oxidising agent.
d. Red colour of ruby is due to the presence of Co^{3+}.

17. The correct shape and I—I—I bond angles respectively in I_3^- ion are

a. distorted trigonal planar, 135° and 90°
b. T-shaped, 180° and 90°
c. Trigonal planar, 120°
d. Linear, 180°

18. Given below are two statements.

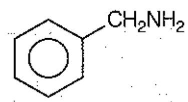
One is labelled as Assertion A and the other is labelled as Reason R.

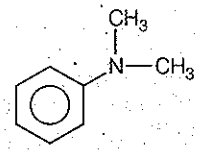
Assertion A Hydrogen is the most abundant element in the universe, but it is not the most abundant gas in the troposphere.

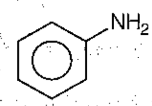
Reason R Hydrogen is the lightest element. In the light of the above statements, choose the correct answer from the options given below.

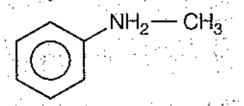
a. A is true but R is false.
b. Both A and R are true and R is the correct explanation of A.
c. A is false but R is true.
d. Both A and R are true but R is not the correct explanation of A.

19. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-naphthol in NaOH ?

a. 

b. 

c. 

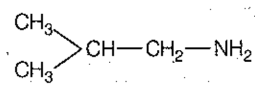
d. 

20. The correct set from the following in which both pairs are in correct order of melting point is

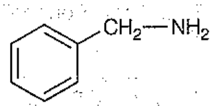
a. LiF > LiCl, MgO > NaCl
b. LiCl > LiF, NaCl > MgO
c. LiF > LiCl, NaCl > MgO
d. LiCl > LiF, MgO > NaCl

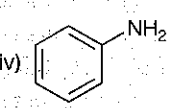
Section B : Numerical Type Questions

21. The total number of amines among the following which can be synthesised by Gabriel synthesis is ....

(i) 

(ii) $CH_3CH_2NH_2$

(iii) 

(iv) 

22. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is ....

(i) α-sulphur (ii) β-sulphur (iii) S_2-form

23. The formula of a gaseous hydrocarbon, which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is ....

24. The volume occupied by 4.75 g of acetylene gas at 50°C and 740 mm Hg pressure is ......L (Rounded off to the nearest integer).
[Given, $R = 0.0826$ L atm K^{-1} mol^{-1}]

25. C_6H_6 freezes at 5.5°C. The temperature at which a solution 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is ....°C. (The molal freezing point depression constant of C_6H_6 is 5.12°C/m.)

26. The magnitude of the change in oxidising power of the MnO_4^-/Mn^{2+} couple is $x \times 10^{-4}$ V, if the H^+ concentration is decreased from 1 M to 10^{-4} M at 25°C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is .... (Rounded off to the nearest integer).

$$\left[\text{Given, } \frac{2.303\,RT}{F} = 0.059\right]$$

27. The solubility product of PbI_2 is 8.0×10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is .... (Rounded off to the nearest integer).
[Given, : $\sqrt{2} = 1.41$]

28. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25°C. After 9 h, the fraction of sucrose remaining is f. The value of $\log_{10}(1/f)$ is ______ $\times 10^{-2}$ (Rounded off to the nearest integer).
[Assume, $\ln 10 = 2.303$, $\ln 2 = 0.693$]

29. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is ______ $\times 10^{-2}$.

30. Assuming ideal behaviour, the magnitude of $\log K$ for the following reaction at 25°C is $x \times 10^{-1}$. The value of x is .... (Integer answer)

$3HC \equiv CH(g) \rightleftharpoons C_6H_6(l)$

[Given, $\Delta_f G^\circ(HC \equiv CH) = -2.04 \times 10^5$ J mol^{-1}, $\Delta_f G^\circ(C_6H_6) = -1.24 \times 10^5$ J mol^{-1}, $R = 8.314$ J K^{-1} mol^{-1}]

MATHEMATICS

Section A : Objective Type Questions

1. For the statements p and q, consider the following compound statements

A. $[\sim q \wedge (p \to q)] \to \sim p$

B. $[(p \vee q) \wedge \sim p] \to q$

Then, which of the following statement(s) is/are correct?

a. (A) and (B) both are not tautologies.
b. (A) and (B) both are tautologies.
c. (A) is a tautology but not (B).
d. (B) is a tautology but not (A).

2. Let $a, b \in R$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a + b|$ is equal to

a. 88 **b.** 86
c. 84 **d.** 90

3. The vector equation of the plane passing through the intersection of the planes $\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$ and $\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = -2$ and the point $(1, 0, 2)$ is

a. $\mathbf{r} \cdot (\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = \frac{7}{3}$ **b.** $\mathbf{r} \cdot (3\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 7$
c. $\mathbf{r} \cdot (\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 7$ **d.** $\mathbf{r} \cdot (\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = \frac{7}{3}$

4. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the coordinates of P are

a. (3, 13) **b.** (1, 5) **c.** (−2, 8) **d.** (2, 8)

5. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 s at the speed of 432 km/h, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is

a. $1800\sqrt{3}$ m **b.** $3600\sqrt{3}$ m
c. $2400\sqrt{3}$ m **d.** $1200\sqrt{3}$ m

6. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2)$ is

a. $\frac{n(n-1)(2n+1)}{6}$

b. $\frac{n(n+1)(2n+1)}{6}$

c. $\frac{n(2n+1)(3n+1)}{6}$

d. $\frac{n(n+1)^2(n+2)}{12}$

7. Let $f: R \to R$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \leq x < 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x \geq 4 \end{cases}$$

Let $A = \{x \in R : f \text{ is increasing}\}$. Then, A is equal to

a. $(-\infty, -5) \cup (4, \infty)$
b. $(-5, \infty)$
c. $(-\infty, -5) \cup (-4, \infty)$
d. $(-5, -4) \cup (4, \infty)$

8. Let f be a twice differentiable function defined on R, such that $f(0) = 1$, $f'(0) = 2$ and $f''(x) \neq 0$ for all $x \in R$.

If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of $f(1)$ lies in the interval

a. (9, 12) **b.** (6, 9)
c. (0, 3) **d.** (3, 6)

9. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

a. $x^2 + y^2 = 7$ **b.** $y^2 = \frac{1}{6\sqrt{3}}x$
c. $2x^2 - 18y^2 = 9$ **d.** $x^2 + 9y^2 = 9$

10. The value of the integral $\int_1^3 [x^2 - 2x - 2]\, dx$, where $[x]$ denotes the greatest integer less than or equal to x, is

a. $-\sqrt{2}-\sqrt{3}+1$ b. $-\sqrt{2}-\sqrt{3}-1$
c. -5 d. -4

11. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

a. $1/\sqrt{7}$ b. $2\sqrt{2}-1$
c. $\sqrt{7}-1$ d. $1/2\sqrt{2}$

12. The negative of the statement $\sim p \wedge (p \vee q)$ is

a. $\sim p \vee q$ b. $p \vee \sim q$
c. $\sim p \wedge q$ d. $p \wedge \sim q$

13. If the curve $y = ax^2 + bx + c, x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are

a. $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$ b. $a = 1, b = 0, c = 1$
c. $a = 1, b = 1, c = 0$ d. $a = -1, b = 1, c = 1$

14. The area of the region $R = \{(x, y): 5x^2 \le y \le 2x^2 + 9\}$ is

a. $11\sqrt{3}$ square units
b. $12\sqrt{3}$ square units
c. $9\sqrt{3}$ square units
d. $6\sqrt{3}$ square units

15. If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)\, dx = \frac{62}{5}$?

a. 5 b. 10
c. 62.5 d. 31/5

16. Let $f(x)$ be a differentiable function defined on [0, 2], such that $f'(x) = f'(2-x)$, for all $x \in (0, 2)$, $f(0) = 1$ and $f(2) = e^2$. Then, the value of $\int_0^2 f(x)\, dx$ is

a. $1 - e^2$ b. $1 + e^2$
c. $2(1 - e^2)$ d. $2(1 + e^2)$

17. Let A and B be 3×3 real matrices, such that A is symmetric matrix and B is skew-symmetric matrix. Then, the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has

a. no solution
b. exactly two solutions
c. infinitely many solutions
d. a unique solution

18. Let a, b, c be in an arithmetic progression. Let the centroid of the triangle with vertices (a, c), $(2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is

a. $\frac{71}{256}$ b. $\frac{69}{256}$ c. $-\frac{69}{256}$ d. $-\frac{71}{256}$

19. For the system of linear equations $x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R$, consider the following statements

(A) The system has unique solution, if $k \ne 2, k \ne -2$.
(B) The system has unique solution, if $k = -2$.
(C) The system has unique solution, if $k = 2$.
(D) The system has no solution, if $k = 2$.
(E) The system has infinite number of solutions, if $k \ne -2$.

Which of the following statements are correct?

a. (C) and (D) b. (B) and (E)
c. (A) and (E) d. (A) and (D)

20. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is

a. $65/2^7$ b. $65/2^8$
c. $135/2^9$ d. $35/2^7$

Section B : Numerical Type Questions

21. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \ge r \ge 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum, $\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to ....

22. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ....

23. If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \ne 0$, then the value of expression $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is ....

24. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (−5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ....

25. If the area of the triangle formed by the positive X-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5, 7) is A, then $24A$ is equal to ....

26. If the variance of 10 natural numbers 1, 1, 1,....., 1, k is less than 10, then the maximum possible value of k is ....

27. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1 and the third term is α, then 2α is ....

28. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then, the total number of possibilities of forming such groups is ....

29. Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$ and $n = [\,|k|\,]$ be the greatest integral part of $|k|$. Then, $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to ....

30. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is ....

Answers

Physics

1. *(b)*	2. *(c)*	3. *(d)*	4. *(d)*	5. *(d)*	6. *(a)*	7. *(a)*	8. *(c)*	9. *(a)*	10. *(b)*
11. *(a)*	12. *(a)*	13. *(a)*	14. *(b)*	15. *(a)*	16. *(c)*	17. *(b)*	18. *(d)*	19. *(d)*	20. *(b)*
21. *(2)*	22. *(5)*	23. *(8)*	24. *(2)*	25. *(400)*	26. *(226)*	27. *(8)*	28. *(900)*	29. *(8)*	30. *(667)*

Chemistry

1. *(b)*	2. *(d)*	3. *(d)*	4. *(d)*	5. *(c)*	6. *(b)*	7. *(d)*	8. *(c)*	9. *(d)*	10. *(b)*
11. *(b)*	12. *(d)*	13. *(a)*	14. *(c)*	15. *(c)*	16. *(d)*	17. *(d)*	18. *(b)*	19. *(c)*	20. *(a)*
21. *(3)*	22. *(1)*	23. *(8)*	24. *(5)*	25. *(1)*	26. *(3776)*	27. *(141)*	28. *(81)*	29. *(243)*	30. *(855)*

Mathematics

1. *(b)*	2. *(a)*	3. *(c)*	4. *(d)*	5. *(d)*	6. *(b)*	7. *(d)*	8. *(b)*	9. *(d)*	10. *(b)*
11. *(a)*	12. *(b)*	13. *(c)*	14. *(b)*	15. *(b)*	16. *(b)*	17. *(c)*	18. *(d)*	19. *(d)*	20. *(c)*
21. *(*)*	22. *(1)*	23. *(2)*	24. *(56.25)*	25. *(1225)*	26. *(11)*	27. *(3)*	28. *(31650)*	29. *(310)*	30. *(2)*

Note *(*) None of the option is correct.*

Solutions

PHYSICS

1. *(b)* Since, the particle is executing SHM.

Therefore, displacement equation of wave will be

$$y = A\sin\omega t$$

$$\Rightarrow \quad y/A = \sin\omega t$$

and wave velocity equation will be

$$v_y = \frac{dy}{dt} = A\omega\cos\omega t$$

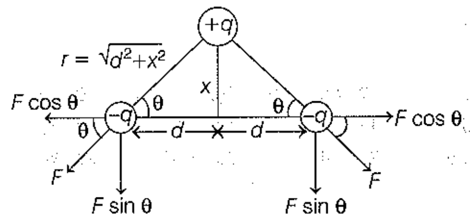
$$\Rightarrow \quad v_y/A\omega = \cos\omega t$$

Now, $\sin^2\omega t + \cos^2\omega t = 1$

$$\therefore \quad (y/A)^2 + (v_y/A\omega)^2 = 1$$

This equation is similar to the equation of ellipse.

2. *(c)* The arrangement of charges is shown below



As we know that,

Coulomb's force between two charges. i.e. q_1 and q_2,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{(d^2+x^2)} \quad \text{...(i)}$$

Here, $q_1 = q_2 = q$

Force in SHM, $F = m\omega^2 x$ ...(ii)

Since, in order to have SHM $+q$ should move downwards and force responsible for this will be only

$$F' = F\sin\theta + F\sin\theta = 2F\sin\theta \quad \text{...(iii)}$$

Using Eqs. (ii) and (iii), we get

$$2F\sin\theta = m\omega^2 x$$

$$\Rightarrow \quad \frac{2}{4\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)}\sin\theta = m\omega^2 x$$

$$\Rightarrow \quad \frac{2}{4\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)}\cdot\frac{x}{(d^2+x^2)^{1/2}} = m\omega^2 x$$

$$\Rightarrow \quad \omega = \left(\frac{1}{2\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)^{3/2}m}\right)^{1/2}$$

As, $x << d$

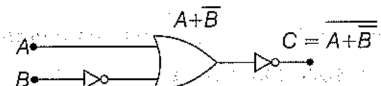
$$\therefore \quad \omega = \left(\frac{1}{2\pi\varepsilon_0}\frac{q^2}{md^3}\right)^{1/2}$$

3. *(d)* On the basis of kinetic theory of gases, the gas exerts pressure because its molecules contain uniform speed, random motion and perform elastic collision with each other, as well as with the walls of container. As a result of which gaseous molecules suffer change in momentum when impinge on the walls of container.

4. *(d)* The magnetic susceptibility (χ) of ferromagnetic material is in the range of 10^3 to 10^4, so it gets easily magnetised and demagnetised in the presence of external field region.

Therefore, when a soft ferromagnetic material is placed in an external magnetic field region, the magnetic domains may increase or decrease in size and change its orientation.

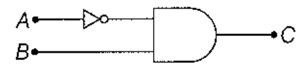
5. *(d)* The logic circuit is given as



By using De-morgan's theorem,

$$C = \overline{A+\overline{B}} = \overline{A}\cdot\overline{\overline{B}} = \overline{A}\cdot B$$

This relation can be shown by the circuit drawn below



6. *(a)* Given, $T = 2\pi\sqrt{\frac{L}{g}}$ ... (i)

where, time period, $T = 1.95$ s

Length of string, $l = 1$ m

Acceleration due to gravity $= g$

Error in time period, $\Delta T = 0.01\text{ s} = 10^{-2}$ s

Error in length, $\Delta L = 1\text{ mm} = 1\times10^{-3}$ m

Squaring Eq. (i) on both sides, we get

$$T^2 = 4\pi^2\frac{L}{g}$$

$$\Rightarrow \quad g = 4\pi^2\frac{L}{T^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} = \frac{10^{-3}}{1} + \frac{2\times10^{-2}}{1.95}$$

$$= 10^{-3} + 1.025\times10^{-2}$$

$$= 10^{-3} + 10.25\times10^{-3}$$

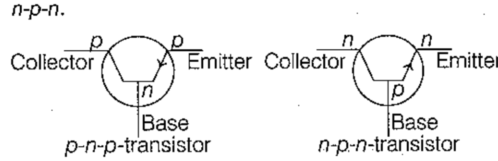
$$= 11.25\times10^{-3}$$

$$\because \quad \Delta g/g\times100 = 11.25\times10^{-3}\times10^2$$

$$= 1.125\% \simeq 1.13\%$$

7. *(a)* Transistor is a device used as switch or amplifier.

It is made by sandwiching three semiconductors, i.e. *p-n-p* and *n-p-n*.



Hence, Statement I is false, because we cannot make transistor from diode.

As we know that, amplification factor (β) is the ratio of collector current to base current.

$$\therefore \quad \beta = \frac{I_C}{I_B}$$

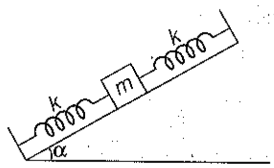
Hence, Statement II is true.

Therefore, option (a) is correct.

8. *(c)* Let T be the time period of oscillation, then

$$T = 2\pi\sqrt{\frac{M}{k_{eq}}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{M}{2k}} \qquad [\because k_{eq} = k + k]$$



and frequency $(f) = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

9. *(a)* Given, resistance, $R = 2\Omega$,

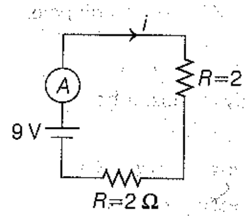
Inductance, $L = 2$ mH,

emf, $E = 9$ V

and i be the current.

$\because$ At $t = 0$ when switch is closed, inductors behave as open circuit.

$\therefore$ Effective circuit will be



By using Ohm's law, $V = i\, R_{eq}$

$$\Rightarrow \quad i = V/R_{eq}$$

where, R_{eq} is equivalent resistance of series resistors,

i.e., $\quad R_{eq} = R + R = 2R = 2 \times 2 = 4\Omega$

$$\therefore \quad i = \frac{9}{4} = 2.25 \text{ A}$$

10. *(b)* Let $\lambda_p, \lambda_\alpha, m_p, m_\alpha, v_p, v_\alpha, p_p$ and p_α be the wavelength, mass, velocity and momentum of proton and α-particle, respectively.

Given, $\lambda_p = \lambda_\alpha$

As we know that,

$$\lambda = h/p$$

$$\therefore \quad \frac{h}{p_p} = \frac{h}{p_\alpha}$$

$$\Rightarrow \quad p_p = p_\alpha$$

$$\Rightarrow \quad m_p v_p = m_\alpha v_\alpha$$

$$\Rightarrow \quad m_p v_p = 4m_p v_\alpha \qquad (\because m_\alpha = 4m_p)$$

$$\Rightarrow \quad \frac{v_p}{v_\alpha} = \frac{4}{1} \text{ or } 4:1$$

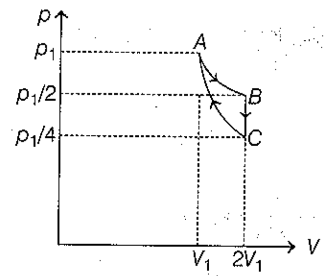
11. *(a)* Let p_i, p_f, V_i and V_f be the initial and final pressure and volume.

Given, AB is isothermal $(\Delta T = 0)$,

BC is isochoric $(\Delta V = 0)$

and CA is adiabatic $(\Delta Q = 0)$

Since, isothermal work $(W_{AB}) = p_1 V_1 \ln\frac{V_f}{V_i}$



where, V_i and V_f are volume at A and B, respectively.

$$\therefore \quad W_{AB} = p_1V_1 \ln\frac{2V_1}{V_1} = p_1V_1 \ln 2$$

Since, at constant volume, work done is zero.

$$\therefore \quad W_{BC} = 0$$

Since, W_{CA} is an adiabatic work done, i.e.

$$W_{CA} = \frac{1}{1-\gamma}(p_fV_f - p_iV_i)$$

$$\Rightarrow \quad W_{CA} = \frac{1}{1-\gamma}\left(p_1V_1 - \frac{p_1}{4} \times 2V_1\right)$$

$$= \frac{1}{1-\gamma}(p_1V_1 - p_1V_1/2) = \frac{1}{1-\gamma}\,\frac{p_1V_1}{2}$$

$\therefore$ Net work done, $W_{net} = W_{AB} + W_{BC} + W_{CA}$

$$= p_1V_1 \ln 2 + 0 + \frac{1}{1-\gamma}\,\frac{p_1V_1}{2}$$

$$= p_1V_1[\ln 2 + 1/2(1-\gamma)]$$

From ideal gas law, $pV = nRT$

$\therefore W_{net} = RT[\ln 2 - 1/2\,(\gamma - 1)] \qquad (\because n = 1)$

12. *(a)* Given, $V = 1.24$ million volt $= 1.24 \times 10^6$ volt

Since, energy $(E) = eV$

where, e is the charge of electron $= 1.6 \times 10^{-19}$ C

$$\therefore \quad E = 1.6 \times 10^{-19} \times 1.24 \times 10^6 \qquad \ldots (i)$$

As we know that,

Energy of photon, $\quad E = \frac{hc}{\lambda} \qquad \ldots (ii)$

Here, Planck's constant, $h = 6.67 \times 10^{-34}$ J-s,

c = speed of light in free space, $c = 3 \times 10^8$ ms^{-1}

Equating Eqs. (i) and (ii), we get

$$1.6 \times 10^{-19} \times 1.24 \times 10^6 = \frac{6.67 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{20.01 \times 10^{-13}}{1.6 \times 1.24} = 10.09 \times 10^{-13}$$

$$= 1.009 \times 10^{-12} \simeq 10^{-3} \times 10^{-9}$$

$$= 10^{-3} \text{ nm}$$

13. *(a)* The equation of a travelling wave in standard form,

$$y = A\sin(\omega t \pm kx)$$

Only option (a), i.e. $y = A\sin(15x - 2t)$ satisfies this equation.

14. *(b)* Let n_f, n_i be the final and initial orbit.

As we know that,

$$\frac{1}{\lambda} = 1.09 \times 10^7 \left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

Now, checking for each option, we get

(a) $\frac{1}{\lambda} \propto \left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \left[\frac{1}{9} - \frac{1}{16}\right] = 0.05$ ... (i)

(b) $\frac{1}{\lambda} \propto \left[\frac{1}{1} - \frac{1}{4}\right] = 0.75$ ... (ii)

(c) $\frac{1}{\lambda} \propto \left[\frac{1}{16} - \frac{1}{25}\right] = 0.0225$ ... (iii)

(d) $\frac{1}{\lambda} \propto \left[\frac{1}{4} - \frac{1}{9}\right] = 0.14$ ... (iv)

The option (b) has highest value.

Since, frequency, $f = \frac{c}{\lambda} \Rightarrow f \propto \frac{1}{\lambda}$

$\therefore$ Frequency will be maximum for transition $n = 2$ to $n = 1$.

15. *(a)* According to Young's double slit experiment,
The distance of nth bright fringe from the centre,

$$y_n = \frac{n\lambda D}{d}$$

Since, $\lambda_{\text{violet}} < \lambda_{\text{red}}$

$\therefore$ $y_{\text{violet}} < y_{\text{red}}$

$\therefore$ Consecutive fringe lines will come closer.

16. *(c)* Given, radius of hole, $r = a/2$
and radius of disc, $R = a$
Let x_{CM} be the centre of mass of system,
m_1, x_1 be the mass and centre of mass of disc
m_2, x_2 be the mass and centre of mass of circular hole.

$\therefore$ $m_2 = \frac{m_1}{\pi R^2}\pi r^2$

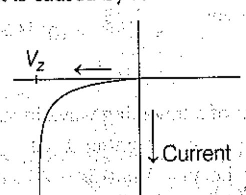
$\Rightarrow$ $m_2 = \frac{m_1 r^2}{R^2} = \frac{m_1 (a/2)^2}{a^2} = \frac{m_1}{4}$

and $x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$\therefore$ $x_{CM} = \frac{m_1 a - (m_1/4)(3a/2)}{m_1 - (m_1/4)}$

$\Rightarrow$ $x_{CM} = \frac{m_1 a(1 - 3/8)}{3m_1/4} \Rightarrow x_{CM} = \frac{5a}{6}$

17. *(b)* As we know that, Zener breakdown takes place, when we supply reverse bias voltage to Zener diode. Due to heavily dopping, the electrons in the valence band of p-type region can jump easily to the conduction band of n-type region, hence due to high electric field, zener breakdown occur. Thus, there is very high sudden increase in Zener current (I_Z) that is caused by reverse breakdown voltage (V_Z).



Hence, Zener breakdown is easily observed in Zener diode which is heavily doped and having narrow depletion region.

18. *(d)* As we know that;

A. Source of microwave frequency is magnetron.

B. Source of infrared frequency is vibration of atoms and molecules.

C. Source of gamma ray is radioactive decay of nucleus.

D. Source of X-ray is transition of electron in inner shells.

$\therefore$ The correct match is A $\rightarrow$ (2), B $\rightarrow$ (4), C $\rightarrow$ (1), D $\rightarrow$ (3).

19. *(d)* Given, speed of projection $= v_0$

Damping force, $F = ma = -\alpha x^2$

$\Rightarrow$ $a = -\alpha x^2 / m$

Also, $a = v\frac{dv}{dx}$

$\Rightarrow$ $v dv = a\,dx = -\frac{\alpha}{m} x^2 dx$

Integrating both sides, we get

$$\int_{v_0}^{v} v\,dv = \int_0^x -\frac{\alpha}{m} x^2 dx$$

$\Rightarrow$ $\left(\frac{v^2}{2}\right)_{v_0}^{0} = -\frac{\alpha}{m}\left(\frac{x^3}{3}\right)_0^x$

$\Rightarrow$ $0 - v_0^2/2 = -\frac{\alpha}{m}\frac{x^3}{3} \Rightarrow x = \left(\frac{3m}{2}\frac{v_0^2}{\alpha}\right)^{1/3}$

20. *(b)* Given, weight of body at North pole,
$w_p = mg = 49$ N
Radius of Earth, $R = 6400$ km
Let weight of body at equator be w_e.
At equator, $g_e = g - R\omega^2$

$\therefore$ $w_e = mg_e = m(g - R\omega^2)$

Since, $w_p > w_e \Rightarrow w_e < 49$ N
Hence, above condition is satisfied by only option (b).

21. *(2)* Let initial length and diameter be l_1 and d_1, whereas final length and diameter be l_2 and d_2.
Given, $l_2 = 2l_1, d_2 = 2d_1, \Delta l_1 = 0.04$ m
By using formula of Young's modulus of elasticity,

$$Y = \frac{F \cdot l}{A\Delta l}$$

$\because$ $Y_1 = Y_2$

$\Rightarrow$ $\frac{Fl_1}{A_1 \times \Delta l_1} = \frac{Fl_2}{A_2 \times \Delta l_2}$

$\Rightarrow$ $\frac{Fl_1}{\pi(d_1/2)^2 \times 0.04} = \frac{F2l_1}{\pi(2d_1/2)^2 \Delta l_2}$

$\Rightarrow$ $\frac{1}{1/4 \times 0.04} = \frac{2}{\Delta l_2}$

$\Rightarrow$ $\Delta l_2 = 0.02 \text{ m} = 2 \text{ cm}$

22. *(5)* Given, radius of cylindrical wire, $r = 0.5 \text{ mm} = 0.5 \times 10^{-3}$ m
Conductivity, $\sigma = 5 \times 10^7$ S/m
Electric field, $E = 10$ mV/m $= 10 \times 10^{-3}$ V/m
We know that current density,

$\therefore$ $J = \sigma E$
$= 5 \times 10^7 \times 10 \times 10^{-3} = 5 \times 10^5 \text{A/m}^2$

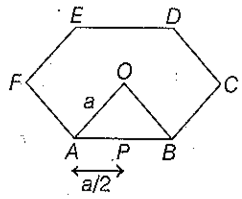
Also, $J = I/A \Rightarrow I = JA$

$\Rightarrow$ $I = 5 \times 10^5 \times \pi \times (0.5 \times 10^{-3})^2$
$= 5 \times 10^5 \times \pi \times 25 \times 10^{-8} = 125\pi \times 10^{-3}$

$\Rightarrow$ $x^3\ \pi\text{mA} = 125\ \pi \text{ mA} \Rightarrow x^3 = 5^3$

$\Rightarrow$ $x = 5$

23. *(8)* Given, mass of uniform bar, $m = 6$ kg
Length of bar, $l = 2.4$ m



Side of hexagon, $a = \frac{2.4}{6} = 0.4$ m

Mass of each side, $m' = 6/6 = 1$ kg

and $OP = \sqrt{a^2 - a^2/4} = a\sqrt{3}/2$

$= 0.2\sqrt{3}$m

Now, by using parallel axes theorem,

$$I_{OP} = \frac{m'a^2}{12} + m'\left(\frac{\sqrt{3}a}{2}\right)^2$$

$$= \frac{a^2}{12} + \frac{3a^2}{4} = \frac{10a^2}{12} = \frac{5a^2}{6}$$

$\Rightarrow$ $I_{OP} = \frac{5}{6} \times 0.4 \times 0.4$

and I_{net} (net moment of inertia at O)

$$= 6 \times \frac{5}{6} \times 0.4 \times 0.4 = 8 \times 10^{-1} \text{ kg-m}^2$$

24. *(2)* Given, $m_A = 1$ kg, $m_B = 2$ kg, $(KE)_A : (KE)_B = A : 1$

Linear momentum of A and B are equal.

$\Rightarrow$ $p_A = p_B$

$\because$ Kinetic energy (KE) $= p^2/2m$

$\therefore$ $\frac{KE_A}{KE_B} = \frac{m_B}{m_A} = \frac{2}{1} = \frac{A}{1} \Rightarrow A = 2$

25. *(400)* Given, $T_1 = 27°C = 27 + 273 = 300$ K,

$p_1 = 1$ atm, $v_1 = 200 \text{ ms}^{-1}$, $T_2 = 127°C = 400$ K,

$p_2 = 2$ atm, $v_2 = ?$

As we know that,

Root mean square speed, $v_{rms} = \sqrt{\frac{3RT}{m}}$

$\therefore$ $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$

$\Rightarrow$ $v_2 = \sqrt{\frac{4}{3}} v_1 = \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}} \text{ ms}^{-1}$

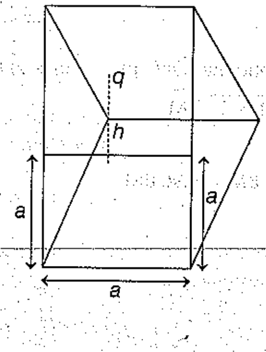
$\Rightarrow$ $\frac{x}{\sqrt{3}} = \frac{400}{\sqrt{3}} \Rightarrow x = 400$

26. *(226)* Given, charge, $q = 12\ \mu C = 12 \times 10^{-6}$ C

Height of charge from surface, $h = 6$ cm $= 6 \times 10^{-2}$ m

and side of square, $a = 12$ cm $= 12 \times 10^{-2}$ m

From figure, it is clear that the given square is one of the face of a cube of side 12 cm and + 12μC charge is placed at its centre. Then, by Gauss's theorem,



Flux through any face, $\phi = \frac{q}{6\varepsilon_0}$

$\Rightarrow$ $\phi = \frac{12 \times 10^{-6}}{6 \times 8.854 \times 10^{-12}} = 0.226 \times 10^6 \text{ N-m}^2/\text{C}$

$= 226 \times 10^3 \text{ N-m}^2/\text{C}$

27. *(8)* Given, power of transmitted signal, $P_i = 0.1$ kW $= 0.1 \times 10^3$ W $= 10^2$ W

Rate of attenuation, $R = -5$ dB/km

Length of cable, $l = 20$ km

Power received at receiver, $P_x = 10^{-x}$ W

Total loss, $\beta = R \times l = -5 \times 20 = -100$ dB

$\because$ Gain (β) $= 10 \log_{10} \frac{P_0}{P_i}$

$\therefore$ $\beta = -100 = -10 \log_{10} \frac{P_0}{P_i}$

$\Rightarrow$ $-10 = \log_{10} \frac{P_0}{P_i} \Rightarrow 10^{-10} = \frac{P_0}{P_i}$

$\Rightarrow$ $P_0 = 10^{-10} P_i = 10^{-10} \times 10^2 = 10^{-8} \Rightarrow P_0 = 10^{-8}$ W

Hence, $x = 8$

28. *(900)* Given, angular frequency at resonance, $\omega_0 = 10^5 \text{rads}^{-1}$

Power drawn from circuit, $P = 16$ W

and supply voltage, $V = 120$ V

Let resistance of circuit $= R$.

As, $P = V^2/R$

$\Rightarrow$ $R = V^2/P = \frac{120 \times 120}{16}$

$= 30 \times 30 = 900\ \Omega$

29. *(8)* Given, $v_A = v_B = 7.2 \text{ kmh}^{-1}$

$= \frac{72}{10} \times \frac{5}{18} = 2 \text{ ms}^{-1}$

Frequency of source, $f_s = 676$ Hz

Speed of sound in air, $v = 340 \text{ ms}^{-1}$

Let f_0 be the frequency heard by each driver.

By using Doppler effect for A,

$(v - v_A) f_s = (v + v_B) f_0$

$\Rightarrow$ $f_0 = \left(\frac{v + v_A}{v - v_B}\right) f_s = \left(\frac{340 + 2}{340 - 2}\right) 676 = \frac{342}{338} \times 676 = 684$ Hz

Now, beat frequency $= f_0 - f_s = 684 - 676 = 8$ Hz

30. *(667)* Given, frequency of wave, $f = 3$ GHz $= 3 \times 10^9$ Hz

Relative permittivity, $\varepsilon_r = 2.25$

Since, $f = c/\lambda$

$\Rightarrow$ $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1$ m

$\because \lambda_m$ (wavelength of wave in a medium) $= \lambda/\mu$

and as we know that, $\mu = \sqrt{\mu_r \varepsilon_r}$

As, dielectric is non-magnetic, $\mu_r = 1$

$\Rightarrow$ $\mu = \sqrt{2.25} = 1.5$

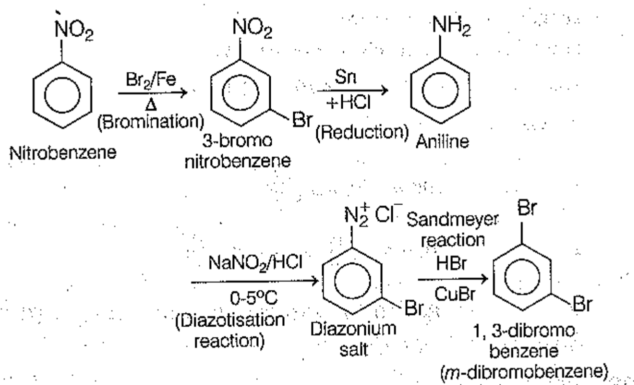
$\Rightarrow$ $\lambda_m = \frac{0.1}{1.5} = \frac{1}{15} = 0.0667$ m

$= 6.67$ cm $= 667 \times 10^{-2}$ cm

CHEMISTRY

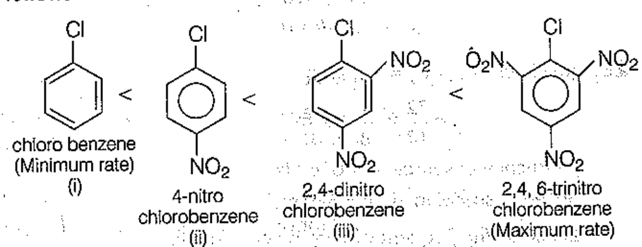
1. (*b*) In first step, NO_2 group is electron withdrawing group, it decreases the electron density in *meta* position and bromination occur in *meta* position. In second step, Sn/HCl used for reduction to convert $—NO_2$ to $—NH_2$ and form aniline. In next step, diazonium salt is formed and at last bromine replace $N_2^+Cl^-$ group to form 1, 3-dibromobenzene.

Complete reaction is as follows



2. (*d*) According to Hardy-Schulze rule, for negatively charged sol, most (+ve) charged ion is needed for efficient coagulation. Blood is a negatively charged sol. Hence $FeCl_3$ can be used for blood clotting and it from Fe^{3+} ion.

3. (*d*) Rate of nucleophilic substitution in aromatic halide are as follows



Greater the number of nitro ($—NO_2$) group in chlorobenzene, greater the rate of nucleophilic substitution reaction in aromatic halides.

When a nitro group is present at *ortho* and *para* position, it can withdraw electrons from the benzene ring. This facilitates, the attack of nucleophiles on haloarenes.

4. (*d*) According to Bohr's theory,

I. $KE \propto \frac{Z^2}{n^2}$ or $13.6 \propto \frac{Z^2}{n^2}\frac{(eV)}{(atom)}$, ($\because$ Correct)

II. Speed of electron $\propto \frac{Z}{n}$

(Here, Z = atomic number, n = number of shells)

$\therefore \quad v \times n \propto Z$ ($\because$ Incorrect)

III. Frequency of revolution of electron $= \frac{v}{2\pi r}$

Frequency $\propto \frac{Z^2}{n^3}$ $\left(\because v \propto \frac{z}{n}, r \propto \frac{n^2}{z}\right)$ ($\because$ Incorrect)

IV. $F = \frac{kq_1q_2}{r^2} = \frac{kZe^2}{r^2}$

$$F = \frac{Z}{\left(\frac{n^2}{Z}\right)^2}$$

$$F \propto \frac{Z^3}{n^4}$$ ($\because$ Correct)

Hence, only I, and IV are correct.

5. (*c*) (A) Alkyl chloride reacts with H_2/Pd-$BaSO_4$ and reduced to alkyl aldehyde. This is known as Rosenmund reduction.

$$\underset{\text{Alkyl chloride}}{R—\overset{\overset{\Large O}{||}}{C}—Cl} \xrightarrow[\text{Pd-BaSO}_4]{H_2} \underset{\text{Alkylal dehyde}}{R—\overset{\overset{\Large O}{||}}{C}—H}$$

(Rosenmund reduction)

(B) Carboxylic acid reacts with Cl_2/P in aqueous medium to form 2-chlorocarboxylic acid. This reaction is known as HVZ reaction.

$$\underset{\text{Alkyl carboxylic acid}}{R—CH_2—COOH} \xrightarrow[H_2O]{Cl_2/P} \underset{\text{2-chlorocarboxylic acid}}{R—\underset{\underset{Cl}{|}}{CH}—COOH}$$

[Hell-Volhard-Zelinsky (HVZ) reaction]

(C) Alkyl amide reacts with Br_2 in presence of NaOH to give alkyl amine. This reaction is known as Hofmann bromide reaction,

$$\underset{\text{Alkyl amide}}{R—\overset{\overset{\Large O}{||}}{C}—NH_2} \xrightarrow[NaOH]{Br_2} \underset{\text{Alkyl amine}}{R—NH_2}$$

(Hofmann bromide reaction)

(D) Ketone reacts with Zn-Hg in presence of conc.HCl to give alkane. This reaction is known as clemmensen reduction.

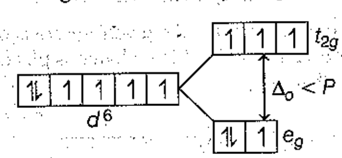
$$\underset{\text{Ketone}}{R—\overset{\overset{\Large O}{||}}{C}—CH_3} \xrightarrow[\text{Conc. HCl}]{\text{Zn(Hg)}} \underset{\text{Alkane}}{R—CH_2—CH_3}$$

(Clemmenson reduction)

Hence, correct match is (A)–2, (B)–4, (C)–1, (D)–3.

6. (*b*) (i) $[FeCl_4]^{2-} \longrightarrow Fe^{2+} \longrightarrow [Ar]\,3d^6$

$\therefore$ Cl is weak field ligand so does not pairing occur.



So, magnetic moment $(\mu) = \sqrt{n(n+2)}$ BM

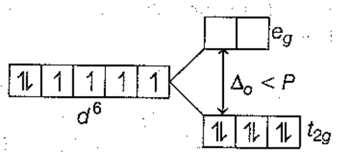
$= \sqrt{4(4+2)}$ BM

(n = Number of total unpaired e^- = 4)

$= \sqrt{24}$ BM = 4.90 BM

(ii) $[Co(C_2O_4)_3]^{3-} \longrightarrow Co^{3+} \longrightarrow [Ar]\,3d^6$

C_2O_4 is strong field ligand so pairing occur.



All electrons are paired, $n = 0$

hence, $\mu = 0$

(iii) $MnO_4^{2-} \rightarrow Mn^{+6} \rightarrow [Ar]3d^1$

$n = 1$

$= \sqrt{n(n+2)}$ BM

$= \sqrt{1(1+2)}$ BM

$= \sqrt{3}$ BM

$= 1.73$ BM

7. (*d*)

Alkali metal	**Colour (Flame)**	**λ (nm)**
Li	Crimson red	670.8
Na	Yellow	589.2
Rb	Red, violet	780.0
Cs	Blue	455.5

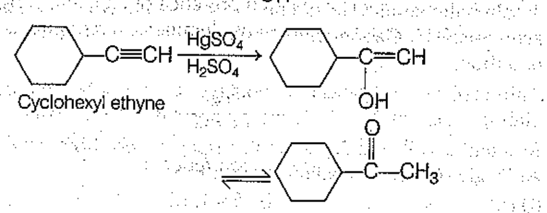
Alkali metals have very low value of ionisation energy as compared to other metals. So, alkali metals easily get excited and impart colour to flame.

Hence, Rb is most excited and having high value of wavelength in all alkali metals.

8. (*c*) Reaction of $HgSO_4$/dil. H_2SO_4 with alkyne result in addition of water as per Markownikoff's rule.

(a) $\underset{\text{Acetylene}}{HC{\equiv}CH} \xrightarrow[H_2SO_4]{HgSO_4} CH_2{=}\underset{OH}{\underset{|}{CH}} \rightleftharpoons \underset{\text{Acetaldehyde}}{CH_3{-}\overset{O}{\overset{\|}{CH}}}$

(b)



Cyclohexyl ethyne

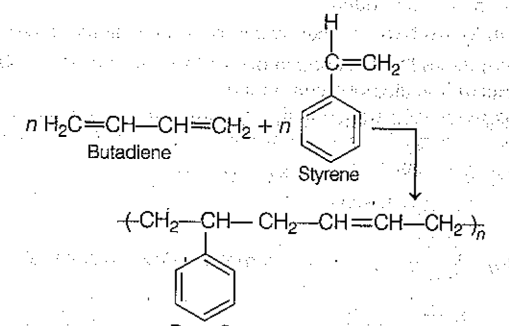
1-cyclo hexylethan-1-one

(c) $\underset{\text{Prop-1-yne}}{CH_3{-}C{\equiv}CH} \xrightarrow[H_2SO_4]{HgSO_4} CH_3{-}\underset{OH}{\underset{|}{C}}{=}CH_2 \rightleftharpoons \underset{\text{Acetone}}{CH_3{-}\overset{O}{\overset{\|}{C}}{-}CH_3}$

Hence $CH_3{-}CO{-}CH_3$ cannot be form.

(d) $\underset{\text{But-2-yne}}{CH_3C{\equiv}C{-}CH_3} \xrightarrow[H_2SO_4]{HgSO_4} CH_3{-}\underset{OH}{\underset{|}{C}}{=}CH{-}CH_3 \rightleftharpoons \underset{\text{But-2-one}}{CH_3{-}\overset{O}{\overset{\|}{C}}{-}CH_2{-}CH_3}$

9. (*d*) Buna-S is the co-polymer of but 1, 3-diene and styrene as follows.



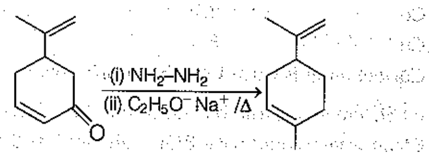
Styrene butadiene rubber is also called buna-S, in which 'Bu' stands for butadiene, 'na' stands for sodium and 'S' stands for styrene.

Since, butadiene and styrene is one of the constituent monomer of given polymer.

10. (*b*) To reduce the carbonyl groups into alkane, wolff-Kishner reduction is used, without affecting the double bond.

Wolff-Kishner reagent It utilises hydrazine ($NH_2{-}NH_2$) as the reducing agent in the presence of strong base KOH or $C_2H_5\bar{O}Na^+$ in a high boiling protic solvent.

The driving force for the reaction is the conversion of hydrazine to nitrogen gas.



11. (*b*) (A) Valium – (4) Tranquilizer

A tranquilizer drug became a standard drug for the treatment of anxiety and one of most commonly prescribed drugs of all time.

(B) Morphine – (3) Analgesic

Morphine is effective for both acute and chronic pain and often used before and after surgery.

(C) Norethindrone – (1) Antifertility drug

It is a form of progesterone, a female hormone important for regulating ovulation and menstruation.

(D) Vitamin B_{12}–(2) Pernicious anaemia

It is a nutrient that helps to keep our body blood cells and nerve cells healthy and help in making DNA.

12. (*d*) (A) Siderite $\rightarrow FeCO_3$

Siderite is a mineral composed of iron (II) carbonate ($FeCO_3$).

(B) Calamine $\rightarrow ZnCO_3$

Calamine is used to treat mild itchiness and contain zinc in its formula.

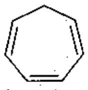
(C) Kaolinite $\rightarrow Al_2(OH)_4 \cdot Si_2O_5$

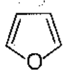
Kaolinite is a type of clay found in nature. It is a dioctahedral pyrosilicate clay containing Si_2O_5.

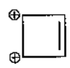
(D) Malachite $\rightarrow Cu(OH)_2 \cdot CuCO_3$

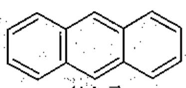
Malachite is a copper carbonate hydroxide mineral, with the formula $Cu_2CO_3(OH)_2$.

13. (*a*) Compound, which obey Huckel rule, $(4n+2)\pi$ is called aromatic compound.
Compound, which obey $4n\pi$ rule, is called anti-aromatic compound.
Compound which contain one or more sp^3 carbon in its structure is called non-aromatic compound.
The nature of following ions/compounds are as follows :

(a)  sp^3 Carbon $\longrightarrow$ Non-aromatic

(b)  $(6e^-)$ $\longrightarrow$ Aromatic (follow Huckel's rule)

(c)  $(2e^-)$ $\longrightarrow$ Aromatic (follow Huckel's rule)

(d)  $\longrightarrow$ Anthracene $\longrightarrow$ Aromatic
$(14e^-)$

Only (a) is non-aromatic and rest all are aromatic.

14. (*c*) Generally, due to decrease in metallic radius and increase in atomic mass density increase across the period from left to right.

Metal	**Density (g/cm^3)**
Zn	7.13
Cr	7.19
Fe	7.8
Co	8.7
Cu	8.9

Correct order is Cu > Co > Fe > Cr > Zn.

15. (*c*) Statement I is true but statement II is false.
Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.
Hence, the value of parameter 'BOD' is important for survival of aquatic life but optimum value of BOD is 17 ppm or more. So, statement II is incorrect.

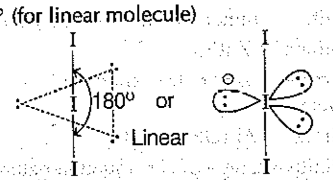
16. (*d*) Red colour of ruby is due to presence of Cr^{3+} ions in Al_2O_3.
Chromium is the trace element that causes ruby's red colour, which ranges from an orange red to a publish red. The strength of ruby's red depends on how much chromium is present.

17. (*d*) Hybridisation of central I in I_3^- is sp^3d with 3 lone pair and 2 bond pair.

Shape Linear $\left[:\ddot{\underset{..}{I}}-\ddot{\underset{..}{I}}-\ddot{\underset{..}{I}}:\right]^-$

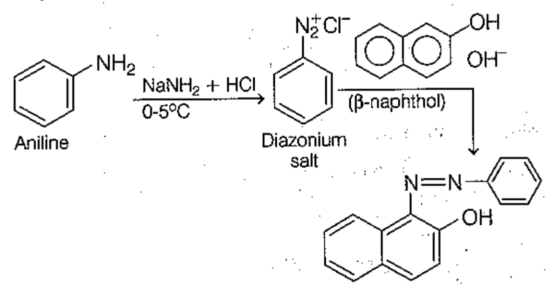
Lone pair 3 lone pair

Bond angle 180° (for linear molecule)



18. (*b*) Both A and R are true and R is the correct explanation of A.
As we know, hydrogen is most abundant element in the universe but the most abundant gas in the troposphere is nitrogen, because hydrogen is lightest element.
Troposphere contains 3-quarters-of-mass-of-the-entire-atmosphere. The air here contain 78% nitrogen and 21% oxygen. The last 1% is made of argon, water vapour and carbon dioxide.

19. (*c*) Initially aniline reacts with diazotisation reagent to form diazonium salt. Then β-naphthol react with salt and orange-red dye is obtained. So, diazonium salt of aniline is used to prepare orange-red dye.



2-naphthol aniline
Orange-red dye

20. (*a*) Correct option is (a) i.e. LiF > LiCl; MgO > NaCl. Melting point is directly proportional to lattice energy. Lattice energy is the energy required to separate a mole of an ionic solid into gaseous ions.
It depends upon charge of ions and size of ions.

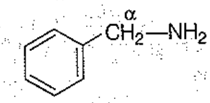
$$\text{M.P} \propto \text{L.E} \propto \frac{\text{Charge}}{\text{Size}}$$

$Li \rightarrow +1$ $\quad$ $Li \rightarrow +1$
$F = -1$ $\quad$ $Cl \rightarrow -1$

Both LiF and LiCl having same charge, so melting point will depend on size.
Larger the size of anion, lesser the lattice energy and hence, melting point order is LiF > LiCl.
Similarly, MgO $\quad$ NaCl
$Mg \rightarrow 2+$ $\quad$ $Na \rightarrow 1+$
$O \rightarrow 2-$ $\quad$ $Cl \rightarrow 1-$
MgO having +2 charge which is greater than NaCl (+1) charge. So, greater the charge on the ions greater will be lattice energy and hence, melting point order is MgO > NaCl.

21. (*3*) Gabriel phthalimide synthesis is used to prepare 1° aliphatic or alicyclic amine. Hence, amine which can be synthesised by Gabriel phthalimide synthesis method contains α-carbon.
Aniline ($C_6H_5NH_2$) does not contain α-C cannot be prepared by Gabriel reaction.
Rest amines all contain α-C in its respective position, hence they can easily give Gabriel phthalimide reaction.

$(CH_3)_2CH-\overset{\alpha}{C}H_2-NH_2$; $CH-\overset{\alpha}{C}H_2-NH_2$;



∴ Three amins out of given four anions can be prepared by Gabriel synthesis.

22. (*1*) Only S_2-form of sulphur is paramagnetic in nature. Because S_2 is like O_2 i.e. paramagnetic as per molecular orbital theory. It contains unpaired electron. While α-sulphur and β-sulphur are diamagnetic as they do not have unpaired electron.

23. (*8*) Combustion reaction :

$$C_xH_y(g) + \left(x+\frac{y}{4}\right)O_2(g) \longrightarrow xCO_2(g) + \frac{y}{z}H_2O(l)$$

Suppose, volume of C_xH_y is V and volume of O_2 is 6 times greater than $C_xH_y = 6\,V$
then volume of $xCO_2 \Rightarrow Vx = 4\,V$
$x = 4$

Since, $V_{O_2} = 6 \times V_{C_xH_y}$

$$V\left(x+\frac{y}{4}\right) = 6\text{ V}$$

$$\left(x+\frac{y}{4}\right) = 6 \quad \ldots \text{(i)}$$

Put value of $x = 4$ in Eq. (i)

We get, $4 + \frac{y}{4} = 6 \Rightarrow y = 8$

24. (*5*) Given, mass of $C_2H_2(g) = 4.75$ g

Molecular weight = 26 g/mol

Temperature = 50 + 273 = 323 K

Pressure = 740 torr/mm of Hg

$$\text{Pressure} = \frac{740}{760}\text{ atm}$$

$$R = 0.0821 \text{ L atm mol}^{-1}\text{ K}^{-1}$$

Hence, no. of mole $n = \frac{4.75}{26}$ mol

Formula used, $pV = nRT$ (ideal gas)

$$\Rightarrow \quad V = \frac{nRT}{p} = \frac{4.75}{26} \times \frac{0.0821 \times 323}{(740/760)}$$

$$= \frac{96314.078}{19240} = 5.0059 \text{ L} = 5 \text{ L}$$

25. (*1*) Pure solvent : $C_6H_6(l)$

Given, $T_f^\circ = 5.5°$C

$K_f = 5.12°$ C/m $\Rightarrow m = 200$ g

$m_{\text{solute}} = 10$ g

Molar mass of solute $C_4H_{10} = 12 \times 4 + 10 = 58$

Solute (C_4H_{10}) is non-dissociative;

$\therefore \quad i = 1$

$\therefore \quad \Delta T_f = iK_f m$

$$\Rightarrow \quad (T_f^\circ - T_f^1) = 1 \times 5.12 \times \frac{(10/58)}{(200/1000)}$$

$$5.5 - T_f^1 = \frac{5.12 \times 5 \times 10}{58} \Rightarrow T_f^1 = 1.086°\text{C}$$

or $\quad T_f^1 \approx 1°$C

26. (*3776*) Reaction, $MnO_4^- + H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$

$n = 5$

Applying Nernst equation, $E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.0591}{n}\log\frac{[P]}{[R]}$

or $\quad E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]}\left[\frac{1}{H^+}\right]^8$

(I) Given, $[H^+] = 1$ M

$$E_1 = E^\circ - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]}$$

(II) Now, $[H^+] = 10^{-4}$ M

$$E_2 = E^\circ - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]} \times \frac{1}{(10^{-4})^8}$$

$\therefore \ |E_1 - E_2|$

$$|E_1 - E_2| = \frac{0.0591}{5} \times 32 = 0.3776 \text{ V} = 3776 \times 10^{-4}$$

$x = 3776$

27. (*141*) Given, $[K_{sp}]_{PbI_2} = 8 \times 10^{-9}$

To calculate solubility of PbI_2 in 0.1 M solution of $Pb(NO_3)_2$.

(I) $Pb(NO_3)_2 \longrightarrow Pb^{2+}(aq) + 2NO_3^-(aq)$

0.1 M $\quad$ 0.1 M $\quad$ 0.2 M

(II) $PbI_2(s) \rightleftharpoons Pb^{2+}(aq) + 2I^-(aq)$

$\qquad\qquad S \qquad 2S$

$\therefore \quad [Pb^{2+}] = S + 0.1 \approx 0.1$

$\because \quad S << 0.1$

Now, $K_{sp} = 8 \times 10^{-9}$

$$[Pb^{2+}][I^-]^2 = 8 \times 10^{-9}$$

$$0.1 \times (2S)^2 = 8 \times 10^{-9}$$

$$4S^2 = 8 \times 10^{-8} \Rightarrow S = 141 \times 10^{-6} \text{ M}$$

$x = 141$

28. (*81*) Given,

$$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{1st order}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

$$t_{1/2} = \frac{10}{3}\text{ h}$$

$t = 0, \quad a = [A]_0$ (initial conc.)

At $\quad t = 9$ h $\ a - x = [A]_t$ [conc. at time t]

For using 1st order equation,

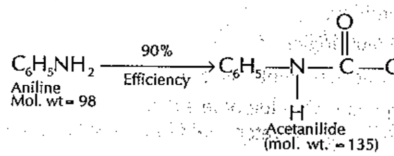
$$K = \frac{2.303}{t}\log\frac{[A]_0}{[A]_t} \Rightarrow \frac{K \times t}{2.303} = \log\frac{[A]_0}{[A]_t}$$

$$\frac{\ln 2 \times 9}{10/3 \times 2.303} = \log\left(\frac{1}{F}\right) \Rightarrow \log\left(\frac{1}{F}\right) = 0.8124 \qquad \left(\because k = \frac{\ln 2}{t_{1/2}}\right)$$

$$\log\left(\frac{1}{F}\right) = 81.24 \times 10^{-2}$$

$x = 81.24$ or $x \approx 81$

29. (*243*) Reaction



Given, weight = 1.86 g

Here, 1 mole of aniline gives 1 mole of acetanilide

$\therefore$ mole of aniline = mole of acetanilide

$$\Rightarrow \quad \frac{1.86}{93} = \frac{W_{\text{Acetanilide}}}{135}$$

$$W_{\text{Acetanilide}} = \frac{1.86 \times 135}{93}\text{ g} = 2.70 \text{ g}$$

But efficiency of reaction is 90% only.

Hence, mass of acetanilide produced

$$= 2.70 \times \frac{90}{100}\text{ g} = 2.43 \text{ g} = 243 \times 10^2 \text{ g}$$

$x = 243$

30. (*855*) Reaction, $\underset{\text{Acetylene}}{3HC \equiv CH(g)} \longrightarrow \underset{\text{Benzene}}{C_6H_6(l)}$

Given, $\Delta G_f^\circ = (CH \equiv CH) = -2.04 \times 10^5$ J mol^{-1}

$\Delta G_f^\circ(C_6H_6) = -1.24 \times 10^5$ J mol^{-1}

Gibb's free energy, $\Delta G^\circ = -nRT\ln K$

$$\Delta G^\circ = \Sigma(\Delta G_f^\circ)_P - \Sigma(\Delta G_f^\circ)_R$$

$$-nRT\ln K = -n'RT\ln K_p - (-n''RT\ln K_r)$$

$$\Rightarrow \quad -RT\ln K = 1 \times (-1.24 \times 10^5) - (-3 \times 2.04 \times 10^5)$$

$$-2.303 \times R \times T\log K = 4.88 \times 10^5$$

$$\log K = -\frac{4.88 \times 10^5}{2.303 \times 8.314 \times 273}$$

$$n\ln K = n'\ln K_p - (-n''\ln K_r)$$

$$K = 85.52 \Rightarrow K = 855 \times 10^{-1}$$

$x = 855$

MATHEMATICS

1. *(b)* Given statements,

(A) $[\sim q \wedge (p \rightarrow q] \rightarrow \sim p$

(B) $[(p \vee q) \wedge \sim p] \rightarrow q$

For statement (A),

$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$
T	F	T
F	F	T
T	F	T
T	T	T

p	q	$\sim p$	$\sim q$
T	T	F	F
F	T	T	F

p	q	$\sim p$	$\sim q$
T	F	F	T
F	F	T	T

$\therefore$ Statement (A) is tautology.

For statement (B),

$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \rightarrow q$
T	F	T
T	F	T
T	T	T
F	F	T

$\therefore$ Statement (B) is tautology.

$\therefore$ (A) and (B) both are tautologies.

2. *(a)* Given, $P(a, 6, 9)$

Equation of line $\dfrac{x-3}{7} = \dfrac{y-2}{5} = \dfrac{z-1}{-9}$

Image of point P with respect to line is point $Q(20, b, -a-9)$

Mid-point of P and $Q = \left(\dfrac{a+20}{2}, \dfrac{6+b}{2}, \dfrac{-a}{2}\right)$

This point lies on line

$$\therefore \quad \frac{\frac{a+20}{2}-3}{7} = \frac{\frac{6+b}{2}-2}{5} = \frac{\frac{-a}{2}-1}{-9}$$

$$\Rightarrow \quad \frac{a+14}{14} = \frac{b+2}{10} = \frac{a+2}{18}$$

$$\Rightarrow \quad \frac{a+14}{14} = \frac{a+2}{18} \text{ and } \frac{b+2}{10} = \frac{a+2}{18}$$

Solving, we get $\quad a = -56, b = -32$

$\therefore \quad |a+b| = |-56-32| = 88$

3. *(c)* Given, point $(1, 0, 2)$

Equation of plane =

$$\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1 \text{ and } \mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = -2$$

Equation of plane passing through the intersection of given planes is

$$[\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1] + \lambda [\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2] = 0$$

$\because$ This plane passes through point $(1, 0, 2)$ i.e.,

vector $(\hat{\mathbf{i}} + 2\hat{\mathbf{k}})$

$\therefore \quad [(\hat{\mathbf{i}} + 2\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1] + \lambda [(\hat{\mathbf{i}} + 2\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2] = 0$

$\Rightarrow \quad (3-1) + \lambda(1+2) = 0$

$\Rightarrow \quad 2 + \lambda \times 3 = 0$

$\Rightarrow \quad \lambda = -2/3$

Hence, equation of required plane is

$$[\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1] + \left(\frac{-2}{3}\right)[\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2] = 0$$

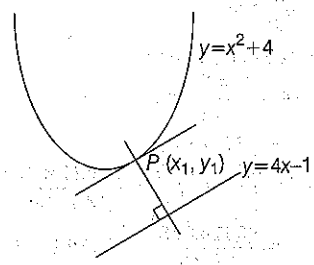
or $\quad 3[\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 1] - 2[\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2] = 0$

or $\quad \mathbf{r} \cdot (\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 7$

4. *(d)* Given, curve $y = x^2 + 4$

and, line $y = 4x - 1$

Here, $y = x^2 + 4$



$\therefore \quad \dfrac{dy}{dx} = 2x \quad \text{...(i)}$

and $y = 4x - 1$

$\dfrac{dy}{dx} = 4 \quad \text{... (ii)}$

Let the required point be $P(x_1, y_1)$.

$\therefore \quad \left.\dfrac{dy}{dx}\right|_P = 2x_1 \quad \text{... (iii)}$

$\because$ Slopes will be equal.

$\therefore \quad 2x_1 = 4 \quad$ [from Eqs. (ii) and (iii)]

$\Rightarrow \quad x_1 = \dfrac{4}{2} = 2$

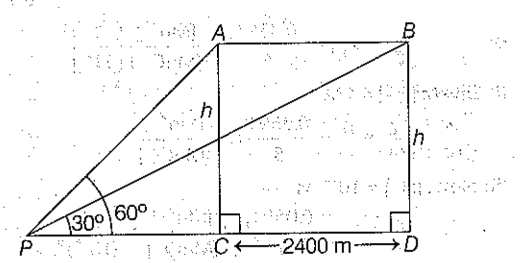
Now, the given point $P(x_1, y_1)$ lies on curve $y = x^2 + 4$,

$\therefore \quad y_1 = x_1^2 + 4$

$\Rightarrow \quad y_1 = 2^2 + 4 = 8$

Hence, required coordinate of $P = (2, 8)$

5. *(d)* Given, angle of elevation are 60° and then 30°.



Also, in 20 sec plane covers the distance from A to B with speed 432 km/h.

$\because \quad 432 \times \dfrac{5}{18}$ m/sec $= 120$ m/sec

$\therefore \quad AB =$ distance $= S \times T = 120 \times 20$

$= 2400$ m

In ΔPBD

$$\tan 30° = \frac{BD}{PD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PD}$$

$$\Rightarrow \quad PD = h\sqrt{3}$$

In ΔPAC

$$\tan 60° = \frac{AC}{PC} \Rightarrow \sqrt{3} = \frac{h}{PC}$$

$$\Rightarrow \quad PC = h/\sqrt{3}$$

Now, $CD = PD - PC$

$$2400 = h\sqrt{3} - \frac{h}{\sqrt{3}} \qquad [\because AB = CD]$$

$$\Rightarrow \quad 2400 = h\left(\frac{3-1}{\sqrt{3}}\right)$$

$$\therefore \quad h = \frac{2400 \times \sqrt{3}}{2} = 1200 \times \sqrt{3}$$

$\therefore$ Required height $= 1200\sqrt{3}$ m

6. *(b)* Given, $n \geq 2$

Let $S = {}^2C_2 + {}^3C_2 + \dots + {}^nC_2 = {}^{n+1}C_3$

Now, ${}^{n+1}C_2 + 2 \times ({}^2C_2 + {}^3C_2 + \dots + {}^nC_2)$

$$= {}^{n+1}C_2 + 2 \times {}^{n+1}C_3$$

$$= ({}^{n+1}C_2 + {}^{n+1}C_3) + {}^{n+1}C_3$$

$$= {}^{n+2}C_3 + {}^{n+1}C_3 = \frac{(n+2)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$$

$$= \frac{(n+2)(n+1)n(n-1)!}{3 \times 2 \times 1 \times (n-1)!} + \frac{(n+1) \times n \times (n-1) \times (n-2)!}{3 \times 2 \times 1 \times (n-2)!}$$

$$= \frac{n(n+1)}{6}[n+2+n-1]$$

$$= \frac{n(n+1)(2n+1)}{6}$$

7. *(d)* Given,

$$f(x) = \begin{cases} -55x &, \quad x < -5 \\ 2x^3 - 3x^2 - 120x &, \quad -5 \leq x < 4 \\ 2x^3 - 3x^2 - 36x + 10 &, \quad x \geq 4 \end{cases}$$

$$\therefore \quad f'(x) = \begin{cases} -55 &, \quad x < -5 \\ 6(x^2 - x - 20) &, \quad -5 \leq x < 4 \\ 6(x^2 - x - 6) &, \quad x \geq 4 \end{cases}$$

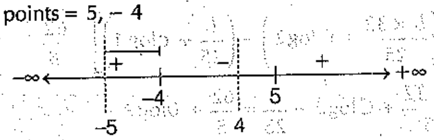
$$f'(x) = \begin{cases} -55 &, \quad x < -5 \\ 6(x-5)(x+4) &, \quad -5 \leq x < 4 \\ 6(x-3)(x+2) &, \quad x \geq 4 \end{cases}$$

For f to be increasing, $f'(x) > 0$

Now, $f'(x) = -55$ is always less than zero.

$f'(x) = 6(x-5)(x+4) < 0, -5 \leq x < 4$

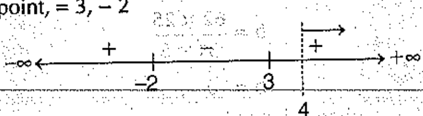
Critical points $= 5, -4$



$$[x \in (-5, -4) \qquad \dots \text{(i)}$$

and $f'(x) = 6(x-3)(x+2) < 0, x \geq 4$

Critical point, $= 3, -2$



$$x \in (4, \infty) \qquad \dots \text{(ii)}$$

From Eqs. (i) and (ii), $f(x)$ is increasing in $x \in (-5, -4) \cup (4, \infty)$

8. *(b)* Given, $f(0) = 1$,

$$f'(0) = 2,$$

$$f''(x) \neq 0$$

$$\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$$

$$\Rightarrow \quad f(x)f''(x) - f'(x)f'(x) = 0$$

$$\Rightarrow \quad \frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$$

$$\Rightarrow \quad \int \frac{f''(x)}{f'(x)}dx = \int \frac{f'(x)}{f(x)}dx$$

$$\Rightarrow \quad \log f'(x) = \log f(x) + \log c \qquad \dots \text{(i)}$$

or $\quad f'(x) = cf(x)$

Now, put $x = 0$, we get

$$f'(0) = cf(0)$$

$$\Rightarrow \quad 2 = c \times 1$$

$$\Rightarrow \quad c = 2$$

Putting the value of $c = 2$ in Eq. (i), we get

$$\log f'(x) = \log f(x) + \log 2$$

$$\Rightarrow \quad f'(x) = 2f(x) \Rightarrow \int \frac{f'(x)}{f(x)}dx = \int 2dx$$

$$\Rightarrow \quad \log f(x) = 2x + D \Rightarrow f(x) = e^{2x+D}$$

$$\Rightarrow \quad f(x) = e^D \cdot e^{2x}$$

$$\Rightarrow \quad f(x) = k \cdot e^{2x} \qquad [\text{Let } k = e^D]$$

Put $x = 0$, we get

$$f(0) = k.e^0$$

$$\Rightarrow \quad 1 = k \Rightarrow f(x) = k \cdot e^{2x}$$

$$\therefore \quad f(x) = e^{2x}$$

Put $x = 1$, we get

$$f(1) = e^2$$

Clearly, e^2 lies in (6, 9).

9. *(d)* Given, line $x + \sqrt{3}y = 2\sqrt{3}$

and point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$

From options, we take the conic $x^2 + 9y^2 = 9$

Equation of any tangent at (x_1, y_1) is

$$xx_1 + 9 \cdot yy_1 = 9$$

$$\Rightarrow \quad \frac{3\sqrt{3}}{2}x + 9 \times \frac{1}{2} \times y = 9 \qquad \left[\because (x_1, y_1) = \left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)\right]$$

$$\Rightarrow \quad \frac{3\sqrt{3}}{2}(x + \sqrt{3}y) = 9$$

$$\Rightarrow \quad x + \sqrt{3}y = \frac{9 \times 2}{3\sqrt{3}}$$

$$\Rightarrow \quad x + \sqrt{3}y = 2\sqrt{3}$$

Clearly, $x + \sqrt{3}y = 2\sqrt{3}$ is a tangent to the curve $x^2 + 9y^2 = 9$.

10. *(b)* Let $I = \int_1^3 [x^2 - 2x - 2]\,dx$

$$= \int_1^3 [x^2 - 2x + 1 - 3]\,dx = \int_1^3 (x-1)^2 - 3]dx$$

$$= \int_1^3 [(x-1)^2]\,dx + \int_1^3 -3dx$$

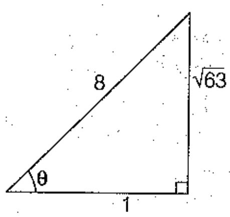
Put $x - 1 = t$; $dx = dt$, when $x = 1, t = 0$ and $x = 3, t = 2$

$$\therefore \quad I = -3[x]_1^3 + \int_0^2 [t^2]\,dt$$

$$= -6 + \int_0^1 0\,dt + \int_1^{\sqrt{2}} 1dt + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dt + \int_{\sqrt{3}}^{2} 3dt$$
$$= -6 + (0) + (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3})$$
$$= -6 + \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$
$$I = -1 - \sqrt{2} - \sqrt{3}$$

11. *(a)* Given, $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta$



$\Rightarrow \quad \sin\theta = \frac{\sqrt{63}}{8} \Rightarrow \cos\theta = \frac{1}{8}$

Also, $\cos\frac{\theta}{2} = \sqrt{\frac{1 + \cos\theta}{2}}$

$$= \sqrt{\frac{1 + \frac{1}{8}}{2}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$\therefore \tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \tan\left(\frac{\theta}{4}\right)$

$$= \sqrt{\frac{1 - \cos\frac{\theta}{2}}{1 + \cos\frac{\theta}{2}}} = \sqrt{\frac{1 - \frac{3}{4}}{1 + \frac{3}{4}}} = \frac{1}{\sqrt{7}}$$

12. *(b)* Given, statement: $\sim p \wedge (p \vee q)$

Negative of given statement

$\sim[\sim p \wedge (p \vee q)]$

$= p \vee \sim [p \vee q)$ [by De Morgan's law]

$= p \vee (\sim p \wedge \sim q)$ [by De Morgan's law]

$= (p \vee \sim p) \wedge (p \vee \sim q)$ [Using distributive property]

$= t \wedge (p \vee \sim q)$

$= p \vee \sim q$

13. *(c)* Given, curve $\Rightarrow y = ax^2 + bx + c, x \in R$ and point (1, 2)

$\because$ The given curve passes through (1, 2).

$\therefore \quad 2 = a + b + c$

Also, slope of tangent of $y = ax^2 + bx + c$ is $\frac{dy}{dx} = 2ax + b$

$\because$ Tangent passes through origin (0, 0).

$\therefore \quad \left.\frac{dy}{dx}\right|_{(0,0)} = 2a \times 0 + b = b$ ... (i)

According to the question, tangent at origin is $y = x$

$\therefore$ Its slope is 1. ... (ii)

From Eqs. (i) and (ii),

$b = 1$

Also, $a + b + c = 2$

$\Rightarrow \quad a + c + 1 = 2 \Rightarrow a + c = 1$

From the option look for $b = 1$ and $a + c = 1$

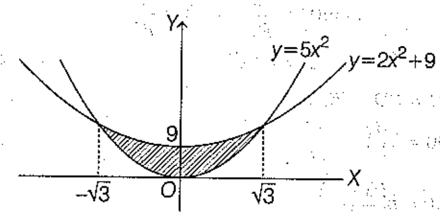
The only correct order triplet is $a = 1, b = 1, c = 0$.

14. *(b)* Given, $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$

Here, we have two curves $y = 5x^2$ and $y = 2x^2 + 9$, point of intersection of both curves is find by solving both equations i.e.

$$5x^2 = 2x^2 + 9$$

$\Rightarrow \quad x^2 = 3 \Rightarrow x = \pm\sqrt{3}$



$\therefore \quad$ Area $= \int_{-\sqrt{3}}^{\sqrt{3}} (2x^2 + 9 - 5x^2)\,dx$

$$= 2\int_0^{\sqrt{3}} (9 - 3x^2)\,dx$$
$$= 2[9x - x^3]_0^{\sqrt{3}}$$
$$= 2[9\sqrt{3} - 3\sqrt{3}]$$
$$= 12\sqrt{3} \text{ sq units}$$

15. *(b)* Given, curve $y = f(x)$ passes through (1, 2) and satisfies

$$x\frac{dy}{dx} + y = bx^4$$

$\Rightarrow \quad x\frac{dy}{dx} + y = bx^4$

$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = bx^3$

$$\text{IF} = e^{\int \frac{1}{x}\cdot dx} = x$$

$\therefore \quad yx = \int bx^4\,dx = \frac{bx^5}{5} + C$

$\Rightarrow \quad y = \frac{bx^4}{5} + \frac{C}{x} = f(x)$ ... (i)

$\because$ This curve passes through (1, 2).

$\therefore \quad 2 \times 1 = \frac{b \times (1)^5}{5} + C$

$\Rightarrow \quad 2 = \frac{b}{5} + C$ ... (ii)

Also, $\int_1^2 f(x)\,dx = \frac{62}{5}$

$\Rightarrow \quad \int_1^2 \left(\frac{bx^4}{5} + \frac{C}{x}\right)dx = \frac{62}{5}$ [from Eq. (i)]

$\Rightarrow \quad \left[b \times \frac{x^5}{25} + C\log x\right]_1^2 = \frac{62}{5}$

$\Rightarrow \quad \left[\left(\frac{b \times 32}{25} + C\log 2\right) - \left(\frac{b}{25} + c\log 1\right)\right] = \frac{62}{5}$

$\Rightarrow \quad \frac{b \times 32}{25} + C\log 2 - \frac{b}{25} = \frac{62}{5} + 0\log 2$

On comparing, we get

$$\frac{b}{25} \times 31 = \frac{62}{5} \text{ and } c = 0$$
$$b = \frac{62 \times 25}{31 \times 5}$$
$$b = 10$$

Hence, the required value of $b = 10$.

16. *(b)* Given, $f(0) = 1$ ... (i)

$f(2) = e^2$ ... (ii)

$f'(x) = f'(2 - x)$

Integrating w.r.t. x,

$f(x) = -f(2 - x) + C$

Put $x = 0$

$f(0) = -f(2) + C$

$\Rightarrow 1 = -e^2 + C$ [from Eqs. (i) and (ii)]

$\Rightarrow C = 1 + e^2$

$\therefore f(x) = -f(2 - x) + 1 + e^2$

or $f(x) + f(2 - x) = 1 + e^2$ ... (iii)

Let $I = \int_0^2 f(x)\, dx$ ... (iv)

Also, $I = \int_0^2 f(2 - x)\, dx$ ... (v)

Now, adding Eqs. (iv) and (v),

$2I = \int_0^2 [f(x) + f(2 - x)]\, dx$

$2I = \int_0^2 (1 + e^2)\, dx$ [from Eq. (iii)]

$2I = 2(1 + e^2)$

$\therefore I = (1 + e^2)$

17. *(c)* Given, A be a 3×3 matrix.

A is symmetric and B is skew-symmetric.

$\therefore A^T = A, B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2 = -(A^2B^2 - B^2A^2) = -P$

$\Rightarrow P^T = -P$

P is skew-symmetric.

$\therefore |P| = 0$

Hence, $PX = 0$ have infinite solutions.

18. *(d)* Given, a, b and c are in AP.

(a, c), $(2, b)$ and (a, b) are vertices of triangle.

Centroid $= \left(\frac{10}{3}, \frac{7}{3}\right)$

α and β are the roots of equation $ax^2 + bx + 1 = 0$

$\because a, b, c$ are in AP.

$\therefore 2b = a + c$ ... (i)

Centroid $= \left(\frac{a + 2 + a}{3}, \frac{c + b + b}{3}\right)$

$= \left(\frac{2a + 2}{3}, \frac{c + 2b}{3}\right) = \left(\frac{10}{3}, \frac{7}{3}\right)$

$\Rightarrow \frac{2a + 2}{3} = \frac{10}{3}$ and $\frac{c + 2b}{3} = \frac{7}{3}$

$\Rightarrow a = 4$

and $c + a + c = 7$ [$\because 2b = a + c$]

$\Rightarrow 2c = 7 - 4$ [$\because a = 4$]

$c = 3/2$

Also, $2b = a + c = 4 + \frac{3}{2}$

$\Rightarrow b = 11/4$

Now, α and β are roots of $ax^2 + bx + 1 = 0$

$\therefore \alpha + \beta = \frac{-b}{a} = \frac{-11/4}{4}$

$\Rightarrow \alpha + \beta = \frac{-11}{16} \Rightarrow \alpha\beta = \frac{1}{a} = \frac{1}{4}$

$\Rightarrow \alpha\beta = \frac{1}{4}$

Now, $\alpha^2 + \beta^2 - \alpha\beta$

$= (\alpha + \beta)^2 - 3\alpha\beta = \left(\frac{-11}{16}\right)^2 - 3 \times \frac{1}{4}$

$= \frac{121 - 192}{256} = \frac{-71}{256}$

19. *(d)* Given, $x - 2y + 0z = 1$

$x - y + kx = -2$

$0x + ky + 4z = 6$

Here, $\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 1(-4 - k^2) + 2(4)$

$= -4 - k^2 + 8 = 4 - k^2$

$\Delta_x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = 1(-4 - k^2) + 2(-8 - 6k)$

$= -4 - k^2 - 16 - 12k = -k^2 - 12k - 20$

If $\Delta \neq 0$, then it has unique solution i.e. $4 - k^2 \neq 0$

$\Rightarrow k \neq \pm 2$ for unique solution.

Also at $k = 2$

$\Delta_x = -2^2 - 12 \times 2 - 20 = -48 \neq 0$

Then, in this case it has no solution.

Hence, statement (A) and statement (D) both are correct.

20. *(c)* Given, set $= \{1, 2, 3, 4, 5\}$

Let the two subsets be A and B.

Then, $n(A \cap B) = 2$ (as given in question)

$\therefore$ Required probability $= \frac{{}^5C_2 \times 3^3}{4^5} = \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$

21. *(*)* Given, $\binom{n}{r} = \begin{cases} {}^nC_r & \text{, if } n \geq r \geq 0 \\ 0 & \text{otherwise} \end{cases}$

and $\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$

$\because (1 + x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + \ldots + {}^{10}C_{10} x^{10}$

and $(1 + x)^{15} = {}^{15}C_0 + {}^{15}C_1 x + {}^{15}C_2 x^2 + \ldots + {}^{15}C_{15} x^{15}$

$\therefore \sum_{i=0}^{k} ({}^{10}C_i)({}^{15}C_{k-i}) = {}^{10}C_0 \cdot {}^{15}C_k + {}^{10}C_1 \cdot {}^{15}C_{k-1} + \ldots + {}^{10}C_k \cdot {}^{15}C_0$

$\Rightarrow$ Coefficient of x^k in $(1 + x)^{25} = {}^{25}C_k$

Also, $\sum_{i=0}^{k+1} ({}^{12}C_i)({}^{13}C_{k+1-i}) = {}^{12}C_0 \cdot {}^{13}C_{k+1}$

$+ {}^{12}C_1 \cdot {}^{13}C_k + \ldots + {}^{12}C_{k+1} \cdot {}^{13}C_0$

$\Rightarrow$ Coefficient of x^{k+1} in $(1 + x)^{25} = {}^{25}C_{k+1}$

$\Rightarrow {}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

So, ${}^{26}C_{k+1}$ always exists.

Now k can be as larger as possible.

22. *(1)* Given, equation of line $\Rightarrow x - \lambda = 2y - 1 = -2z$

$\Rightarrow \frac{x - \lambda}{1} = \frac{y - 1/2}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

or $\frac{x - \lambda}{2} = \frac{y - 1/2}{1} = \frac{z}{-1}$ ... (i)

Point on this line through which it passes is $\left(\lambda, \frac{1}{2}, 0\right)$.

Equation of another line $\Rightarrow x = y + 2\lambda = z - \lambda$

$$\Rightarrow \quad \frac{x}{1} = \frac{y - (-2\lambda)}{1} = \frac{z - \lambda}{1} \quad \text{... (ii)}$$

A point through which this line passes is $(0, -2\lambda, \lambda)$.

Now, distance between two skew lines

$$= \frac{|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$$

$$= \frac{\begin{vmatrix} \lambda & \frac{1}{2} + 2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}} = \frac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}}$$

According to the question, $\dfrac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}} = \dfrac{\sqrt{7}}{2\sqrt{2}}$

$$\Rightarrow \quad |10\lambda + 3| = 7$$

$$\Rightarrow \quad 10\lambda + 3 = \pm 7$$

$$\Rightarrow \quad 10\lambda = 4, -10$$

$$\therefore \quad \lambda = \frac{2}{5} \text{ and } \lambda = -1$$

($\lambda = \frac{2}{5}$ is not possible as λ is an integer)

$$\therefore \quad \lambda = -1.$$

Hence, $|\lambda| = |-1| = 1$

23. *(2)* Given, $a + \alpha = 1$

$$b + \beta = 2$$

$$\because \quad a \cdot f(x) + \alpha \cdot f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x} \quad \text{... (i)}$$

Replace x by $\frac{1}{x}$,

$$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x \quad \text{... (ii)}$$

Adding Eqs. (i) and (ii), we get

$$(a + \alpha)\left[f(x) + f\left(\frac{1}{x}\right)\right] = \left(x + \frac{1}{x}\right)(b + \beta)$$

$$\Rightarrow \quad \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b + \beta}{a + \alpha} = \frac{2}{1} = 2$$

24. *(56.25)* Let P be (h, k), $A(5, 0)$ and $B(-5, 0)$.

Given, $\quad PA = 3PB$

$$\Rightarrow \quad PA^2 = 9PB^2$$

$$\Rightarrow (h - 5)^2 + k^2 = 9[(h + 5)^2 + k^2]$$

$$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$$

$\therefore$ Locus of P is $x^2 + y^2 + \frac{25}{2}x + 25 = 0$

$$\text{Centre} = \left(\frac{-25}{4}, 0\right)$$

$$\text{and } r^2 = \left(\frac{-25}{4}\right)^2 + 0^2 - 25$$

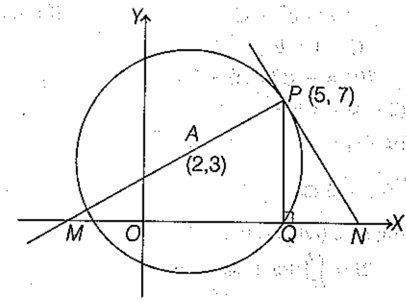
$$= \frac{625}{16} - 25 = \frac{225}{16}$$

$$\therefore \quad 4r^2 = 4 \times \frac{225}{16} = \frac{225}{4} = 56.25$$

25. *(1225)* Given, circle $(x - 2)^2 + (y - 3)^2 = 5^2$

$$c = (2, 3)$$

$$r = 5$$



Equation of normal at P (i.e. PA line)

$$\Rightarrow \quad (y - 7) = \left(\frac{7 - 3}{5 - 2}\right)(x - 5)$$

$$\Rightarrow \quad 3y - 21 = 4x - 20$$

$$\Rightarrow \quad 4x - 3y + 1 = 0$$

Therefore, $M = \left(\frac{-1}{4}, 0\right)$ [Put $y = 0$ in above equation]

Now, equation of tangent at P.

$$y - 7 = \frac{-3}{4}(x - 5) \quad \left[\because \text{slope of } PN = \frac{-1}{\text{Slope of } PA}\right]$$

$$\Rightarrow 4y - 28 = -3x + 15$$

$$\Rightarrow 3x + 4y = 43$$

Therefore, $N = \left(\frac{43}{3}, 0\right)$ [Put $y = 0$ in above equation]

$$\therefore \text{Area } (A) = \frac{1}{2} \times MN \times PQ$$

$$= \frac{1}{2} \times \left(\frac{43}{3} + \frac{1}{4}\right) \times 7$$

$$= \frac{1}{2} \times \frac{175}{12} \times 7$$

$$\therefore \quad 24A = 24 \times \frac{1}{2} \times \frac{175}{12} \times 7 = 1225$$

But this question is wrong as in question. It is mentioned that the triangle is formed with the positive X-axis which contradicts the solution.

26. *(11)* Given, 10 natural numbers $= 1, 1, 1, \ldots, 1, k$

According to the question, variance $(\sigma^2) < 10$

$$\because \quad \sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

$$\sigma^2 = \frac{(9 + k^2)}{10} - \left(\frac{9 + k}{10}\right)^2 < 10$$

$$\Rightarrow \quad 10(9 + k^2) - (81 + k^2 + 18k) < 1000$$

$$\Rightarrow \quad 90 + 10k^2 - 81 - k^2 - 18k < 1000$$

$$\Rightarrow \quad 9k^2 - 18k + 9 < 1000$$

$$\Rightarrow \quad (k - 1)^2 < \frac{1000}{9}$$

$$\Rightarrow \quad k - 1 < \frac{10\sqrt{10}}{3}$$

and $k-1 > \frac{-10\sqrt{10}}{3} \Rightarrow k < \frac{10\sqrt{10}}{3} + 1$

It is not possible because $k \in N$.

$\therefore$ Maximum possible integral value of k is 11.

27. *(3)* Let four numbers in GP be a, ar, ar^2, ar^3.

According to the question, $a + ar + ar^2 + ar^3 = \frac{65}{12}$ ...(i)

and $\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$\Rightarrow \frac{1}{a}\left(\frac{1+r+r^2+r^3}{r^3}\right) = \frac{65}{18}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{a(1+r+r^2+r^3)}{\frac{1}{a}\frac{(1+r+r^2+r^3)}{r^3}} = \frac{65/12}{65/18}$$

$\Rightarrow a^2r^3 = \frac{18}{12} \Rightarrow a^2r^3 = \frac{3}{2}$

Also, product of first three terms = 1

$a \times ar \times ar^2 = 1$

$\Rightarrow a^3r^3 = 1$

$\Rightarrow a^3 \times \frac{3}{2a^2} = 1$ $\left[\because r^3 = \frac{3/2}{a^2}\right]$

$\Rightarrow a = \frac{2}{3}$

and $r^3 = \frac{3/2}{(2/3)^2} = \left(\frac{3}{2}\right)^3 \Rightarrow r = \frac{3}{2}$

According to the question,

third term $= \alpha = ar^2 = \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} = \frac{3}{2}$

$\therefore 2\alpha = 2 \times \frac{3}{2} = 3$

28. *(31650)* Given, total students = 10.

Number of groups = 3 (i.e. A, B and C)

Each group has atleast one student but group C has atmost 3 students.

$\therefore$ There are 3 cases depending on number of students in group C.

Case I C has 1 student, then $\left.\begin{matrix}A\\B\end{matrix}\right] \leftarrow$ 9 students.

$\therefore$ Number of ways $= {}^{10}C_1 \times [2^9 - 2]$

Case II C has 2 students, then $\left.\begin{matrix}A\\B\end{matrix}\right] \leftarrow$ 8 Students.

$\therefore$ Number of ways $= {}^{10}C_2 \times [2^8 - 2]$

Case III C has 3 students, then $\left.\begin{matrix}A\\B\end{matrix}\right] \leftarrow$ 7 Students.

$\therefore$ Number of ways $= {}^{10}C_3 \times [2^7 - 2]$

$\therefore$ Required number of possibilities

$= {}^{10}C_1(2^9 - 2) + {}^{10}C_2(2^8 - 2) + {}^{10}C_3(2^7 - 2)$

$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3] - 20 - 90 - 240$

$= 128[40 + 90 + 120] - 350$

$= (128 \times 250) - 350 = 31650$

29. *(310)* Given, $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$

$\because -1 + i\sqrt{3} = 2e^{i2\pi/3}$

$1 + i\sqrt{3} = 2e^{i\pi/3}$

$1 - i = \sqrt{2}e^{-i\pi/4}$

$1 + i = \sqrt{2}e^{i\pi/4}$

Now, $\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{(\sqrt{2}e^{-i\pi/4})^{24}} + \frac{(2e^{i\pi/3})^{21}}{(\sqrt{2}e^{i\pi/4})^{24}}$

$= \frac{2^{21}\cdot e^{i14\pi}}{2^{12}\cdot e^{-i6\pi}} + \frac{2^{21}\cdot e^{i7\pi}}{2^{12}\cdot e^{i6\pi}}$

$= 2^9 \cdot e^{i20\pi} + 2^9 \cdot e^{i\pi}$

$= 2^9(1) + 2^9(-1)$

$\Rightarrow 2^9 - 2^9 = 0 = k$ (given)

$\therefore n = [|k|] = 0$

Now, $\sum_{j=0}^{5}(j+5)^2 - \sum_{j=0}^{5}(j+5)$ $[\because n = 0]$

$= [5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2] - [5 + 6 + 7 + 8 + 9 + 10]$

$= [(1^2 + 2^2 + 3^2 + \ldots + 10^2) - (1^2 + 2^2 + \ldots + 4^2)]$

$- [(1 + 2 + 3 + \ldots + 10) - (1 + 2 + 3 + 4)]$

$= \left[\frac{10 \times 11 \times 21}{6} - \frac{4 \times 5 \times 9}{6}\right] - \left[\frac{10 \times 11}{2} - \frac{4 \times 5}{2}\right]$

$= (385 - 30) - (55 - 10)$

$= 355 - 45 = 310$

30. *(2)* Given, equation $(x+1)^2 + |(x-5)| = \frac{27}{4}$

Case I For $x \geq 5$

$\Rightarrow (x+1)^2 + (x-5) = \frac{27}{4}$

$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$

$\Rightarrow 4x^2 + 12x - 43 = 0$

$\therefore x = \frac{-12 \pm \sqrt{144 + 688}}{8}$

$= \frac{-12 \pm \sqrt{832}}{8}$

$= \frac{-12 \pm 28.8}{8}$

$x = \frac{-3 \pm 7.2}{2}$

$x = \frac{-3 + 7.2}{2}, \frac{-3 - 7.2}{2}$

Both the values are less than 5.

$\therefore$ No solution from here.

Case II $x < 5$

$\Rightarrow (x+1)^2 - (x-5) = \frac{27}{4}$

$\Rightarrow x^2 + x + 6 - \frac{27}{4} = 0$

$\Rightarrow 4x^2 + 4x - 3 = 0$

$\Rightarrow x = \frac{-4 \pm \sqrt{16 + 48}}{8}$

$= \frac{-4 \pm 8}{8}$

$\Rightarrow x = \frac{-12}{8}, \frac{4}{8}$, both are less than 5.

$\therefore$ These values must be the solution.

Hence, here 2 real roots are possible.

JEE Main 2021

25 FEBRUARY SHIFT I

PHYSICS

Section A : Objective Type Questions

1. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A) When a rod lying freely is heated, no thermal stress is developed in it.

Reason (R) On heating, the length of the rod increases.

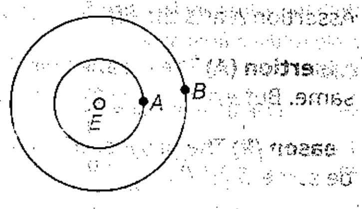
In the light of the above statements, choose the correct answer from the options given below

a. Both A and R are true but R is not the correct explanation of A.
b. A is false but R is true.
c. A is true but R is false.
d. Both A and R are true and R is the correct explanation of A.

2. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is

a. 13 cm **b.** 16.6 cm
c. 18.4 cm **d.** 14.8 cm

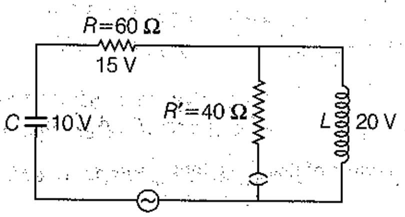
3. Two satellites A and B of masses 200 kg and 400 kg are revolving around the Earth at height of 600 km and 1600 km, respectively. If T_A and T_B are the time periods of A and B respectively, then the value of $T_B - T_A$ is



(Given, radius of Earth = 6400 km, mass of Earth $= 6 \times 10^{24}$ kg)

a. 1.33×10^3 s **b.** 3.33×10^2 s
c. 4.24×10^3 s **d.** 4.24×10^2 s

4. The angular frequency of alternating current in an L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



a. 0.8 H and 150 μF **b.** 0.8 H and 250 μF
c. 1.33 H and 250 μF **d.** 1.33 H and 150 μF

5. A proton, a deuteron and an α-particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is .... and their speed is .... in the ratio.

a. 1 : 2 : 4 and 2 : 1 : 1 **b.** 2 : 1 : 1 and 4 : 2 : 1
c. 4 : 2 : 1 and 2 : 1 : 1 **d.** 1 : 2 : 4 and 1 : 1 : 2

6. Given, below are two statements

Statement I A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.

Statement II The side band frequencies are 1002 kHz and 998 kHz. In the light of the above statements, choose the correct answer from the options given below

a. Statement I is true but Statement II is false.
b. Statement I is false but Statement II is true.
c. Both Statement I and Statement II are true.
d. Both Statement I and Statement II are false.

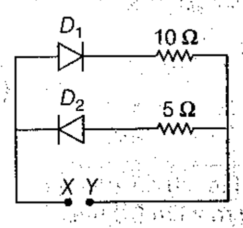
7. If the time period of a 2 m long simple pendulum is 2 s, the acceleration due to gravity at the place, where pendulum is executing SHM is

a. $\pi^2 \text{ms}^{-2}$ **b.** 9.8 ms^{-2}
c. $2\pi^2$ ms^{-2} **d.** 16 ms^{-2}

8. The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72nd division on circular scale coincides with the reference line. The radius of the wire is

a. 1.64 mm **b.** 0.82 mm **c.** 1.80 mm **d.** 0.90 mm

9. A 5 V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery, if the positive terminal of the battery is connected to point X.



a. ~ 0.5 A b. ~ 1.5 A
c. ~ 0.86 A d. ~ 0.43 A

10. An α-particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de-Broglie wavelengths are λ_α and λ_p, respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is

a. 3.8 b. 8 c. 7.8 d. 2.8

11. A diatomic gas having $C_p = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$, is heated at constant pressure. The ratio $dU : dQ : dW$ is

a. 5 : 7 : 3 b. 5 : 7 : 2
c. 3 : 7 : 2 d. 3 : 5 : 2

12. An engine of a train moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is

a. $\sqrt{\frac{v^2+u^2}{2}}$ b. $\frac{v-u}{2}$ c. $\frac{u+v}{2}$ d. $\sqrt{\frac{v^2-u^2}{2}}$

13. Match List-I with List-II

List-I	List-II
A. h (Planck's constant)	1. $[M L T^{-1}]$
B. E (kinetic energy)	2. $[M L^2 T^{-1}]$
C. V (electric potential)	3. $[M L^2 T^{-2}]$
D. P (linear momentum)	4. $[M L^2 I^{-1} T^{-3}]$

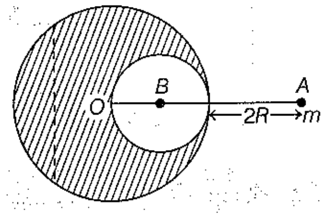
Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	3	4	2	1	b.	2	3	4	1
c.	1	2	4	3	d.	3	2	4	1

14. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8 : 1. The radius of coil is

a. 0.2 m b. 0.1 m
c. 0.15 m d. 1.0 m

15. A solid sphere of radius R gravitationally attracts a particle placed at $3R$ from its centre with a force F_1. Now, a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is

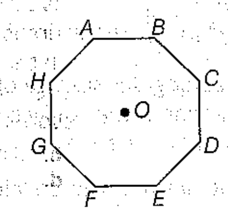


a. 25 : 36 b. 36 : 25
c. 50 : 41 d. 41 : 50

16. Two radioactive substances X and Y originally have N_1 and N_2 nuclei, respectively. Half-life of X is half of the half-life of Y. After three half-lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to

a. $\frac{1}{8}$ b. $\frac{3}{1}$
c. $\frac{8}{1}$ d. $\frac{1}{3}$

17. In an octagon $ABCDEFGH$ of equal side, what is the sum of $\mathbf{AB} + \mathbf{AC} + \mathbf{AD} + \mathbf{AE} + \mathbf{AF} + \mathbf{AG} + \mathbf{AH}$ if, $\vec{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$?



a. $-16\hat{i} - 24\hat{j} + 32\hat{k}$ b. $16\hat{i} + 24\hat{j} - 32\hat{k}$
c. $16\hat{i} + 24\hat{j} + 32\hat{k}$ d. $16\hat{i} - 24\hat{j} + 32\hat{k}$

18. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

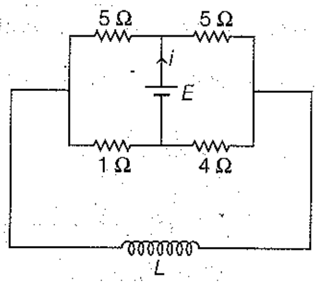
Assertion (A) The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason (R) The product of their mass and radius must be same, $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below.

a. Both A and R are correct but R is not the correct explanation of A.
b. A is correct but R is not correct.
c. Both A and R are correct and R is the correct explanation of A.
d. A is not correct but R is correct.

19. The current (i) at time $t = 0$ and $t = \infty$ respectively for the given circuit is



a. $\frac{18E}{55}, \frac{5E}{18}$ b. $\frac{10E}{33}, \frac{5E}{18}$ c. $\frac{5E}{18}, \frac{18E}{55}$ d. $\frac{5E}{18}, \frac{10E}{33}$

20. Two coherent light sources having intensity in the ratio $2x$ produce an interference pattern. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be

a. $\frac{2\sqrt{2x}}{x+1}$ b. $\frac{\sqrt{2x}}{2x+1}$ c. $\frac{\sqrt{2x}}{x+1}$ d. $\frac{2\sqrt{2x}}{2x+1}$

Section B : Numerical Type Questions

21. A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self-inductance of coil necessary for resonance is .... $\times 10^{-8}$ H.

22. The electric field in a region is given

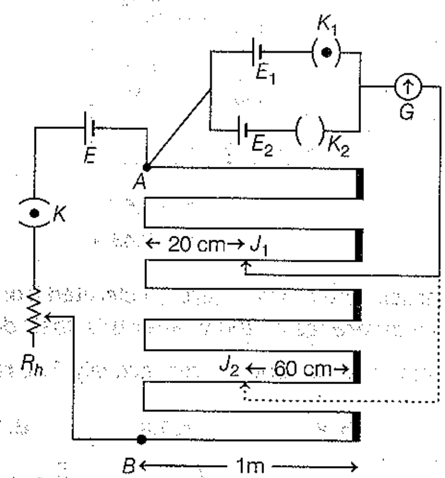
$\mathbf{E} = \left(\frac{3}{5}E_0\hat{\mathbf{i}} + \frac{4}{5}E_0\hat{\mathbf{j}}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to YZ-plane) to that of the surface of area 0.3 m^2 (parallel to XZ- plane) is $a : b$, where a = ...

[Here $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along X, Y and Z-axes, respectively]

23. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be .... nR, where n denotes number of moles of a gas.

24. A small bob tied at one end of a thin string of length 1 m is describing a vertical circle, so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is .... m/s. (Take, $g = 10$ m/s^2)

25. In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1(closed), the jockey is adjusted to touch the wire at point J_1, so that there is no deflection in the galvanometer. Now, the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = ...



26. The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is .... cm.

27. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is .... V.

28. A coil of inductance 2H having negligible resistance is connected to a source of supply whose voltage is given by $V = 3t$ V (where, t is in second). If the voltage is applied when $t = 0$, then the energy stored in the coil after 4 s is .... J.

29. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped, then change in temperature of the gas (R = gas constant) is $\frac{x}{3R}$. Value of x is ....

30. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as

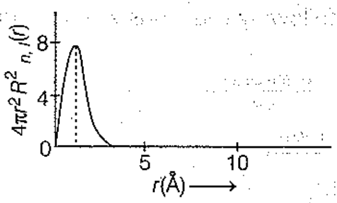
$U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$

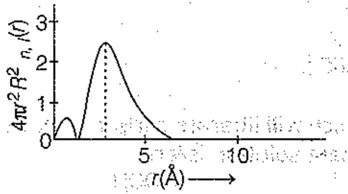
where, α and β are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = ...

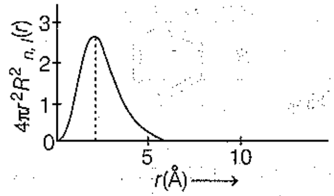
CHEMISTRY

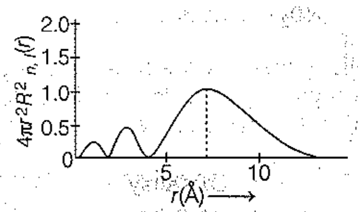
Section A : Objective Type Questions

1. The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below.

(A)


(B)


(C)


(D)


The correct plot for 3s–orbital is

a. (A) **b.** (B) **c.** (C) **d.** (D)

2. According to molecular orbital theory, the species among the following that does not exist is

a. O_2^{2-} **b.** He_2^-
c. Be_2 **d.** He_2^+

3. The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is...... .

[Assume : No cyano complex is formed; $K_{sp}(AgCN) = 2.2 \times 10^{-16}$ and $K_a(HCN) = 6.2 \times 10^{-10}$]

a. 0.625×10^{-6} **b.** 1.6×10^{-6}
c. 2.2×10^{-16} **d.** 1.9×10^{-5}

4. In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to p^x. The value of x is

a. 1 **b.** zero
c. ∞ **d.** $\frac{1}{n}$

5. Ellingham diagram is a graphical representation of

a. ΔG vs T **b.** ΔH vs T
c. ΔG vs p **d.** $(\Delta G - T\Delta S)$ vs T

6. Which of the following equation depicts the oxidising nature of H_2O_2?

a. $KIO_4 + H_2O_2 \longrightarrow KIO_3 + H_2O + O_2$
b. $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$
c. $2I^- + H_2O_2 + 2H^+ \longrightarrow I_2 + 2H_2O$
d. $Cl_2 + H_2O_2 \longrightarrow 2HCl + O_2$

7. The correct statement about B_2H_6 is

a. all B—H—B angles are of 120°
b. the two B—H—B bonds are not of same length
c. terminal B—H bonds have less p-character when compared to bridging bonds
d. Its fragment, BH_3, behaves as a Lewis base

8. Given below are two statements:

Statement I CeO_2 can be used for oxidation of aldehydes and ketones.

Statement II Aqueous solution of $EuSO_4$ is a strong reducing agent.

In the light of the above statements, choose the correct answer from the options given below.

a. Both statement I and statement II are true.
b. Both statement I and statement II are false.
c. Statement I is true but statement II is false.
d. Statement I is false but statement II is true.

9. In which of the following pairs, the outer most electronic configuration will be the same?

a. V^{2+} and Cr^+ **b.** Cr^+ and Mn^{2+}
c. Ni^{2+} and Cu^+ **d.** Fe^{2+} and Co^+

10. The hybridisation and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are

a. d^2sp^3 and paramagnetic
b. sp^3d^2 and diamagnetic
c. d^2sp^3 and diamagnetic
d. sp^3d^2 and paramagnetic

11. Given below are two statements:

Statement I An allotrope of oxygen is an important intermediate in the formation of reducing smog.

Statement II Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog.

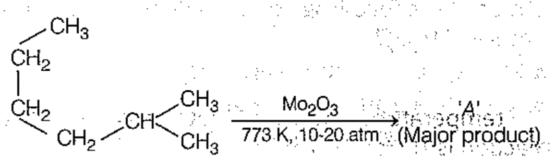
In the light of the above statements, choose the correct answer from the options given below.

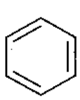
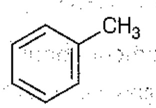
a. Both statement I and statement II are true.
b. Both statement I and statement II are false.
c. Statement I is true but statement II is false.
d. Statement I is false but statement II is true.

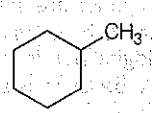
12. Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is

a. 50.33 b. 53.33
c. 63.53 d. 51.63

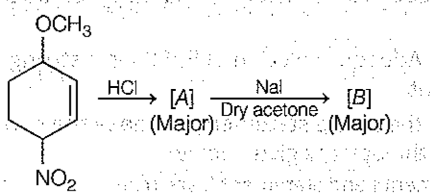
13. Identify A in the given chemical reaction.

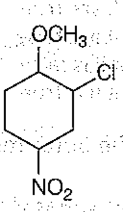
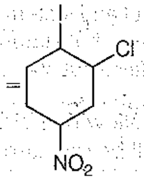


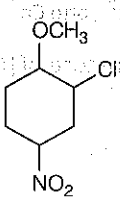
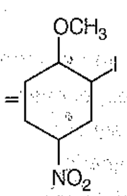
a.  b. 

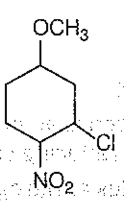
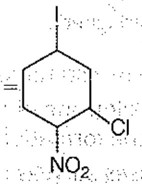
c.  d. 

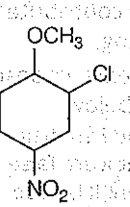
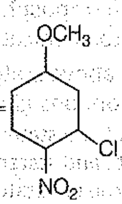
14. Identify A and B in the chemical reaction.



a. $A =$  $B =$ 

b. $A =$  $B =$ 

c. $A =$  $B =$ 

d. $A =$  $B =$ 

15. The major product of the following chemical reaction is

$$CH_3CH_2CN \xrightarrow[\text{(iii) } Pd/BaSO_4,\, H_2]{\text{(i) } H_3O^+,\, \Delta \quad \text{(ii) } SOCl_2} ?$$

a. $CH_3CH_2CH_2OH$ b. $(CH_3CH_2CO)_2O$
c. $CH_3CH_2CH_3$ d. CH_3CH_2CHO

16. Which one of the following reactions will not form acetaldehyde?

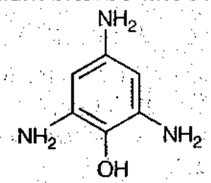
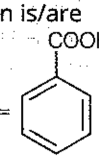
a. $CH_2 = CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$

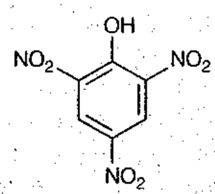
b. $CH_3CH_2OH \xrightarrow{CrO_3\text{-}H_2SO_4}$

c. $CH_3CN \xrightarrow[\text{(ii) } H_2O]{\text{(i) DIBAL-H}}$

d. $CH_3CH_2OH \xrightarrow[573\text{ K}]{Cu}$

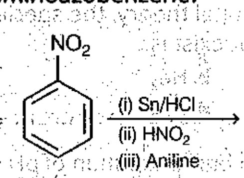
17. Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/are

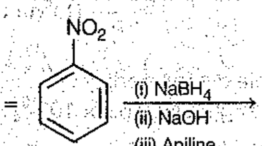
$A =$  $B =$ 

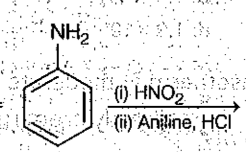
$C =$ 

a. A and B only b. C only
c. B and C only d. B only

18. Which of the following reaction(s) will not give p-aminoazobenzene?

$A =$ 

$B =$ 

$C =$ 

a. A only b. C only
c. B only d. A and B

19. Which statement is correct?
a. Buna-N is a natural polymer.
b. Buna-S is a synthetic and linear thermosetting polymer.
c. Neoprene is an addition copolymer used in plastic bucket manufacturing.
d. Synthesis of buna-S needs nascent oxygen.

20. Which of the glycosidic linkage between galactose and glucose is present in lactose?
a. C-1 of galactose and C-4 of glucose
b. C-1 of galactose and C-6 of glucose
c. C-1 of glucose and C-4 of galactose
d. C-1 of glucose and C-6 of galactose

Section B : Numerical Type Questions

21. 0.4 g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with N / 10 HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is .... (Rounded off to the nearest integer).

22. A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is .... (Rounded-off to the nearest integer).

23. The reaction of cyanamide, $NH_2CN(s)$ with oxygen was run in a bomb calorimeter and ΔU was found to be -742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$NH_2CN(s) + \frac{3}{2}O_2(g) \longrightarrow N_2(g) + O_2(g) + H_2O(l)$ is .... kJ

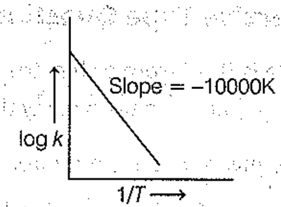
(Rounded off to the nearest integer). [Assume ideal gases and $R = 8.314$ J mol^{-1} K^{-1}]

24. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is .... K (Rounded off to the nearest integer). [Given, K_b for $(H_2O) = 0.52$ K kg mol^{-1}]

25. In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is .... mL

(Rounded off to the nearest integer).

26. For the reaction, $aA + bB \rightarrow cC + dD$, the plot of $\log k$ vs $\frac{1}{T}$ is given below



The temperature at which the rate constant of the reaction is 10^{-4} s^{-1} is .... K

(Rounded off to the nearest integer).

[Given : The rate constant of the reaction is 10^{-5} s^{-1} at 500 K]

27. The ionisation enthalpy of Na^+ formation from $Na(g)$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is -325.0 kJ mol^{-1}. Given, the lattice enthalpy of NaBr is -728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is (–) .... $\times 10^{-1}$ kJ mol^{-1}.

28. Among the following, the number of halide(s) which is/are inert to hydrolysis is ....

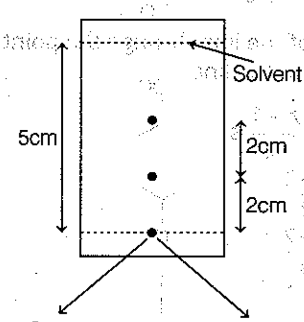
(i) BF_3 (ii) $SiCl_4$ (iii) PCl_5 (iv) SF_6

29. Consider the following chemical reaction.

$CH \equiv CH \xrightarrow[\text{(2) CO, HCl, AlCl}_3]{\text{(1) Red hot Fe tube, 873 K}} \text{Product}$

The number of sp^2 hybridised carbon atom(s) present in the product is ....

30. Using the provided information in the following paper chromatogram.



Paper chromatography for compounds A and B

The calculated R_f value of A .... $\times 10^{-1}$.

MATHEMATICS

Section A : Objective Type Questions

1. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$.

If three missiles are fired independently from the ship, then the probability that all three hit the target, is

a. $\frac{1}{27}$ **b.** $\frac{3}{4}$

c. $\frac{1}{8}$ **d.** $\frac{3}{8}$

2. If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$, then

a. $xy - z = (x+y)z$

b. $xy + yz + zx = z$

c. $xyz = 4$

d. $xy + z = (x+y)z$

3. Let $f, g : N \to N$, such that $f(n+1) = f(n) + f(1)\ \forall\ n \in N$ and g be any arbitrary function. Which of the following statements is not true?

a. if fog is one-one, then g is one-one

b. if f is onto, then $f(n) = n\ \forall\ n \in N$

c. f is one-one

d. if g is onto, then fog is one-one

4. The equation of the line through the point (0,1,2) and perpendicular to the line

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is

a. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

b. $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

c. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

d. $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

5. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then, the value of $\sin^4\alpha + \cos^4\alpha$ is

a. $\frac{3}{4}$ **b.** $\frac{3}{8}$

c. $\frac{5}{8}$ **d.** $\frac{1}{2}$

6. The value of the integral

$$\int \left[\frac{\sin\theta \cdot \sin 2\theta\,(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} \right] d\theta$$ is (where, c is a constant of integration)

a. $\frac{1}{18}[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta]^{\frac{3}{2}} + c$

b. $\frac{1}{18}[9 - 2\cos^6\theta\ 3\cos^4\theta - 6\cos^2\theta]^{\frac{3}{2}} + c$

c. $\frac{1}{18}[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta]^{\frac{3}{2}} + c$

d. $\frac{1}{18}[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta]^{\frac{3}{2}} + c$

7. The value of $\int_{-1}^{1} x^2 e^{[x^3]}\,dx$, where $[t]$ denotes the greatest integer $\le t$, is

a. $\frac{e-1}{3e}$ **b.** $\frac{e+1}{3}$ **c.** $\frac{e+1}{3e}$ **d.** $\frac{1}{3e}$

8. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (ignore man's height). After sailing for 20 s, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then, the time taken (in seconds) by the boat from B to reach the base of the tower is

a. 10 **b.** $10\sqrt{3}$

c. $10(\sqrt{3}+1)$ **d.** $10(\sqrt{3}-1)$

9. A tangent is drawn to the parabola $y^2 = 6x$, which is perpendicular to the line $2x + y = 1$. Which of the following points does not lie on it?

a. (–6, 0) **b.** (4, 5) **c.** (5, 4) **d.** (0, 3)

10. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in

a. $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

b. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

c. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

d. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

11. Let the lines $(2-i)z=(2+i)\bar{z}$ and $(2+i)z+(i-2)\bar{z}-4i=0$, (here $i^2=-1$) be normal to a circle C. If the line $iz+\bar{z}+1+i=0$ is tangent to this circle C, then its radius is

a. $\frac{3}{\sqrt{2}}$ **b.** $\frac{1}{2\sqrt{2}}$ **c.** $3\sqrt{2}$ **d.** $\frac{3}{2\sqrt{2}}$

12. The image of the point (3, 5) in the line $x-y+1=0$, lies on

a. $(x-2)^2+(y-2)^2=12$

b. $(x-4)^2+(y+2)^2=16$

c. $(x-4)^2+(y-4)^2=8$

d. $(x-2)^2+(y-4)^2=4$

13. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$, intersect each other at an angle of 90°, then which of the following relations is true?

a. $a+b=c+d$ **b.** $a-b=c-d$

c. $a-c=b+d$ **d.** $ab=\frac{c+d}{a+b}$

14. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+\ldots\ldots+\frac{1}{n}}{n^2}\right)^n$ is equal to

a. $\frac{1}{2}$ **b.** 0

c. $\frac{1}{e}$ **d.** 1

15. The coefficients a, b and c of the quadratic equation, $ax^2+bx+c=0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is

a. $\frac{1}{72}$ **b.** $\frac{5}{216}$

c. $\frac{1}{36}$ **d.** $\frac{1}{54}$

16. The total number of positive integral solutions (x, y, z), such that $xyz=24$ is

a. 36 **b.** 24

c. 45 **d.** 30

17. The integer 'k', for which the inequality $x^2-2(3k-1)x+8k^2-7>0$ is valid for every x in R, is

a. 3 **b.** 2

c. 0 **d.** 4

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point

a. (5, 4) **b.** (4, 5)

c. (4, 4) **d.** (5, 5)

19. The statement $A\to(B\to A)$ is equivalent to

a. $A\to(A\wedge B)$ **b.** $A\to(A\to B)$

c. $A\to(A\leftrightarrow B)$ **d.** $A\to(A\vee B)$

20. If Rolle's theorem holds for the function $f(x)=x^3-ax^2+bx+4$, $x\in[1,2]$ with $f'\left(\frac{4}{3}\right)=0$, then ordered pair (a, b) is equal to

a. (5, 8) **b.** (−5, 8)

c. (5, −8) **d.** (−5, −8)

Section B : Numerical Type Questions

21. Let $f(x)$ be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at $x=-1$ and $x=1$. If $\lim_{x\to 0}\frac{f(x)}{x^3}=1$, then $5f(2)$ is equal to ....

22. The number of points at which the function $f(x)=|2x+1|-3|x+2|+|x^2+x-2|$, $x\in R$ is not differentiable, is ....

23. The graph of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to ....

24. Let $A_1, A_2, A_3, \ldots\ldots$ be squares, such that for each $n\geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ....

25. Let $A=\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers, such that $x+y+z>0$ and $xyz=2$. If $A^2=I_3$, then the value of $x^3+y^3+z^3$ is ....

26. If $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2+A)(I_2-A)^{-1}=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2+b^2)$ is equal to ....

27. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is ....

28. Let $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$, $\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and $\mathbf{c}=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ be three given vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r}\times\mathbf{a}=\mathbf{c}\times\mathbf{a}$ and $\mathbf{r}\cdot\mathbf{b}=0$, then $\mathbf{r}\cdot\mathbf{a}$ is equal to ....

29. If the system of equations
$kx + y + 2z = 1$ $3x - y - 2z = 2$
$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to ....

30. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is ....

Answers

Physics

1. (a)	2. (d)	3. (a)	4. (b)	5. (b)	6. (c)	7. (c)	8. (b)	9. (d)	10. (d)
11. (b)	12. (a)	13. (b)	14. (b)	15. (c)	16. (c)	17. (b)	18. (b)	19. (d)	20. (d)
21. (10)	22. (1)	23. (50)	24. (5)	25. (1)	26. (15)	27. (128)	28. (144)	29. (3600)	30. (1)

Chemistry

1. (d)	2. (c)	3. (d)	4. (d)	5. (a)	6. (c)	7. (c)	8. (a)	9. (b)	10. (a)
11. (d)	12. (b)	13. (b)	14. (b)	15. (d)	16. (b)	17. (c)	18. (c)	19. (d)	20. (a)
21. (4)	22. (70)	23. (741)	24. (375)	25. (173)	26. (526)	27. (5576)	28. (1)	29. (7)	30. (4)

Mathematics

1. (c)	2. (d)	3. (d)	4. (d)	5. (c)	6. (d)	7. (c)	8. (c)	9. (c)	10. (d)
11. (d)	12. (d)	13. (b)	14. (d)	15. (b)	16. (d)	17. (a)	18. (d)	19. (d)	20. (a)
21. (144)	22. (2)	23. (64)	24. (9)	25. (7)	26. (13)	27. (32)	28. (12)	29. (21)	30. (2)

Solutions

PHYSICS

1. *(a)* Thermal stress is defined as the stress, experienced by any rod on heating between two fixed rigid supports. On heating, the size of the rod increases but, if the two ends are free, rod will not experience any stress. i.e, there is no thermal stress will be produced in it.

Hence, option (a) is the correct.

2. *(d)* Given, diameter of the column tube,

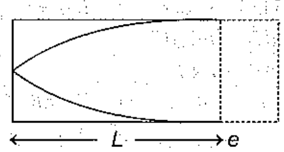
$$d = 6\text{ cm} = 6 \times 10^{-2}\text{ m}$$

Frequency of tuning fork, $f = 504$ Hz

Speed of sound at given temperature, $v = 336\text{ ms}^{-1}$

As, this is a closed organ pipe.

Let L be the length of tube and λ be the wavelength, then



$$L + e = \lambda/4 = v/4f \qquad \left(\because \lambda = \frac{v}{f}\right)$$

and $\quad e = 0.6 \times d/2$

$$= \frac{6}{10} \times \frac{6 \times 10^{-2}}{2} = 0.018$$

$$\Rightarrow \quad L + e = \frac{v}{4f} = \frac{336}{4 \times 504}$$

$$\Rightarrow \quad L = \frac{336}{2016} - 0.018 = 0.1667 - 0.018$$

$$= 0.1487\text{ m} = 14.87\text{ cm}$$

$$\simeq 14.8\text{ cm}$$

3. *(a)* Given,

$M_A = 200$ kg, $M_B = 400$ kg, $H_A = 600$ km, $H_B = 1600$ km

and $\quad R_A = R_E + H_A = 6400 + 600 = 7000$ km

$\quad R_B = R_E + H_B = 6400 + 1600 = 8000$ km

Let $T_A, T_B, \omega_A, \omega_B, R_A$ and R_B be the time period, angular frequencies and radii of satellites A and B, respectively.

Force on satellite A, $F_A = m_A \omega_A^2 R_A = \dfrac{GMm_A}{R_A^2}$

$$\Rightarrow \quad \omega_A^2 = \frac{GM}{R_A^3}$$

but, $\quad \omega_A = \dfrac{2\pi}{T_A}$

$$\therefore \quad \left(\frac{2\pi}{T_A}\right)^2 = \frac{GM}{R_A^3} \Rightarrow T_A = \sqrt{\frac{4\pi^2 R_A^3}{GM}}$$

Similarly, $\quad T_B = \sqrt{\dfrac{4\pi^2 R_B^3}{GM}}$

$$\therefore \quad T_B - T_A = \sqrt{\frac{4\pi^2}{GM}}\left(\sqrt{R_B^3} - \sqrt{R_A^3}\right)$$

$$= \frac{2\pi}{\sqrt{GM}}\left[\sqrt{(8 \times 10^3)^3} - \sqrt{(7 \times 10^3)^3}\right]$$

$$= \frac{2\pi \times 10^9}{\sqrt{GM}}(8\sqrt{8} - 7\sqrt{7})$$

$$= \frac{2\pi \times 10^9}{\sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}}}(4.107)$$

$$= \frac{2\pi}{\sqrt{4 \times 10^{14}}} \times 10^9 (4.107)$$

$$= \pi \times 10^2 \times 4.107 = 12.9 \times 10^2\text{ s}$$

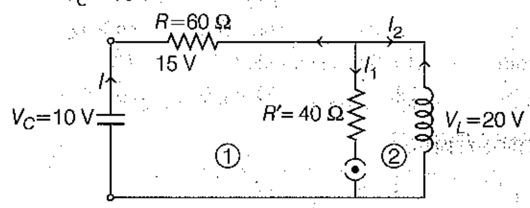
$$\simeq 1.33 \times 10^3\text{ s}$$

4. *(b)* Given, angular frequency, $\omega = 100\text{ rads}^{-1}$

$R = 60\Omega, V_R = 15$ V,

$R' = 40\Omega, V_{R'} = V_L = 20$ V

and $\quad V_C = 10$ V



By using Ohm's law,

$V = IR \Rightarrow I = V/R$

$\Rightarrow \quad I = 15/60 = 1/4$ A $\quad$ ... (i)

and $\quad I_1 = \dfrac{V_{R'}}{R'} = 20/40 = 1/2$ A $\quad$ ... (ii)

As, $\quad X_C = \dfrac{V_C}{I} = \dfrac{10}{1/4} = 40\,\Omega$

and $\quad X_C = \dfrac{1}{\omega C}$

$$\Rightarrow \quad C = \frac{1}{X_C\omega} = \frac{1}{40 \times 100}$$

$$= 0.25 \times 10^{-3}\text{ F} = 0.25\text{ mF}$$

$$= 250\,\mu\text{F}$$

By using KCL in loop 2,

$$I_2 = I - I_1$$

$$= 1/4 - 1/2 = -1/4\text{ A}$$

$$\therefore \quad X_L = \frac{V_L}{|I_2|} = \frac{20}{1/4} = 80\Omega \Rightarrow \omega L = 80$$

$$\Rightarrow \quad L = \frac{80}{\omega} = \frac{80}{100} = 0.8\text{ H}$$

5. *(b)* Let F_p, F_d and F_α be the forces and v_p, v_d and v_α be the velocities of proton, deuteron and α-particle, respectively.

Since, $\quad F = Bqv$

On dividing and multiplying F by m, we get

$$F = Bqv\frac{m}{m}$$

$$\Rightarrow \quad F = B\frac{q}{m}p \qquad (\because p = mv)$$

$$\Rightarrow \quad F \propto q/m \qquad (\because p \text{ and } B \text{ are same})$$

$$\therefore \quad F_p : F_d : F_\alpha = \frac{+q}{m} : \frac{+q}{2m} : \frac{+2q}{4m}$$

$$= 1 : 1/2 : 1/2 = 2 : 1 : 1$$

$\therefore \quad F_p : F_d : F_\alpha = \frac{+q}{m} : \frac{+q}{2m} : \frac{+2q}{4m}$

$= 1 : 1/2 : 1/2 = 2 : 1 : 1$

and $\quad F = Bqv$

$\Rightarrow \quad v \propto F/q$

$\therefore \quad v_p : v_d : v_\alpha = \frac{F_p}{q_p} : \frac{F_d}{q_d} : \frac{F_\alpha}{q_\alpha} = \frac{2x}{q} : \frac{x}{q} : \frac{x}{2q}$

$= 2 : 1 : 1/2 = 4 : 2 : 1$

6. *(c)* Given, frequency of modulated signal,

$f_m = 2 \text{ kHz} = 2 \times 10^3 \text{ Hz}$

Frequency of carrier signal, $f_c = 1 \text{ MHz}$

$= 1 \times 10^6 \text{ Hz}$

$= 1000 \text{ kHz}$

Then, bandwidth $= 2f_m = 4$ kHz

and side band frequency $= f_c \pm f_m$

$= (1000 \pm 2)$ kHz

$= 1002$ kHz and 998 kHz

Hence, option (c) is the correct.

7. *(c)* Given, length of simple pendulum, $l = 2$ m

Time period, $T = 2$ s

Let g_{eff} be the acceleration due to gravity.

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$

$\Rightarrow \quad g_{\text{eff}} = 4\pi^2 \frac{l}{T^2}$

$= 4\pi^2 \cdot \frac{2}{4} = 2\pi^2 \text{ ms}^{-2}$

8. *(b)* Given, pitch of screw gauge, $P = 1$ mm

Number of division, $n = 100$

$\therefore$ Least count (LC) $= \frac{P}{n} = 1/100 = 0.01$ mm

As, zero of circular division lies 8 divisions below.

$\therefore$ Zero error $= 8 \times \text{LC} = 8 \times 0.01 = 0.08$ mm

Since, 1st linear scale division coinside with 72nd circular scale division.

$\therefore$ Radius $(r) = \frac{[P + (72 \times \text{LC}) - \text{Zero error}]}{2}$

$= [1 + (72 \times 0.01) - 0.08]/2$

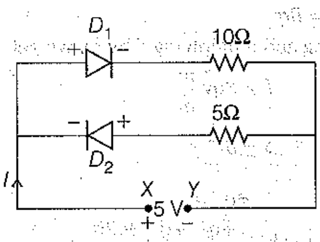
$= (1.72 - 0.08)/2 = 1.64/2$

$= 0.82$ mm

9. *(d)* Given, supply voltage,

$V = 5$ V

The circuit diagram, when positive terminal of the battery is connected to X is as shown below



Let I current is coming from battery.

$\therefore D_1$ will act as closed circuit as forward biased and D_2 will act as open circuit as reverse biased.

Now, by using Kirchhoff's voltage law,

$5 - V_{D_1} - 10I = 0$

$\Rightarrow \quad 5 - 0.7 - 10I = 0 \quad (\because V_{D_1} = 0.7 \text{ V})$

$\Rightarrow \quad 4.3 = 10I$

$\Rightarrow \quad I = 0.43$ A

10. *(d)* de-Broglie wavelength,

$\lambda = \frac{h}{\sqrt{2mqV}}$

$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p \cdot 2e}{m_p \cdot e}}$

$= \frac{2\sqrt{2}}{1} = 2 \times 1.414 = 2.8$

11. *(b)* Given, $C_p = 7/2\, R$, $C_V = 5/2\, R$

Since, change in internal energy $(dU) = nC_V dT$

Heat change $(dQ) = nC_p dT$

Work done $(dW) = nRdT$

$\therefore \quad dU : dQ : dW = nC_V dT : nC_p dT : nRdT$

$= C_V : C_p : R = \frac{5}{2}R : \frac{7}{2}R : R$

$= 5R : 7R : 2R$

$= 5 : 7 : 2$

12. *(a)* Given, initial speed of engine $= u$

Speed of engine last compartment $= v$

Let the length of train be l

and the speed of mid-point of train be v'.

Using third equation of motion,

$v^2 = u^2 + 2as$

$\Rightarrow \quad v'^2 = u^2 + 2al/2$

$\Rightarrow \quad v'^2 - u^2 = al \quad \ldots \text{(i)}$

Also, $\quad v^2 - u^2 = 2al \Rightarrow a = \frac{v^2 - u^2}{2l}$

Substituting the value of a in Eq. (i), we get

$v'^2 - u^2 = \frac{v^2 - u^2}{2l} \cdot l = \frac{v^2 - u^2}{2}$

$\Rightarrow \quad v' = \sqrt{\frac{v^2 - u^2}{2} + u^2} = \sqrt{\frac{v^2 + u^2}{2}}$

13. *(b)* The dimensional formulae of given terms are

Planck's constant $(h) = [ML^2T^{-1}]$

Kinetic energy $(E) = [ML^2T^{-2}]$

Electric potential $(V) = [ML^2I^{-1}T^{-3}]$

Linear momentum $(p) = [MLT^{-1}]$

So, the correct match is

A $\rightarrow$ 2, B $\rightarrow$ 3, C $\rightarrow$ 4, D $\rightarrow$ 1.

14. *(b)* Given, $d_1 = 0.05$ m, $d_2 = 0.2$ m

and $\quad B_1 : B_2 = 8 : 1$

Let radius of coil be r.

As we know that, magnetic field at a point on axis of a coil,

$B = \frac{\mu_0}{2} \cdot \frac{Ir^2}{(d_1^2 + r^2)^{3/2}}$

$$\therefore \quad \frac{B_1}{B_2} = \frac{r^2 \cdot (d_2^2 + r^2)^{3/2}}{(d_1^2 + r^2)^{3/2} \cdot r^2} = \frac{(d_2^2 + r^2)^{3/2}}{(d_1^2 + r^2)^{3/2}}$$

$$\Rightarrow \quad \frac{8}{1} = \left(\frac{d_2^2 + r^2}{d_1^2 + r^2}\right)^{3/2} \Rightarrow \frac{2^3}{1} = \left(\frac{d_2^2 + r^2}{d_1^2 + r^2}\right)^{3/2}$$

$$\Rightarrow \quad \frac{4}{1} = \frac{d_2^2 + r^2}{d_1^2 + r^2}$$

$$\Rightarrow \quad 4\, d_1^2 + 4r^2 = d_2^2 + r^2$$

$$\Rightarrow \quad 4(0.05)^2 + 4r^2 = (0.2)^2 + r^2 \Rightarrow 3r^2 = 0.03$$

$$\Rightarrow \quad r = 0.1\text{ m}$$

15. *(c)* Given, radius of sphere $= R$

Distance of particle from centre of Earth $= 3R$

Force between sphere and particle $= F_1$

Radius of cavity $= R/2$

Let mass of sphere $= M$

Mass of sphere with cavity $= M'$

Mass of particle $= m$

Now, $M' = \rho \cdot \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$

By using concept of gravitational force,

$$F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2} \quad \text{... (i)}$$

and $$F_{\text{cavity}} = \frac{GM'm}{(AB)^2} = \frac{G(M/8)m}{(2R + R/2)^2}$$

$$= \frac{4GMm}{25 \times 8\, R^2} \quad \text{... (ii)}$$

$$\therefore \quad F_2 = F_1 - F_{\text{cavity}} = \frac{GMm}{9R^2} - \frac{4}{25 \times 8}\frac{GMm}{R^2}$$

$$= \frac{GMm}{R^2}\left(\frac{1}{9} - \frac{1}{50}\right) = \frac{41}{50 \times 9}\frac{GMm}{R^2}$$

$$\because \quad \frac{F_1}{F_2} = \frac{50}{41}$$

16. *(c)* Given, initial amount of X and Y be N_1 and N_2.

Let half-life of X be t_x and y be t_y.

According to question, $t_x = \frac{t_y}{2} = t$

$$\Rightarrow \quad t_x = t \text{ and } t_y = 2t$$

After 3 half-lives of Y,

$$3t_y = 6t$$

As we know that,

$$N = N_0 e^{-\lambda t}$$

where, N is the number of nuclei left undecayed.

and $$t_{1/2} = \frac{0.693}{\lambda}$$

or $$\lambda = \frac{0.693}{t_{1/2}}$$

$$\Rightarrow \quad \lambda_1 = \frac{0.693}{t} \text{ and } \lambda_2 = \frac{0.693}{2t}$$

Since, after 3 half-lives of Y number of nuclei of both become equal.

$$\therefore \quad N_1 e^{-\lambda_1 6t} = N_2 e^{-\lambda_2 6t}$$

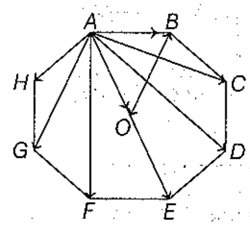
$$\Rightarrow \quad N_1/N_2 = e^{6t(-\lambda_2 + \lambda_1)}$$

$$\Rightarrow \quad N_1/N_2 = e^{6t\left(\frac{0.693}{t} - \frac{0.693}{2t}\right)}$$

$$= e^{0.693\left(\frac{6t}{t} - \frac{6t}{2t}\right)} = e^{0.693 \times 3} = 7.9 \simeq 8$$

$$\frac{N_1}{N_2} = \frac{8}{1}$$

17. *(b)* Given, $\mathbf{AO} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$



By using triangle law,

$$\mathbf{AB} = \mathbf{AO} + \mathbf{OB}$$

Similarly, $\mathbf{AC} = \mathbf{AO} + \mathbf{OC}$

$$\mathbf{AD} = \mathbf{AO} + \mathbf{OD}$$
$$\mathbf{AE} = \mathbf{AO} + \mathbf{OE}$$
$$\mathbf{AF} = \mathbf{AO} + \mathbf{OF}$$
$$\mathbf{AG} = \mathbf{AO} + \mathbf{OG}$$
$$\mathbf{AH} = \mathbf{AO} + \mathbf{OH}$$

Now, adding all vectors

$$\mathbf{AB} + \mathbf{AC} + \mathbf{AD} + \mathbf{AE} + \mathbf{AF} + \mathbf{AG} + \mathbf{AH}$$
$$= 7\,\mathbf{AO} + (\mathbf{OB} + \mathbf{OC} + \mathbf{OD} + \mathbf{OE} + \mathbf{OF} + \mathbf{OG} + \mathbf{OH}) \quad \text{... (i)}$$

By using cyclic vector,

$$\mathbf{OA} + \mathbf{OB} + \mathbf{OC} + \mathbf{OD} + \mathbf{OE} + \mathbf{OF} + \mathbf{OG} + \mathbf{OH} = 0$$
$$\Rightarrow \mathbf{OB} + \mathbf{OC} + \mathbf{OD} + \mathbf{OE} + \mathbf{OF} + \mathbf{OG} + \mathbf{OH}$$
$$= 0 - \mathbf{OA} = 0 + \mathbf{AO}$$

Substituting in Eq. (i), we get

$$\mathbf{AB} + \mathbf{AC} + \mathbf{AD} + \mathbf{AE} + \mathbf{AF} + \mathbf{AG} + \mathbf{AH} = 7\,\mathbf{AO} + \mathbf{AO} = 8\,\mathbf{AO}$$
$$= 8(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$$
$$= 16\hat{\mathbf{i}} + 24\hat{\mathbf{j}} - 32\hat{\mathbf{k}}$$

18. *(b)* Let v_A, v_B, M_A, M_B and R_A, R_B be the escape velocities, masses and radii of planet A and B, respectively.

As we know that,

$$v = \sqrt{2gR} = \sqrt{\frac{2GM}{R^2} \cdot R} = \sqrt{\frac{2GM}{R}}$$

where, G is the gravitational constant.

$$\Rightarrow \quad v \propto \sqrt{\frac{M}{R}}$$

Since, $v_A = v_B$

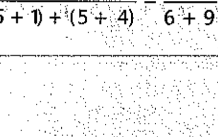
$$\because \quad \frac{M_A}{R_A} = \frac{M_B}{R_B} \Rightarrow M_A R_B = M_B R_A$$

or $M_1 R_2 = M_2 R_1$

Hence, option (b) is the correct.

19. *(d)* As we know that at time $t = 0$, inductor acts as open circuit.

Then, the circuit becomes



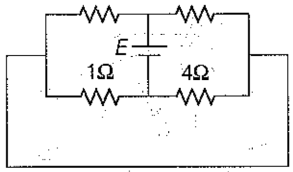
Therefore, $$R_{\text{eq}} = \frac{(5+1)(5+4)}{(5+1)+(5+4)} = \frac{6 \times 9}{6+9} = \frac{54}{15} = \frac{18}{5}$$

By using Ohm's law,

$$V = IR_{eq} \qquad [\because V = E]$$

$$\Rightarrow \quad I = \frac{E \times 5}{18} = \frac{5E}{18}$$

At $t = \infty$, inductor will act as short circuit. It is shown below



Therefore,

$$R_{eq} = \frac{5 \times 5}{5+5} + \frac{1 \times 4}{1+4} = \frac{25}{10} + \frac{4}{5} = \frac{5}{2} + \frac{4}{5} = \frac{33}{10}\,\Omega$$

and $I = \dfrac{E}{\frac{33}{10}} = \dfrac{10E}{33}$

20. *(d)* Given, $\dfrac{I_1}{I_2} = 2x$

$$\Rightarrow \quad I_1 = 2I_2 x$$

As we know,

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$$\therefore \quad \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} \qquad \text{(using dividendo rule)}$$

$$= \frac{2\sqrt{2I_2 x \cdot I_2}}{2I_2 x + I_2} = \frac{2\sqrt{2x}}{2x+1}$$

21. *(10)* Given, wavelength of transmission signal,

$$\lambda = 960 \text{ m}$$

Capacitance, $C = 2.56\ \mu\text{F} = 2.56 \times 10^{-6}\text{F}$

As we know resonance frequency,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Also, frequency $(f) = \dfrac{\text{speed } (v)}{\text{wavelength } (\lambda)}$

$$\therefore \quad \frac{v}{\lambda} = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \quad \sqrt{LC} = \frac{\lambda}{v \times 2\pi}$$

On squaring both sides, we get

$$\Rightarrow \quad LC = \frac{\lambda^2}{v^2 \times 4\pi^2} \Rightarrow L = \frac{\lambda^2}{v^2 \times 4\pi^2 \times C}$$

$$\Rightarrow \quad L = \frac{(960)^2}{(3 \times 10^8)^2 \times 4\pi^2 \times 2.56 \times 10^{-6}}$$

$$\therefore \quad L = 10 \times 10^{-8}\text{ H}$$

22. *(1)* Given, $\mathbf{E} = \dfrac{3E_0}{5}\,\hat{\mathbf{i}} + \dfrac{4}{5}E_0\hat{\mathbf{j}}$,

$$\mathbf{A}_1 = 0.2\text{ m}^2\ \hat{\mathbf{i}} \ \text{ and } \ \mathbf{A}_2 = 0.3\text{ m}^2\ \hat{\mathbf{j}}$$

Let ϕ_1 and ϕ_2 be the flux linked with area A_1 and A_2, respectively.

As we know that,

$$\phi = \oint \mathbf{E} \cdot d\mathbf{S} = \mathbf{E} \cdot \mathbf{A}$$

$$\Rightarrow \quad \phi_1 = (3/5E_0\hat{\mathbf{i}} + 4/5E_0\hat{\mathbf{j}}) \cdot 0.2\ \hat{\mathbf{i}} = 3/5\,E_0 \times 0.2$$

and similarly, $\phi_2 = 4/5\,E_0 \times 0.3$

Now, $$\frac{\phi_1}{\phi_2} = \frac{3/5E_0 \times 0.2}{4/5\,E_0 \times 0.3} = \frac{0.6}{1.2} = \frac{1}{2}$$

$$\therefore \quad a = 1$$

23. *(50)* Given, pressure $(p) \propto kV^3$

$$T_1 = 100^\circ\text{C},\ T_2 = 300^\circ\text{C}$$

$$\Rightarrow \quad \Delta T = T_2 - T_1 = 300 - 100 = 200^\circ\text{C}$$

By using ideal gas equation,

$$pV = nRT$$

$$\Rightarrow \quad kV^3 \cdot V = nRT \Rightarrow kV^4 = nRT$$

On differentiating both sides w.r.t temperature, we get

$$4kV^3\frac{dV}{dT} = nR$$

$$\Rightarrow \quad 4kV^3 dV = nRdT \Rightarrow kV^3 dV = nRdT/4$$

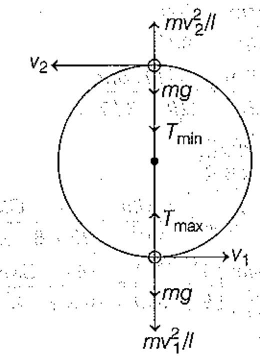
$$\Rightarrow \quad pdV = nRdT/4$$

As, work done $(W) = pdV = nRdT/4$

$$= \frac{nR}{4}\Delta T = \frac{nR}{4} \times 200 = 50nR$$

24. *(5)* Given, length of string, $l = 1$ m

T_{max} and T_{min} be the tension in string and v_1 and v_2 be the velocities of bob at bottom and top in vertical circle.



$$T_{max} = mg + mv_1^2/l$$

and $$T_{min} = mv_2^2/l - mg$$

$$\because \quad \frac{T_{max}}{T_{min}} = \frac{mg + mv_1^2/l}{mv_2^2/l - mg} = \frac{5}{1} \qquad \text{(given)}$$

$$\Rightarrow \quad mg + mv_1^2/l = 5mv_2^2/l - 5mg$$

Here, $v_1 = \sqrt{v_2^2 + 4gl}$

$$\Rightarrow \quad mg + \frac{m}{l}(v_2^2 + 4gl) = \frac{5mv_2^2}{l} - 5mg$$

$$\Rightarrow \quad g + \frac{v_2^2 + 4gl}{l} = \frac{5v_2^2}{l} - 5g$$

$$\Rightarrow \quad 6gl = 5v_2^2 - v_2^2 - 4gl \Rightarrow 10gl = 4v_2^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{10\,gl}{4}} = \sqrt{\frac{10 \times 10 \times 1}{4}}$$

$$= \sqrt{25} = 5 \text{ m/s}$$

25. *(1)* Given, length of $AB = 10$ m $= 1000$ cm

Length of one arm $= \dfrac{1000}{10} = 100$ cm

For no deflection,

In first case, $l_1 = 3 \times 100 + 80 = 380$ cm

In 2nd case, $l_2 = 7 \times 100 + 60 = 760$ cm

As we know that in balanced potentiometer,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{380}{760} = \frac{1}{2}$$

$$\therefore \quad a = 1$$

26. **(15)** Let v be the position of image, h_i be the height of image and h_o be the height of object.

Given, $h_i = h_o$

Since, magnification, $m = h_i/h_o = h_i/h$ ... (i)

By using lens formula,

$$1/f = 1/v - 1/u \Rightarrow 1/v = 1/f + 1/u$$

$$\Rightarrow \quad v = \frac{fu}{u+f}$$

$$\therefore \quad m = \frac{v}{u} = \frac{f}{f+u}$$

Now, from Eq. (i), m can be ± 1.

For, $m = +1 = \frac{f}{-10+f}$ ... (ii)

For $m = -1 = \frac{f}{-20+f}$ ... (iii)

On dividing Eq. (iii) by Eq. (ii), we get

$$-1 = \frac{-10+f}{-20+f}$$

$$\Rightarrow \quad 20 - f = -10 + f \Rightarrow 30 = 2f$$

$$\Rightarrow \quad f = 15 \text{ cm}$$

27. **(128)** Given, number of mercury drops, $n = 512$

Voltage of each drop, $V = 2V$

Let r, R be the radius of drop small and combined spherical drop, respectively.

Now, when all drops are joined into single drop, volume remains constant,

i.e. $512 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$$R^3 = (512\, r)^3 \Rightarrow R = 8r$$

$$\because \quad V = \frac{kq}{r} = 2 \quad \text{... (i)}$$

$$\therefore \quad V_{\text{net}} = \frac{kQ}{R}$$

where, Q be the charge of bigger sphere.

$$\Rightarrow \quad V_{\text{net}} = \frac{k \times 512q}{8r} \quad \text{... (ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{V_{\text{net}}}{V} = \frac{k \times 512\, q \times r}{8r \times kq} = \frac{512}{8}$$

$$\Rightarrow \quad V_{\text{net}} = 2 \times \frac{512}{8} = 128 \text{ V}$$

28. **(144)** Given, inductance of coil, $L = 2\text{H}$

Supply voltage, $V = 3t$ V

Let E be the energy stored in the coil.

Since, emf $V = L \cdot \frac{dI}{dt}$

$$3t = L\frac{dI}{dt} \Rightarrow 3tdt = LdI$$

On integrating both sides, we get

$$3\frac{t^2}{2} = LI$$

At $t = 4$ s,

$$\frac{3}{2} \times 4^2 = LI \Rightarrow \frac{3}{2} \times \frac{16}{L} = I$$

$$\Rightarrow \quad \frac{24}{L} = I \quad \text{...(i)}$$

As, $E = 1/2\, LI^2$

$$\therefore \quad E = 1/2\, L\frac{24^2}{L^2} \quad \text{[From Eq. (i)]}$$

$$= \frac{1}{2}\frac{24^2}{L} = \frac{24^2}{2 \times 2}$$

$$= 144 \text{ J}$$

29. **(3600)** Given, atomic mass of monoatomic gas,

$$m = 4\text{u}$$

Velocity of container, $u = 30 \text{ ms}^{-1}$

Final velocity of container, $v = 0 \text{ ms}^{-1}$

Let ΔT be the change in temperature and ΔU be the internal energy change.

Therefore,

$$\Delta \text{KE} = \Delta U$$

$$= (1/2)\, m(v^2 - u^2) = nC_V\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{1}{2}\frac{m(v^2 - u^2)}{n \cdot 3/2R} \quad \left(\because C_V = \frac{3}{2}R\right)$$

$$\Rightarrow \quad \frac{1}{2} \times m[v^2 - u^2] = n \cdot \frac{3}{2} \cdot R\Delta T$$

Now, $\frac{1}{2} \times 4\,|0^2 - 30^2| = 1 \times \frac{3}{2}R\Delta T$

$$\Delta T = \frac{4 \times 900}{3R} = \frac{x}{3R}$$

$$\therefore \quad x = 3600$$

30. **(1)** Given, potential energy, $U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$

As we know for equilibrium, differentiation of potential energy with respect to distance $\left(\frac{du}{dr}\right) = 0$

$$\Rightarrow \quad \frac{d}{dr}\left(\frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3\right) = 0$$

$$\Rightarrow \quad \frac{d}{dr}(\alpha r^{-10} - \beta r^{-5} - 3) = 0$$

$$\Rightarrow \quad -10\,\alpha r^{-11} + 5\beta r^{-6} - 0 = 0$$

$$\Rightarrow \quad 10\,\alpha r^{-11} = 5\,\beta\, r^{-6}$$

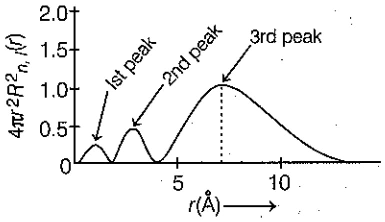
$$\Rightarrow \quad 2\alpha r^{-11} = \beta r^{-6}$$

$$\Rightarrow \quad \frac{2\alpha}{\beta} = \frac{r^{-6}}{r^{-11}} = r^5 \Rightarrow r = \left(\frac{2\alpha}{\beta}\right)^{1/5}$$

$$\therefore \quad a = 1$$

CHEMISTRY

1. *(d)* The correct plot for 3s-orbital is



For 3s,

value of $l = 0$

value of $n = 3$

Number of peak $= n - l = 3 - 0 = 3$

In graph *D*, three peaks are present, so this is the correct plot for 3s-orbital.

2. *(c)* According to Molecular Orbital Theory (MOT), electronic configuration and their bond order of given options are as follows

- $O_2^{2-} = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2, \pi 2p_y^2, \pi^* 2p_x^2, \pi^* 2p_y^2$

$$\text{Bond order} = \frac{[\text{No. of bonding electrons} - \text{No. of antibonding electrons}]}{2}$$

$$= \frac{10-8}{2} = \frac{2}{2} = 1$$

- $He_2^- = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^1$

$$\text{Bond order} = \frac{3-2}{2} = \frac{1}{2} = 0.5$$

- $Be_2 = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$

$$\text{Bond order} = \frac{4-4}{2} = 0$$

- $He_2^+ = \sigma 1s^2, \sigma^* 1s^1$

$$\text{Bond order} = \frac{2-1}{2} = \frac{1}{2} = 0.5$$

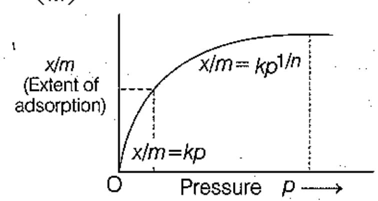
If bond order of chemical species is zero then that chemical species does not exist.

Therefore, Be_2 does not exist.

3. *(d)* pH of AgCN buffer solution = 3

$$[H^+] = 10^{-3}$$
$$K_{sp}(AgCN) = 2.2 \times 10^{-16}$$
$$K_a[HCN] = 6.2 \times 10^{-10}$$
$$AgCN \rightleftharpoons Ag^+ + CN^- \;......\; K_{sp}$$
$$CN^- + H^+ \rightleftharpoons HCN \;......\; \frac{1}{K_a}$$
$$AgCN + H^+ \rightleftharpoons HCN + Ag^+$$
$$K_{sp} \times \frac{1}{K_a} = \frac{[Ag^+][CN^-][HCN]}{[H^+][CN^-]}$$
$$[S] = \sqrt{\frac{K_{sp}[H^+]}{K_a}} \Rightarrow \frac{2.2 \times 10^{-16}}{6.2 \times 10^{-10}} = \frac{[S][S]}{10^{-3}}$$
$$[S]^2 = \frac{2.2 \times 10^{-16}}{6.2 \times 10^{-10}} \times 10^{-3}$$
$$S = 1.9 \times 10^{-5}$$

4. *(d)* According to Freundlich isotherm, at moderate pressure, extent of adsorption $\left(\frac{x}{m}\right) \propto (p)^{\frac{1}{n}}$



At moderate pressure,

$$\frac{x}{m} = k(p)^{1/n}$$
$$\frac{x}{m} \propto (p)^{1/n} \quad \text{... (i)}$$
$$\frac{x}{m} \propto p^x \text{ (Given in question)} \quad \text{... (ii)}$$

Compare Eqs. (i) and (ii),

$$(p)^{1/n} \propto p^x$$
$$\therefore \quad x = \frac{1}{n}$$

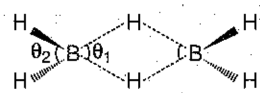
5. *(a)* Ellingham diagram is the graphical representation of ΔG *vs* *T*. For the formation of oxides of elements similar diagrams can also be constructed for sulphides and halides. Such diagrams can help us in pridicting the feasibility of thermal reduction of an ore.

6. *(c)* Oxidation involves increase in oxidation number and reduction involves decrease in oxidation number.

$$2I^- + 2H^+ + \overset{+1}{H_2}\overset{-1}{O_2} \longrightarrow \overset{0}{I_2} + 2\overset{+1}{H_2}\overset{-2}{O_2}$$

In this reaction, H_2O_2 oxidises I^- to I_2 and itself gets reduced to H_2O, so the reaction depicts oxidising nature of H_2O_2. While in other reactions H_2O_2 does not oxidise KIO_4, I_2 and Cl_2.

7. *(c)* Statement (c) is correct, whereas all other statements are incorrect.



$\theta_2 < \theta_1$

Correct statements are as follows

- Both B—H—B bridge bond having same bond length.
- B—H—B bond angle is 90°.
- BH_3 is electron deficient species and therefore act as Lewis acid.

8. *(a)* Both statement I and statement II are true. The +3 oxidation state of lanthanide is most stable and therefore lanthanide in +4 oxidation state has strong tendency to gain electrons and converted into +3 and therefore act as strong oxidising agent, e.g. Ce^{4+}.

$\therefore$ CeO_2 is used to oxidised alcohol, aldehyde and ketones.

Lanthanides in +2 oxidation state has strong tendency to loss electron and converted into +3 oxidation state e.g. Eu^{+2}.

$\therefore$ $EuSO_4$ acts as strong reducing agent.

9. *(b)* (a) V^{2+} - [Ar] $3d^3$; Cr^+ - [Ar] $3d^5$

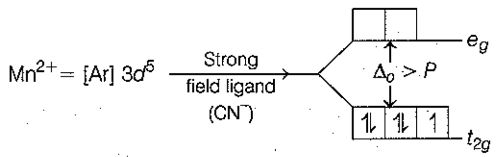
(b) Cr^+ - [Ar] $3d^5$; Mn^{2+} - [Ar] $3d^5$

(c) Ni^{2+} - [Ar] $3d^8$; Cu^+ - [Ar] $3d^{10}$

(d) Fe^{2+} - [Ar] $3d^6$; Co^+ - [Ar] $3d^7 4s^1$

Thus, in option (b), both ions have same outer most electronic configuration.

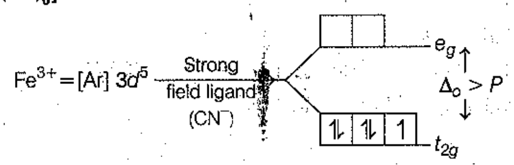
10. *(a)* $[Mn(CN)_6]^{4-}$ In this complex, oxidation state of Mn is + 2.



Hybridisation- d^2sp^3

Magnetic nature-Paramagnetic

$[Fe(CN)_6]^{-3}$



Hybridisation- d^2sp^3

Magnetic nature-Paramagnetic

11. *(d)* Reducing smog is a mixture of smoke, fog and sulphur dioxide. Therefore, statement I is false.

Tropospheric pollutants such as nitrogen oxide and sulphur oxide contribute to the formation of photochemical smog. So, statement II is true.

12. *(b)* $\underset{1.8\,g}{C_xH_yO_z} + O_2 \longrightarrow \underset{2.64\,g}{CO_2} + \underset{1.08\,g}{H_2O}$

$$n_C = \underset{(\text{Moles})}{n_{CO_2}} = \frac{2.64 \text{ (Given mass)}}{44 \text{ (Molecular mass)}} = 0.06$$

$$n_H = 2 \times n_{H_2O} = \frac{1.08}{18} \times 2 = 0.12$$

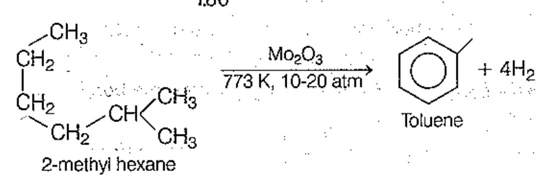
Weight of oxygen in $C_xH_yO_z$

$$= 1.80 - 12 \times \frac{2.64}{44} - \frac{1.08}{18} \times 2$$

$$= 1.80 - 0.72 - 0.12 = 0.96 \text{ g}$$

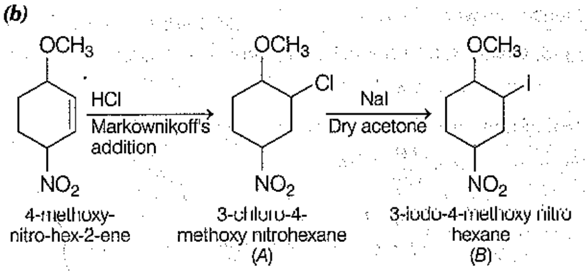
$$\% \text{ of oxygen by weight} = \frac{0.96}{1.80} \times 100 = 53.33\%$$

13. *(b)*



Oxides of molybdenum, vanadium and chromium, i.e. Mo_2O_3, V_2O_5 and Cr_2O_3 work as catalyst. When they are operated under certain temperature and pressure with *n*-heptane then they form toluene which is aromatic compound. Mo_2O_3 behave as aromatising agent.

14. *(b)*



Ist reaction is Markownikoff's addition of HCl on double bond while 2nd reaction is halide substitution by Finkelstein reaction in which chlorine get displaced by iodine.

15. *(d)* The complete reaction take place as follows

$$\underset{\text{Ethyl cyanide}}{CH_3CH_2C{\equiv}N} \xrightarrow[\Delta]{\text{(i) } H_3O^+} \underset{\text{Propanoic acid}}{CH_3{-}CH_2{-}\overset{\overset{O}{\|}}{C}{-}OH}$$

$$\xrightarrow{\text{(ii) } SOCl_2} \underset{\text{Propanoyl chloride}}{CH_3{-}CH_2{-}\overset{\overset{O}{\|}}{C}{-}Cl} \xrightarrow[\text{Rosenmund's reaction}]{\text{(iii) } Pd/BaSO_4, H_2} \underset{\text{Propanaldehyde}}{CH_3{-}CH_2{-}\overset{\overset{O}{\|}}{C}{-}H}$$

Final product of reaction is propanaldehyde.

16. *(b)* $\underset{\text{Ethene}}{CH_2{=}CH_2} + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)} \underset{\text{Ethanal}}{CH_3CHO}$

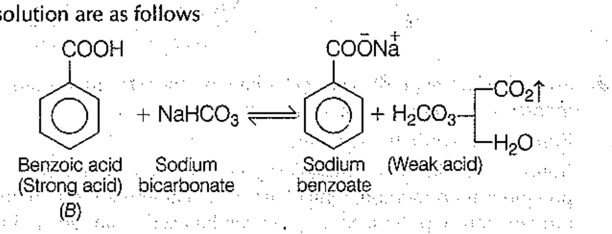
$$\underset{\text{Ethanol}}{CH_3CH_2OH} \xrightarrow{CrO_3\cdot H_2SO_4} \underset{\text{Ethanoic acid}}{CH_3{-}\overset{\overset{O}{\|}}{C}{-}OH}$$

$$\underset{\text{Acetonitrile}}{CH_3CN} \xrightarrow[H_2O]{\text{DIBAL-H}} \underset{\text{Ethanal}}{CH_3{-}\overset{\overset{O}{\|}}{C}{-}H}$$

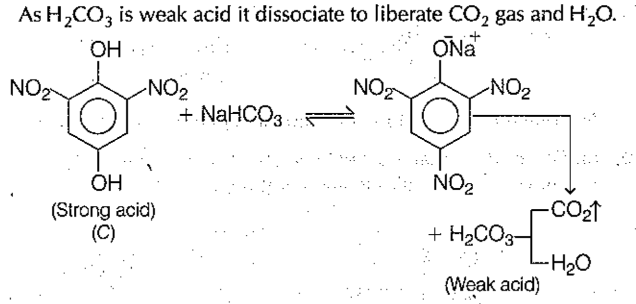
$$\underset{\text{Ethanol}}{CH_3CH_2OH} \xrightarrow[573\text{ K}]{Cu} \underset{\text{Ethanal}}{CH_3CHO}$$

Since, $CrO_3 \cdot H_2SO_4$ behave as strong oxidising agent and it converts alcohol directly to carboxylic acid. Thus, reaction (b) will not form acetaldehyde.

17. *(c)* The reactions of given compound with sodium bicarbonate solution are as follows

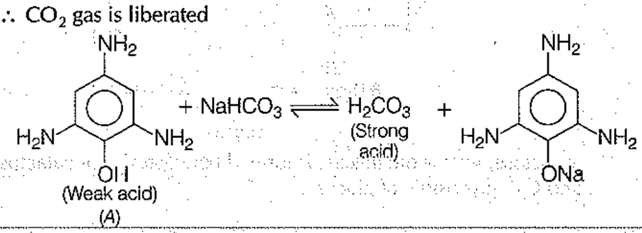


As H_2CO_3 is weak acid it dissociate to liberate CO_2 gas and H_2O.



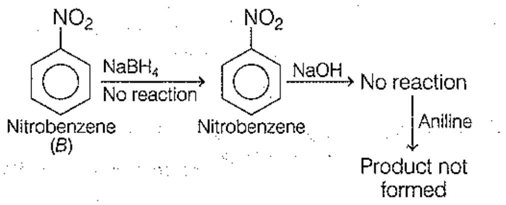
Equilibrium favours forward direction and CO_2 is liberated. In the above two reactions, H_2CO_3 is comparatively weak acid.

$\therefore$ CO_2 gas is liberated

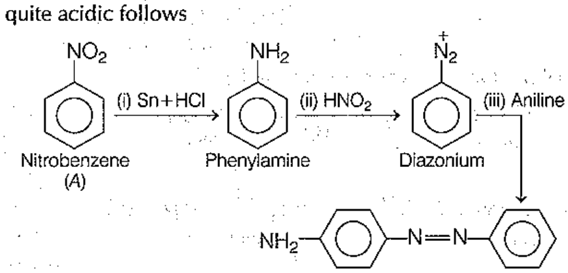


Equilibrium favours backward direction and CO_2 is not liberated. Thus, only B and C will liberate carbon dioxide with sodium bicarbonate solution.

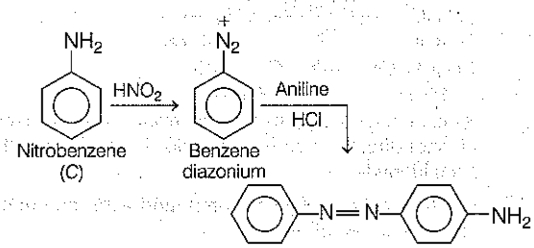
18. *(c)* Nitrobenzene in presence of $NaBH_4$, NaOH and aniline will not give p-amino azobenzene.



In case of (A) and (C), coupling reaction takes place as the medium is quite acidic follows



p-aminoazo benzene

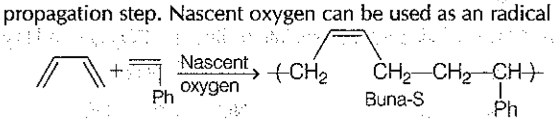


p-aminoazo benzene

19. *(d)* Only statement (d) is correct whereas other statements are incorrect. The correct statements are as follows

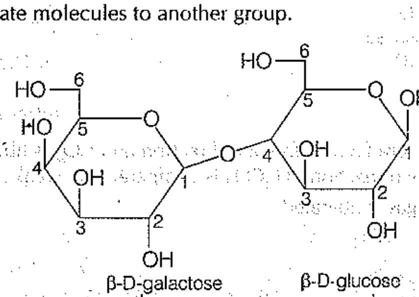
Buna-N is a synthetic polymer.

Buna-S is an elastomer. Buna-S is polymerised by addition polymerisation method, which needs radical initiator for chain propagation step. Nascent oxygen can be used as an radical initiator.



Neoprene is a synthetic rubber not an addition copolymer.

20. *(a)* Glycosidic linkage is a type of covalent bond that joins carbohydrate molecules to another group.



Lactose

In lactose, glycosidic linkage is formed between C-1 of galactose and C-4, glycosidic of glucose.

21. *(4)* As given, NaOH and Na_2CO_3 is titrated with N/10 HCl.

For NaOH,

Equivalents of NaOH = Equivalents of HCl

Equivalents of HCl = Normality × Volume (L)

$$= 0.1 \times \frac{17.5}{1000}$$

Equivalent of HCl $= 1.75 \times 10^{-3}$

Equivalents of NaOH $= 1.75 \times 10^{-3}$

Weight of NaOH = Equivalent of NaOH × equivalent weight of NaOH

$$= 40 \times 1.75 \times 10^{-3} = 0.07 \text{ g}$$

Now, weight % of NaOH

$$= \frac{0.07}{0.4} \times 100 = \frac{70}{4} = 17.5\%$$

Similarly for Na_2CO_3,

Equivalent of HCl = Equivalent of Na_2CO_3

Equivalent of $Na_2CO_3 = 0.1 \times \frac{1.5}{1000}$

$$= 0.15 \times 10^{-3}$$

Weight of Na_2CO_3 = Equivalent of Na_2CO_3 × equivalent weight of Na_2CO_3

$$= 0.15 \times 10^{-3} \times 106 = 15.9 \times 10^{-3} \text{ g}$$

Weight % of $Na_2CO_3 = \frac{15.9 \times 10^{-3}}{0.4} \times 100$

$$= 0.039 \times 100 = 3.9\% \simeq 4\%$$

22. *(70)* $p_1 = 35$ psi, $T_1 = 27°C = 300$ K

$p_2 = 40$ psi, $T_2 = ?$

According to Charle's law,

$$p \propto T$$

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} \Rightarrow \frac{40}{35} = \frac{T_2}{300}$$

$$\Rightarrow \quad T_2 = \frac{300 \times 40}{35} = 342.85 \text{ K} = 69.7°C$$

$$T_2 \simeq 70°C$$

Hence, answer is 70.

23. *(741)* $\Delta U = -742.24$ kJ mol^{-1}

Δn_g = [Number of gaseous molecules of products − Number of gaseous molecules of reactants]

$$NH_2CN(s) + \frac{3}{2}O_2(g) \longrightarrow N_2(g) + O_2(g) + H_2O(l)$$

$$\Delta n_g = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -742.24 + \frac{1}{2} \times \frac{8.314}{1000} \times 298$$

$$= -741 \text{ kJ/mol}$$

Hence, answer is 741.

24. *(375)* Given, $K_b(H_2O) = 0.52$ K kg mol^{-1}

$$A_2B_3 \longrightarrow 2A^{+3} + 3B^{-2}$$

No. of ions, $n = 5$, concentration, $m = 1$ molal (Given)

$$\alpha = \frac{60}{100} = 0.6$$

$$\Delta T_b = i \cdot K_f \cdot m$$

$$= [(1 + (n-1)\alpha] \times K_f \times m$$

$$\Delta T_b = [(1 + (5-1)0.6] \times 0.52 \times 1$$
$$= (1 + 2.4) \times 0.52$$
$$\Delta T_b = 1.768$$
$$T_b = 1.768 + 373.15$$
$$= 374.91 = 375 \text{ K}$$

25. *(173)* Given, Molarity of $CrO_4^{2-}(M_1) = 0.154$ M

Molarity of $S_2O_3^{2-}(M_2) = 0.25$M

Volume of $S_2O_3^{2-}(V_2) = 40$ mL

Volume of $CrO_4^{2-}(V_1) = ?$

$$\overset{+6}{Cr}\overset{2-}{O_4} + \overset{+6}{S_2}\overset{2-}{O_3} \longrightarrow \overset{+6}{S}O_4^{2-} + \overset{+3}{Cr}(OH)_4^-$$

Gram equivalent of CrO_4^{2-} = Gram equivalent of $S_2O_3^{2-}$

$$N_1V_1 = N_2V_2$$

Normality = Molarity × n factor

n for Cr = 6 − 3 = 3

n for S $\Rightarrow S_2O_3^{2-} \longrightarrow 2\,SO_4^{2-} + 8e^-$

$= 8$

$$0.154 \times 3 \times V_1 = 0.25 \times 40 \times 8$$
$$V_1 = 173 \text{ mL}$$

26. *(526)* According to Arrhenius equation,

$$\log K = \log A - \frac{E_a}{2.303RT}$$

Given, $\text{Slope} = \dfrac{E_a}{2.303\,R} = 10{,}000$

$$K_1 = 10^{-5}, K_2 = 10^{-4}$$
$$T_1 = 500 \text{ K}$$
$$\log_{10}\left[\frac{K_1}{K_2}\right] = \frac{Ea}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$
$$\log\left[\frac{10^{-4}}{10^{-5}}\right] = 10000\left[\frac{1}{500} - \frac{1}{T_2}\right]$$
$$\log 10 = 10000\left[\frac{1}{500} - \frac{1}{T_2}\right]$$
$$T_2 = 526.31 \simeq 526 \text{ K}$$

27. *(5576)* $Na(g) \xrightarrow{IE_1} Na^+(g)$ [$IE_1 = 495.8$ kJ mol^{-1}]

$Br(g) \xrightarrow[\text{gain enthalpy}]{\text{Electron}} (EG)\ Br^-(g)$ [Gain enthalpy = − 325 kJ mol^{-1}]

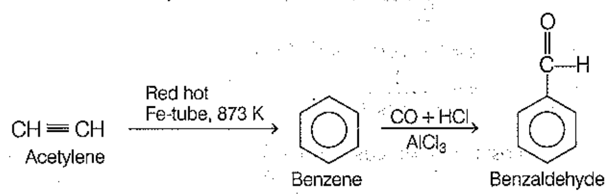
$Na^+ + Br^- \xrightarrow[\text{energy}]{\text{Lattice}} (LE)\ NaBr(s)$ [Lattice energy = − 728.4 kJ mol^{-1}]

$$\Delta H_{\text{Formation}} = IE_1 + \text{Gain enthalpy} + \text{Lattice energy}$$
$$\Delta H = 495.8 + (-325) + (-728.4)$$
$$= -557.6 \text{ kJ/mol}$$
$$= -5576 \times 10^{-1} \text{ kJ}$$

28. *(1)* BF_3, $SiCl_4$ and PCl_5 are easily hydrolysed while SF_6 is inert to hydrolysis due to the presence of sterically protected sulphur atom by six F atoms, which does not permit the reactions like hydrolysis to take place.

Thus, only 1 halide is inert to hydrolysis.

29. *(7)* The reaction take place as follows



Every carbon atom found in benzaldehyde (6 carbon in benzene ring +1 in aldehyde group), forms three bonds with neighbouring atoms-(one double bond + two single bond). Therefore, in benzaldehyde total number of sp^2 'C' are 7.

30. *(4)* The retention factor (R_f) value can be used to identify the components of a mixture (solutes).

$$R_f = \frac{\text{Distance travelled by solute}}{\text{Distance travelled by solvent front}}$$

On chromatogram, distance travelled by compound is 2 cm.

Distance travelled by solvent = 5 cm

$$\therefore \quad R_f = \frac{2}{5} = 4 \times 10^{-1}$$

MATHEMATICS

1. (c) Probability of missile to get intercepted $= \frac{1}{3}$

$\therefore$ Probability of missile to not get intercepted $= 1 - \frac{1}{3} = \frac{2}{3}$

Probability of missile to hit the target $= \frac{3}{4}$

$\therefore$ Probability of three missiles to hit the target

$$= \left(\frac{2}{3} \times \frac{3}{4}\right) \times \left(\frac{2}{3} \times \frac{3}{4}\right) \times \left(\frac{2}{3} \times \frac{3}{4}\right) = \frac{1}{8}$$

2. (d) Given, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$

$$y = \sum_{n=0}^{\infty} \sin^{2n}\phi$$

$$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$$

$\Rightarrow \quad x = 1 + \cos^2\theta + \cos^4\theta + \ldots\infty$

$\therefore \quad x = \frac{1}{1 - \cos^2\theta} \qquad \ldots \text{(i)}$

$1 - \cos^2\theta = \frac{1}{x} \Rightarrow \cos^2\theta = 1 - \frac{1}{x}$

$\Rightarrow \quad y = 1 + \sin^2\phi + \sin^4\phi + \ldots\infty$

$\therefore \quad y = \frac{1}{1 - \sin^2\phi} \qquad \ldots \text{(ii)}$

$1 - \sin^2\phi = \frac{1}{y} \Rightarrow \sin^2\phi = 1 - \frac{1}{y}$

$\Rightarrow \quad z = 1 + \cos^2\theta \cdot \sin^2\phi + \cos^4\theta \sin^4\phi + \ldots\infty$

$\therefore \quad z = \frac{1}{1 - \cos^2\theta \sin^2\phi} \qquad \ldots \text{(iii)}$

From Eqs. (i), (ii) and (iii), we get

$$z = \frac{1}{1 - \left(1 - \frac{1}{x}\right)\left(1 - \frac{1}{y}\right)} \qquad \left[\begin{array}{l} \because \cos^2\theta = 1 - \frac{1}{x} \\ \because \sin^2\phi = 1 - \frac{1}{y} \end{array}\right]$$

$$z = \frac{xy}{xy - (x-1)(y-1)}$$

$$z = \frac{xy}{xy - xy + x + y - 1}$$

$\Rightarrow \quad xz + yz - z = xy$

$\Rightarrow \quad xy + z = (x + y)z$

3. (d) Given, $f(n+1) = f(n) + f(1), \forall\, n \in N$

$\Rightarrow \quad f(n+1) - f(n) = f(1)$

It is an AP with common difference $= f(1)$

Also, general term $= T_n = f(1) + (n-1)f(1) = nf(1)$

$\Rightarrow \quad f(n) = nf(1)$

Clearly, $f(n)$ is one-one.

For fog to be one-one, g must be one-one.

For f to be onto, $f(n)$ should take all the values of natural numbers.

As, $f(x)$ is increasing, $f(1) = 1$

$\Rightarrow \quad f(n) = n$

If g is many-one, then fog is many one.

So, if g is onto, then fog is onto.

4. (d) Given, line $\Rightarrow \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = \lambda$ (let)

Any point on this line is $B(2\lambda + 1, 3\lambda - 1, -2\lambda + 1)$ and direction ratios of this line $= (2, 3, -2) = \mathbf{d}_1$

Let given point be $A(0, 1, 2)$.

Then direction ratio of

$$AB = (2\lambda + 1, 3\lambda - 2, -2\lambda - 1) = \mathbf{d}_2$$

$\because$ Both lines are perpendicular to each other.

$\therefore \quad \mathbf{d}_1 \cdot \mathbf{d}_2 = 0$

$2(2\lambda + 1) + 3(3\lambda - 2) - 2(-2\lambda - 1) = 0$

$\Rightarrow \quad 4\lambda + 2 + 9\lambda - 6 + 4\lambda + 2 = 0$

$\Rightarrow \quad 17\lambda = 2$

$\lambda = 2/17$

$\therefore$ Direction ratio of required line $\mathbf{d}_2 = (21, -28, -21)$

$= (3, -4, -3) = (-3, 4, 3)$

This line passes through $A(0, 1, 2)$.

$\therefore$ Required equation of line $\Rightarrow \frac{x-0}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

5. (c) Given, $l + m - n = 0 \qquad \ldots \text{(i)}$

and $l^2 + m^2 - n^2 = 0 \qquad \ldots \text{(ii)}$

On squaring Eq. (i), we get

$(l + m)^2 = n^2$

$\Rightarrow \quad l^2 + m^2 + 2lm = n^2 \qquad \ldots \text{(iii)}$

From Eqs. (ii) and (iii),

$$\begin{array}{r} l^2 + m^2 - n^2 = 0 \\ l^2 + m^2 + 2lm = n^2 \\ - \quad - \quad - \quad\quad - \\ \hline -n^2 - 2lm = -n^2 \end{array}$$

$\Rightarrow \quad 2lm = 0 \Rightarrow lm = 0$

$\Rightarrow \quad l = 0$ or $m = 0$

Case I When $l = 0$

$\Rightarrow \quad 0 + m - n = 0$

$\Rightarrow \quad m = n$

and $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad m^2 + m^2 = 1 \qquad [\because n = m \text{ and } l = 0]$

$\Rightarrow \quad m^2 = \frac{1}{2}$

$\therefore \quad m = \pm \frac{1}{\sqrt{2}} = n$

$\therefore \quad (l, m, n) = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

Case II When $m = 0$

then, $l + m - n = 0$

$\Rightarrow \quad l = n$

and $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad l^2 + 0 + l^2 = 1 \qquad [\because n = l \text{ and } m = 0]$

$\Rightarrow \quad l = \pm \frac{1}{\sqrt{2}}$

$\therefore \quad (l, m, n) = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ or $\left(\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$

$\Rightarrow \quad \mathbf{a} = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\mathbf{b} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

Then $\cos\alpha = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{1}{2}$

$\therefore \sin\alpha = \pm\frac{\sqrt{3}}{2}$

Now, $\cos^4\alpha + \sin^4\alpha = \frac{1}{16} + \frac{9}{16} = \frac{10}{16} = \frac{5}{8}$

6. *(d)* Let $I = \int \left[\frac{\dfrac{\sin\theta\cdot\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)}{\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}}{1-\cos 2\theta}\right] d\theta$

$\because \sin 2A = 2\sin A\cos A$ and $1 - \cos 2A = 2\sin^2 A$

$$I = \int \left[\frac{\dfrac{\sin\theta\cdot 2\sin\theta\cos\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)}{\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}}{2\sin^2\theta}\right] d\theta$$

$$I = \int [\cos\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}\,d\theta]$$

Put $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

$\therefore I = \int (t^6 + t^4 + t^2)\sqrt{2t^4 + 3t^2 + 6}\,dt$

$= \int (t^5 + t^3 + t)\sqrt{2t^6 + 3t^4 + 6t^2}\,dt$

Let $2t^6 + 3t^4 + 6t^2 = z$

$\therefore \quad dz = (12t^5 + 12t^3 + 12t)\,dt$

$\therefore \quad dz = 12(t^5 + t^3 + t)\,dt$

Now, $\frac{1}{12}\int\sqrt{z}\,dz = \frac{1}{12}\times\frac{z^{3/2}}{3/2} + c = \frac{1}{18}z^{3/2} + c$

$= \frac{1}{18}[2t^6 + 3t^4 + 6t^2]^{3/2} + c$

$= \frac{1}{18}[2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta]^{3/2} + c$

$= \frac{1}{18}[(1-\cos^2\theta)\{2(1-\cos^2\theta)^3 + 3 - 3\cos^2\theta + 6\}]^{3/2} + c$

$= \frac{1}{18}[(1-\cos^2\theta)(2\cos^4\theta - 7\cos^2\theta + 11)]^{3/2} + c$

$= \frac{1}{18}[-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + c$

$= \frac{1}{18}[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta]^{3/2} + c$

7. *(c)* Given, $\int_{-1}^{1} x^2 e^{[x^3]}dx$, where $[t]$ is greatest integer function.

$\because \quad [x^3] = 0\ \forall\ x \in (0,1)$

and $[x^3] = -1\ \forall\ x \in (-1, 0)$

So, $\int_{-1}^{1} x^2 e^{[x^3]}dx = \int_{-1}^{0} x^2 e^{-1}dx + \int_0^1 x^2 e^0 dx$

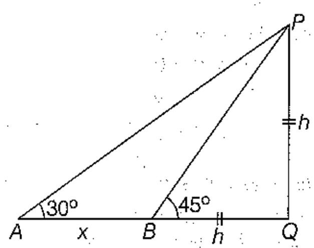
$= \frac{1}{e}\int_{-1}^{0} x^2 dx + \int_0^1 x^2 dx = \frac{1}{e}\times\left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{e}\times\left[0 + \frac{1}{3}\right] + \left[\frac{1}{3}\right] = \frac{1}{3e} + \frac{1}{3} = \left(\frac{1+e}{3e}\right)$

8. *(c)* In ΔPQB, $\tan 45° = \frac{PQ}{BQ}$

$1 = \frac{PQ}{BQ}$

$\Rightarrow \quad PQ = BQ = h$ (let)



In ΔPAQ, $\tan 30° = \frac{PQ}{AQ}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+h} \Rightarrow x + h = \sqrt{3}h$

$\Rightarrow \quad x = (\sqrt{3}-1)h$

$\because \quad$ Speed $= \frac{\text{Distance}}{\text{Time}}$

$\therefore$ Speed from A to $B = \frac{AB}{20} = \frac{x}{20}$

Also, distance from B to $Q = h$

$\therefore$ Time taken to reach Q from $B = \frac{BQ}{\text{Speed}}$

$= \frac{h}{x/20} = \frac{h\times 20}{x}$

$= \frac{h}{(\sqrt{3}-1)h}\times 20 \qquad [\because x = (\sqrt{3}-1)h]$

$= \frac{(\sqrt{3}+1)\times 20}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{(\sqrt{3}+1)\times 20}{2}$

$= 10(\sqrt{3}+1)$

9. *(c)* Given, parabola $\Rightarrow y^2 = 6x$

$\Rightarrow \quad y^2 = 4\left(\frac{3}{2}\right)x \qquad [\therefore y^2 = 4ax]$

and given, line $\Rightarrow 2x + y = 1$.

$\because$ Equation of any tangent to the parabola having slope m is

$y = mx + 3/2m \qquad [\because a = 3/2]$

Slope of line $2x + y = 1$ is $m_1 = -2$

$\because$ Tangent is perpendicular to this line,

$\therefore$ Slope of tangent $= m_2 = -\frac{1}{m_1} = \frac{1}{2}$

$\therefore$ Equation of tangent will be

$y = \frac{1}{2}x + \frac{3}{2}\times 2$

$\Rightarrow \quad y = \frac{x}{2} + 3$

or $\quad 2y = x + 6$

or $\quad x - 2y + 6 = 0$

Clearly, on putting the coordinates of point (5, 4), the equation of tangent is not satisfied.

$\therefore$ Point (5, 4) does not lie on this tangent.

10. *(d)* $\sin 2\theta + \tan 2\theta > 0$

$\Rightarrow \quad \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$

$\Rightarrow \quad \sin 2\theta\left(1 + \frac{1}{\cos 2\theta}\right) > 0$

$\Rightarrow \quad \sin 2\theta\left(\frac{\cos 2\theta + 1}{\cos 2\theta}\right) > 0$

$\Rightarrow \quad \tan 2\theta(2\cos^2\theta) > 0$

$\because \quad \cos 2\theta \neq 0$

$\Rightarrow \quad 1 - 2\sin^2\theta \neq 0$

$\therefore \quad \sin\theta \neq \pm \dfrac{1}{\sqrt{2}}$

Now, $\tan 2\theta(1 + \cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0$ as $1 + \cos 2\theta > 0$

$\Rightarrow 2\theta \in \left(0, \dfrac{\pi}{2}\right) \cup \left(\pi, \dfrac{3\pi}{2}\right) \cup \left(2\pi, \dfrac{5\pi}{2}\right) \cup \left(3\pi, \dfrac{7\pi}{2}\right)$

$\therefore \theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right) \cup \left(\pi, \dfrac{5\pi}{4}\right) \cup \left(\dfrac{3\pi}{2}, \dfrac{7\pi}{4}\right)$

$\left[\text{Since, } \sin\theta \neq \pm \dfrac{1}{\sqrt{2}}\right]$

11. *(d)* Given, $(2 - i)z = (2 + i)\bar{z}$

Let $z = x + iy$, then $\bar{z} = x - iy$

$\Rightarrow \quad (2 - i)(x + iy) = (2 + i)(x - iy)$

$\Rightarrow \quad 2x - ix + 2iy + y = 2x + ix - 2iy + y$

$\Rightarrow \quad 2ix - 4iy = 0$

$\therefore$ Equation of line $L_1 \Rightarrow x - 2y = 0$ ... (i)

Also, $(2 + i)z + (i - 2)\bar{z} - 4i = 0$

$\Rightarrow \quad (2 + i)(x + iy) + (i - 2)(x - iy) - 4i = 0$

$\Rightarrow \quad 2x + ix + 2iy - y + ix - 2x + y + 2iy - 4i = 0$

$\Rightarrow \quad 2ix + 4iy - 4i = 0$

$\therefore$ Equation of line $L_2 \Rightarrow x + 2y - 2 = 0$ ... (ii)

From Eqs. (i) and (ii),

$4y = 2$ or $y = 1/2$ and $x = 1$

Hence, centre $= (1, 1/2)$

Equation of third line $L_3 \Rightarrow iz + \bar{z} + 1 + i = 0$

$\Rightarrow \quad i(x + iy) + (x - iy) + 1 + i = 0$

$\Rightarrow \quad ix - y + x - iy + 1 + i = 0$

$\Rightarrow \quad (x - y + 1) + i(x - y + 1) = 0$

$\therefore$ Radius = Distance of point $(1, 1/2)$ to the line $x - y + 1 = 0$

$$\therefore \quad r = \frac{\left|1 - \frac{1}{2} + 1\right|}{\sqrt{1^2 + 1^2}} = \frac{3}{2\sqrt{2}}$$

12. *(d)* Image of $P(3, 5)$ on the line $x - y + 1 = 0$ is

$$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2}$$

$\Rightarrow \quad \dfrac{x-3}{1} = \dfrac{y-5}{-1} = 1$

$\Rightarrow \quad \dfrac{x-3}{1} = 1$ and $\dfrac{y-5}{-1} = 1$

$x = 4, y = 4$

$\therefore$ Required image is at $(4, 4)$.

Clearly, this point lies on $(x - 2)^2 + (y - 4)^2 = 4$ as $(4, 4)$ satisfies this equation.

13. *(b)* Given, curves $\dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$ and $\dfrac{x^2}{c} + \dfrac{y^2}{d} = 1$

$\because \quad \dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$

On differentiating both sides w.r.t. x, we get

$$\frac{2x}{a} + \frac{2y}{b} \cdot \frac{dy}{dx} = 0$$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{-bx}{ay}$ ... (i)

Also, $\dfrac{x^2}{c} + \dfrac{y^2}{d} = 1$

On differentiating both sides w.r.t. x, we get

$$\frac{2x}{c} + \frac{2y}{d} \cdot \frac{dy}{dx} = 0$$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{-dx}{cy}$ ... (ii)

$\because$ Both the curves intersect each other at 90°.

$\therefore$ Tangents at point of intersection must be perpendicular to each other.

$\therefore$ Product of slope of tangents $= -1$

$\dfrac{-bx}{ay} \times \dfrac{-dx}{cy} = -1$ [from Eqs. (i) and (ii)]

$\Rightarrow \quad bdx^2 = -acy^2$ ... (iii)

Also, on subtracting the equation of given curves, we get

$$\left(\frac{x^2}{a} + \frac{y^2}{b} - 1\right) - \left(\frac{x^2}{c} + \frac{y^2}{d} - 1\right) = 0$$

$\Rightarrow \quad x^2\left(\dfrac{1}{a} - \dfrac{1}{c}\right) + y^2\left(\dfrac{1}{b} - \dfrac{1}{d}\right) = 0$

or $\quad x^2\left(\dfrac{1}{a} - \dfrac{1}{c}\right) = -y^2\left(\dfrac{1}{b} - \dfrac{1}{d}\right)$ ... (iv)

Dividing Eq. (iii) by Eq. (iv),

$$\frac{bd}{\left(\frac{1}{a} - \frac{1}{c}\right)} = \frac{ac}{\left(\frac{1}{b} - \frac{1}{d}\right)}$$

$\Rightarrow \quad \dfrac{bd \times ac}{c - a} = \dfrac{ac \times bd}{d - b}$

$\Rightarrow \quad c - a = d - b$

or $\quad c - d = a - b$

or $\quad a - b = c - d$

14. *(d)* Given, limit form is 1^∞.

$$L = e^{\lim_{n \to \infty}\left(\frac{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}}{n}\right)}$$

Let $\quad S = 1 + \left(\dfrac{1}{2} + \dfrac{1}{3}\right) + \left(\dfrac{1}{4} + \dfrac{1}{5} + \dfrac{1}{6} + \dfrac{1}{7}\right) + \dots$

Clearly, $S < 1 + \left(\dfrac{1}{2} + \dfrac{1}{2}\right) + \left(\dfrac{1}{4} + \dfrac{1}{4} + \dfrac{1}{4} + \dfrac{1}{4}\right) + \dots + \underbrace{\left(\dfrac{1}{2^n} + \dots + \dfrac{1}{2^n}\right)}_{2^n \text{ times}}$

$S < 1 + 1 + 1 + 1 + \dots + 1$

$S < n + 1$

$\therefore \quad L = e^{\lim_{n \to \infty}\left(\frac{n+1}{2^{n+1} - 1}\right)} \Rightarrow L = e^0$

$\therefore \quad L = 1$

15. *(b)* Given, $ax^2 + bx + c = 0$

According to the question, $a, b, c \in \{1, 2, 3, 4, 5, 6\}$

$\therefore \quad n(S) = 6 \times 6 \times 6 = 216$

For equal roots

$\because \quad D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad b^2 = 4ac$

$\Rightarrow \quad ac = \dfrac{b^2}{4}$

If $b = 2, ac = 1 \Rightarrow (a = 1, c = 1)$

If $b = 4, ac = 4 \Rightarrow (a = 1, c = 4)$ or $(a = 4, c = 1)$

or $(a = 2, c = 2)$

If $b = 6, ac = 9 \Rightarrow (a = 3, c = 3)$

$\therefore$ Required probability $= 5/216$

16. *(d)* Given, $xyz = 24$

$\Rightarrow \quad xyz = 2^3 \cdot 3^1$

Let $\quad x = 2^{a_1} \cdot 3^{b_1}$,

$y = 2^{a_2} \cdot 3^{b_2}$,

$z = 2^{a_3} \cdot 3^{b_3}$

where, $a_1, a_2, a_3 \in \{0, 1, 2, 3\}$

$b_1, b_2, b_3 \in \{0, 1\}$

Case I $a_1 + a_2 + a_3 = 3$

$\therefore$ Non-negative solution $= {}^{3+3-1}C_{3-1} = {}^5C_2 = 10$

Case II $b_1 + b_2 + b_3 = 1$

$\therefore$ Non-negative solution $= {}^{1+3-1}C_{3-1} = {}^3C_2 = 3$

$\therefore$ Total solutions $= 10 \times 3 = 30$

17. *(a)* Given, $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$

Here, $a > 0$

$\therefore D < 0$

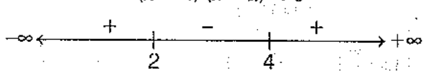
$\Rightarrow \quad [2(3k-1)]^2 - 4(8k^2 - 7) < 0$

$4(9k^2 + 1 - 6k) - 4(8k^2 - 7) < 0$

$\Rightarrow \quad k^2 - 6k + 8 < 0$

$\Rightarrow \quad (k - 4)(k - 2) < 0$

$\therefore$



$k \in (2, 4)$

$\therefore$ Required integer, $k = 3$

18. *(d)* Given, slope $= \dfrac{x^2 - 4x + y + 8}{x - 2}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2 - 4x + y + 8}{x - 2} = \frac{(x-2)^2 + (y+4)}{(x-2)}$$

$$= (x - 2) + \frac{y + 4}{x - 2}$$

Let $(x - 2) = t \Rightarrow dx = dt$

and $(y + 4) = u \Rightarrow dy = du$

$$\therefore \quad \frac{dy}{dx} = \frac{du}{dt}$$

Now, $\dfrac{dy}{dx} = (x - 2) + \dfrac{(y + 4)}{(x - 2)}$

$$\Rightarrow \quad \frac{du}{dt} = t + \frac{u}{t} \Rightarrow \frac{du}{dt} - \frac{u}{t} = t$$

Here, integrating factor (IF) $= 1/t$

$$\Rightarrow \quad u \cdot \left(\frac{1}{t}\right) = \int t\left(\frac{1}{t}\right) dt \Rightarrow u/t = t + c$$

$$\Rightarrow \quad \frac{(y + 4)}{(x - 2)} = (x - 2) + c$$

$\because$ It passes through origin [i.e. (0, 0)], then

$$\therefore \quad \frac{4}{-2} = -2 + c$$

$\Rightarrow \quad -2 + 2 = c \Rightarrow c = 0$

Hence, $\dfrac{(y + 4)}{(x - 2)} = (x - 2) + 0 \qquad [\because c = 0]$

$\Rightarrow \quad y + 4 = (x - 2)^2$

Clearly, this curve passes through (5, 5) as it satisfies the equation.

19. *(d)* Given, statement $A \to (B \to A)$

$\equiv A \to (\sim B \vee A)$

$\equiv \sim A \vee (\sim B \vee A)$

$\equiv (\sim A \vee A) \vee \sim B$

$\equiv T \vee \sim B \equiv T$

$\therefore T \vee B = T$

$\equiv (\sim A \vee A) \vee B$

$\equiv \sim A \vee (A \vee B)$

$\equiv A \to (A \vee B)$

20. *(a)* Given, $f(x) = x^3 - ax^2 + bx + 4, x \in [1, 2]$

Here, $f(1) = f(2)$

$\Rightarrow \quad 1 - a + b - 4 = 8 - 4a + 2b - 4$

$\Rightarrow \quad 3a - b = 7 \quad$ ... (i)

Also, $f'(x) = 3x^2 - 2ax + b$

According to the question, $f'\left(\dfrac{4}{3}\right) = 0$

$$\Rightarrow \quad 3 \times \left(\frac{4}{3}\right)^2 - 2a\left(\frac{4}{3}\right) + b = 0$$

$\Rightarrow \quad -8a + 3b = -16 \quad$ ... (ii)

From Eqs. (i) and (ii),

$a = 5, b = 8$

$\therefore \quad (a, b) = (5, 8)$

21. *(144)* $f(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

As, $\lim\limits_{x \to 0} \dfrac{f(x)}{x^3} = 1$ non-zero finite

So, $d = e = f = 0$

and $f(x) = x^3(x^3 + ax^2 + bx + c)$

Hence, $\lim\limits_{x \to 0} \dfrac{f(x)}{x^3} = c = 1$

Now, as $f(x) = x^6 + ax^5 + bx^4 + x^3$

and $f'(x) = 0$ at $x = 1$ and $x = -1$

i.e. $f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

Now, $f'(1) = 0$

$\Rightarrow \quad 6 + 5a + 4b + 3 = 0$

$\Rightarrow \quad 5a + 4b = -9 \quad$ ... (i)

and $f'(-1) = 0$

$\Rightarrow \quad -6 + 5a - 4b + 3 = 0$

$\Rightarrow \quad 5a - 4b = 3 \quad$ ... (ii)

From Eqs. (i) and (ii),

$a = -3/5$ and $b = -3/2$

$$\therefore \quad f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$$

$$\therefore \quad 5f(2) = 5\left[2^6 - \frac{3}{5}(2)^5 - \frac{3}{2}(2)^4 + (2)^3\right]$$

$$= 5\left[64 - \frac{3 \times 32}{5} - \frac{3 \times 16}{2} + 8\right]$$

$= 320 - 96 - 120 + 40$

$= 144$

22. *(2)* Given,

$f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$= |2x + 1| - 3|x + 2| + |x + 2||x - 1|$

Here, critical points are $x = \dfrac{-1}{2}, -2, 1$

$$\therefore f(x) = \begin{cases} x^2 + 2x + 3 & ; \quad x < -2 \\ -x^2 - 6x - 5 & ; \quad -2 < x < \frac{-1}{2} \\ -x^2 - 2x - 3 & ; \quad \frac{-1}{2} < x < 1 \\ x^2 - 7 & ; \quad x > 1 \end{cases}$$

$$\text{Now, } f'(x) = \begin{cases} 2x + 2 & ; \quad x < -2 \\ -2x - 6 & ; \quad -2 < x < \frac{-1}{2} \\ -2x - 2 & ; \quad \frac{-1}{2} < x < 1 \\ 2x & ; \quad x > 1 \end{cases}$$

Now, $f'(x)$ at $1, -2$ and $-1/2$ are

For $x = 1$,

$$f'(x) = 2x = 2 \times 1 = 2$$

and $-2x - 2 = -(2 \times 1) - 2 = -4$
both are not equal.

$\therefore$ Non-differentiable at $x = 1$

Similarly, for $x = -2$, $f'(x) = 2x + 2 = 2 \times (-2) + 2 = -2$ and $-2x - 6 = -2 \times (-2) - 6 = -2$ both are equal.

$\therefore$ Differentiable at $x = -2$

and for $x = -1/2$, $f'(x) = -2x - 6$

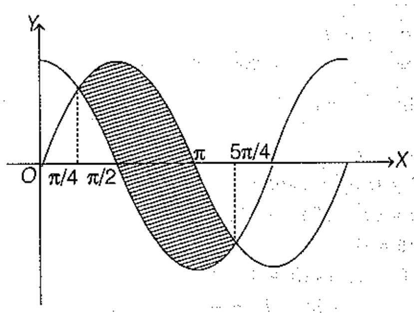
$$= -2 \times \left(\frac{-1}{2}\right) - 6 = -5 \text{ and}$$

$$-2x - 2 = -2 \times \left(\frac{-1}{2}\right) - 2 = -1 \text{ both are not equal.}$$

$\therefore$ Non-differentiable at $x = -1/2$

$\therefore$ The number of points at which $f(x)$ is non-differentiable is 2.

23. *(64)*



Required area of shaded region

$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx$$

$$= [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$$

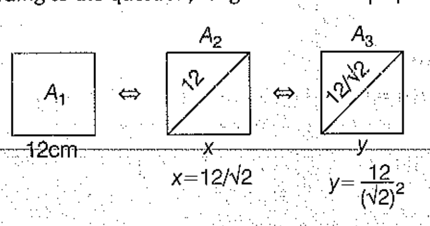
$$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$\therefore \quad A = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow \quad A^4 = (2\sqrt{2})^4 = 64$$

24. *(9)* According to the question, length of side of A_1 square is 12 cm.



$\because$ Side lengths are in GP.

$$\therefore \quad T_n = \frac{12}{(\sqrt{2})^{n-1}} \qquad \text{(Side of } n\text{ th square i.e. } A_n)$$

$$\therefore \text{Area} = (\text{Side})^2 = \left(\frac{12}{(\sqrt{2})^{n-1}}\right)^2 = \frac{144}{2^{n-1}}$$

According to the question, the area of A_n square < 1

$$\frac{144}{2^{n-1}} < 1$$

$$\Rightarrow \quad 2^{n-1} > 144$$

Here, the smallest possible value of n is 9.

25. *(7)* Here, $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = x^3 + y^3 + z^3 - 3xyz$$

Also, $A^2 = I_3$

$|A^2| = |I_3| = 1$

$\therefore \quad |(x^3 + y^3 + z^3 - 3xyz)^2| = 1$

$\Rightarrow \quad x^3 + y^3 + z^3 - 3xyz = 1 \qquad [\because x + y + z > 0]$

$\Rightarrow \quad x^3 + y^3 + z^3 = 1 + 3xyz$

$= 1 + 3(2) \qquad [\because xyz = 2]$

$= 7$

26. *(13)* $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$

and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \quad |(I_2 + A)(I_2 - A)^{-1}| = a^2 + b^2$

$$\Rightarrow \quad a^2 + b^2 = \frac{|I_2 + A|}{|I_2 - A|} \qquad \text{... (i)}$$

$$\text{Now, } I_2 + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$$

$$\text{Similarly, } I_2 - A = \begin{bmatrix} 1 & \tan\left(\frac{\theta}{2}\right) \\ -\tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$$

$$\text{Here, } |I_2 + A| = |I_2 - A| = \left(1 + \tan^2\left(\frac{\theta}{2}\right)\right)$$

$$\Rightarrow \quad \frac{|I_2 + A|}{|I_2 - A|} = 1 \qquad \text{... (ii)}$$

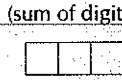
From Eqs. (i) and (ii),

$$a^2 + b^2 = 1$$

Now, $13(a^2 + b^2) = 13 \times 1 = 13$

27. *(32)* Given, digits $= \{1, 2, 3, 4, 5\}$

Numbers divisible by 3 (sum of digits divisible by 3).



Case I When sum is 12 $\rightarrow 3, 4, 5 \rightarrow 3! = 6$
Case II When sum is 9 $\rightarrow 2, 3, 4 \rightarrow 3! = 6$
Case III When sum is 9 $\rightarrow 1, 3, 5 \rightarrow 3! = 6$
Case IV When sum is 6 $\rightarrow 1, 2, 3 \rightarrow 3! = 6$

So, total numbers divisible by 3 = $6 \times 4 = 24$
Numbers divisible by 5 (ending with 5)

$$\underbrace{\boxed{}\,\boxed{}}_{4\times 3}\,\underset{\underset{1}{\uparrow}}{\boxed{5}} = 4\times 3 = 12$$

So, total numbers divisible by 5 = 12
Numbers divisible by 15, are 145, 415, 345, 435
i.e. total 4 numbers are divisible by both 3 and 5.
i.e. divisible by 15.
Hence, the required numbers which are divisible by 3 or 5
$= 24 + 12 - 4 = 32$

28. *(12)* Given, $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$,

$$\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}},$$
$$\mathbf{c} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$$
$$\mathbf{r} \times \mathbf{a} = \mathbf{c} \times \mathbf{a}$$
$\Rightarrow$ $\mathbf{r} \times \mathbf{a} - \mathbf{c} \times \mathbf{a} = 0$
$\Rightarrow$ $(\mathbf{r} - \mathbf{c}) \times \mathbf{a} = 0$
$\therefore$ $\mathbf{r} - \mathbf{c} = \lambda\, \mathbf{a}$
$\Rightarrow$ $\mathbf{r} = \lambda \mathbf{a} + \mathbf{c}$
$\Rightarrow$ $\mathbf{r}\cdot\mathbf{b} = \lambda \mathbf{a}\cdot\mathbf{b} + \mathbf{c}\cdot\mathbf{b}$ (taking dot with **b**)
$\Rightarrow$ $0 = \lambda \mathbf{a}\cdot\mathbf{b} + \mathbf{c}\cdot\mathbf{b}$ $[\because \mathbf{r}\cdot\mathbf{b} = 0]$
$\Rightarrow$ $\lambda\,(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}}) + (\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}}) = 0$
$\Rightarrow$ $\lambda(1 - 2) + 2 = 0$
$\Rightarrow$ $\lambda = 2$
$\therefore$ $\mathbf{r} = 2\mathbf{a} + \mathbf{c}$
$\Rightarrow$ $\mathbf{r}\cdot\mathbf{a} = 2\mathbf{a}\cdot\mathbf{a} + \mathbf{c}\cdot\mathbf{a}$ [taking dot with **a**]
$= 2|\mathbf{a}|^2 + \mathbf{a}\cdot\mathbf{c}$
$= 2(1 + 4 + 1) + (1 - 2 + 1)$
$\Rightarrow$ $\mathbf{r}\cdot\mathbf{a} = 12$

29. *(21)* Given equations, $kx + y + 2z = 1$
$$3x - y - 2z = 2$$
$$-2x - 2y - 4z = 3$$
For infinitely many solutions,
$$\Delta = \Delta x = \Delta y = \Delta z = 0$$
Here, $\Delta y = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$

$\Rightarrow$ $k(-8 + 6) - 1(-12 - 4) + 2(9 + 4) = 0$
$\Rightarrow$ $-2k + 16 + 26 = 0$
$\Rightarrow$ $2k = 42$
$\therefore$ $k = 21$

30. *(2)* Given, lines are $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ ... (i)
$$\sqrt{3}x - y - 4\sqrt{3}\,k = 0 \quad \text{... (ii)}$$
Multiply Eq. (ii) $\times\, k$ and then adding Eqs. (i) and (ii),
$$\sqrt{3}kx + ky - 4\sqrt{3} = 0$$
$$\underline{\sqrt{3}kx - ky - 4\sqrt{3}k^2 = 0}$$
$$(2\sqrt{3}x)k = 4\sqrt{3} + 4\sqrt{3}k^2$$
$\therefore$ $x = \dfrac{4\sqrt{3}(1 + k^2)}{2\sqrt{3}k} = 2\left(k + \dfrac{1}{k}\right)$

Subtracting Eq. (i) from Eq. (ii),
$$\sqrt{3}kx + ky - 4\sqrt{3} = 0$$
$$\sqrt{3}kx - ky - 4\sqrt{3}k^2 = 0$$
$$\underline{-\qquad +\qquad +}$$
$$2ky = 4\sqrt{3} - 4\sqrt{3}k^2$$
$$y = \frac{4\sqrt{3}(1 + k^2)}{2k} = 2\sqrt{3}\left(k + \frac{1}{k}\right)$$
We have, $x = 2\left(k + \dfrac{1}{k}\right)$ and $y = 2\sqrt{3}\left(\dfrac{1}{k} - k\right)$
$$\frac{x}{2} = \left(k + \frac{1}{k}\right) \quad \text{... (iii)}$$
$$\frac{y}{2\sqrt{3}} = \left(\frac{1}{k} - k\right) \quad \text{... (iv)}$$
Squaring and subtracting Eq. (iii) from Eq. (iv),
$$\frac{x^2}{4} - \frac{y^2}{12} = \left(k^2 + \frac{1}{k^2} + 2\right) - \left(\frac{1}{k^2} + k^2 - 2\right)$$
$$\frac{x^2}{4} - \frac{y^2}{12} = 4$$
or $\dfrac{x^2}{16} - \dfrac{y^2}{48} = 1$

Clearly, this is a hyperbola.
$\therefore$ $e^2 = 1 + \dfrac{b^2}{a^2}$
$= 1 + \dfrac{48}{16} = 1 + 3$
$e^2 = 4$
$\therefore$ $e = \sqrt{4} = 2$ ($\because$ e is positive)

JEE Main 2021

25 FEBRUARY SHIFT II

PHYSICS

Section A : Objective Type Questions

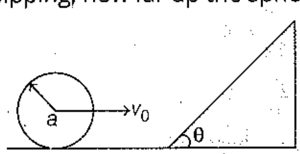
1. If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of

a. $[MLT^0]$ **b.** $[MLT^{-1}]$
c. $[M^0L^0T^0]$ **d.** $[LC^{-1}]$

2. A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is

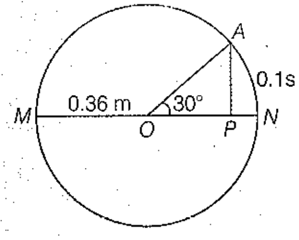
a. 45 m **b.** 25 m **c.** 35 m **d.** 50 m

3. A sphere of radius a and mass m rolls along a horizontal plane with constant speed v_0. It encounters an inclined plane at angle θ and climbs upwards. Assuming that it rolls without slipping, how far up the sphere will travel?



a. $\frac{v_0^2}{2g\sin\theta}$ **b.** $\frac{v_0^2}{5g\sin\theta}$ **c.** $\frac{10v_0^2}{7g\sin\theta}$ **d.** $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$

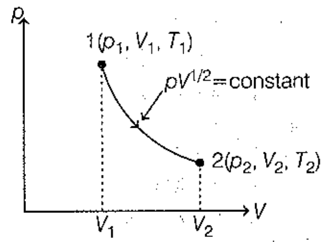
4. The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection P from A on the diameter MN represents the simple harmonic motion of P. The restoration force per unit mass when P touches M will be



a. 100 N **b.** 9.87 N
c. 50 N **d.** 0.49 N

5. Thermodynamic process is shown below on a p-V diagram for one mole of an ideal gas.

If $V_2 = 2V_1$, then the ratio of temperature $\frac{T_2}{T_1}$ is



a. $\frac{1}{\sqrt{2}}$ **b.** $\sqrt{2}$ **c.** $\frac{1}{2}$ **d.** 2

6. Given below are two statements:

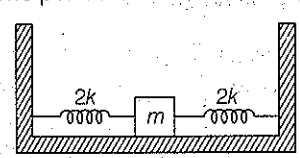
Statement I In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II In a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below.

a. Both Statement I and Statement II are true.
b. Both Statement I and Statement II are false.
c. Statement I is true but Statement II is false.
d. Statement I is false but Statement II is true.

7. Two identical springs of spring constant $2k$ are attached to a block of mass m and to fixed support (see figure).When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is

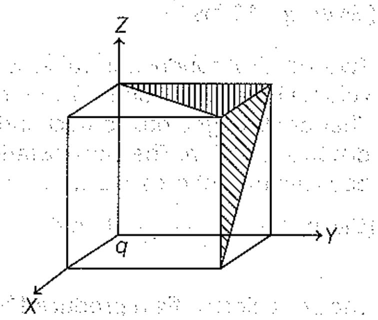


a. $2\pi\sqrt{\frac{m}{2k}}$ **b.** $2\pi\sqrt{\frac{m}{k}}$
c. $\pi\sqrt{\frac{m}{k}}$ **d.** $\pi\sqrt{\frac{m}{2k}}$

8. $Y = A\sin(\omega t + \phi_0)$ is the time-displacement equation of SHM. At $t = 0$, the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then, the initial phase angle ϕ_0 will be

a. $\frac{\pi}{3}$ **b.** $\frac{5\pi}{6}$
c. $\frac{\pi}{6}$ **d.** $\frac{2\pi}{3}$

9. A charge q is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\mathbf{E}$ through the shaded area is



a. $\frac{q}{48\varepsilon_0}$ **b.** $\frac{q}{4\varepsilon_0}$
c. $\frac{q}{8\varepsilon_0}$ **d.** $\frac{q}{24\varepsilon_0}$

10. An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle α with the plates. It leaves the plates at angle β with kinetic energy K_2. Then, the ratio of kinetic energies $K_1 : K_2$ will be

a. $\frac{\cos\beta}{\cos\alpha}$ **b.** $\frac{\cos\beta}{\sin\alpha}$
c. $\frac{\sin^2\beta}{\cos^2\alpha}$ **d.** $\frac{\cos^2\beta}{\cos^2\alpha}$

11. In a ferromagnetic material, below the Curie temperature, a domain is defined as

a. a macroscopic region with zero magnetisation
b. a macroscopic region with saturation magnetisation
c. a macroscopic region with randomly oriented magnetic dipoles
d. a macroscopic region with consecutive magnetic dipoles oriented in opposite direction

12. An L-C-R circuit contains resistance of $110\,\Omega$ and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be

a. 1A **b.** 1.5 A
c. 2A **d.** 2.5 A

13. The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. The new wavelength is

a. 309 nm **b.** 329 nm **c.** 382 nm **d.** 400 nm

14. Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1µm. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that

a. its size increases and intensity increases
b. its size increases, but intensity decreases
c. its size decreases, but intensity increases
d. its size decreases and intensity decreases

15. An electron of mass m_e and a proton of mass $m_p = 1836\, m_e$ are moving with the same speed. The ratio of their de-Broglie wavelength $\frac{\lambda_{\text{electron}}}{\lambda_{\text{proton}}}$ will be

a. 1 **b.** 1836 **c.** $\frac{1}{1836}$ **d.** 918

16. The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from $n = 2$ to $n = 1$ state is

a. 121.8 nm **b.** 194.8 nm **c.** 490.7 nm **d.** 913.3 nm

17. If a message signal of frequency f_m is amplitude modulated with a carrier signal of frequency f_c and radiated through an antenna, the wavelength of the corresponding signal in air is

a. $\frac{c}{f_c - f_m}$ **b.** $\frac{c}{f_c + f_m}$ **c.** $\frac{c}{f_c}$ **d.** $\frac{c}{f_m}$

18. For extrinsic semiconductors when doping level is increased,

a. Fermi level of p-type semiconductor will go upward and Fermi level of n-type semiconductors will go downward
b. Fermi level of p-type semiconductors will go downward and Fermi level of n-type semiconductor will go upward
c. Fermi level of p and n-type semiconductors will not be affected
d. Fermi level of both p-type and n-type semiconductors will go upward for $T > T_F$ K and downward for $T < T_F$ K, where T_F is Fermi temperature

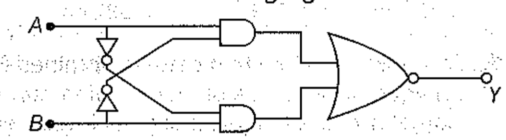
19. Match List-I with List-II.

List-I		List-II
A. Rectifier	1.	Used either for stepping up or stepping down the AC voltage
B. Stabiliser	2.	Used to convert AC voltage into DC voltage
C. Transformer	3.	Used to remove any ripple in the rectified output voltage
D. Filter	4.	Used for constant output voltage even when the input voltage or load current change

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
a.	2	1	3	4	**b.**	2	4	1	3
b.	2	1	4	3	**c.**	3	4	1	2

20. The truth table for the followng logic circuit is



a.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

b.

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

c.

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

d.

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Section B : Numerical Type Questions

21. Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is $n : 2$. The value of n will be ....

22. The initial velocity v_i required to project a body vertically upward from the surface of the Earth to reach a height of $10R$, where R is the radius of the Earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be ....

23. The percentage increase in the speed of transverse waves produced in a stretched string, if the tension is increased by 4%, will be .... %.

24. If $\mathbf{P} \times \mathbf{Q} = \mathbf{Q} \times \mathbf{P}$, the angle between **P** and **Q** is θ $(0° < \theta < 360°)$. The value of θ will be ....°.

25. A reversible heat engine converts one-fourth of the heat input into work. When the temperature of the sink is reduced by 52 K, its efficiency is doubled. The temperature in kelvin of the source will be ....

26. Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m. The charge on each of the sphere is $\frac{a}{21} \times 10^{-8}$ C.

The value of a will be ....

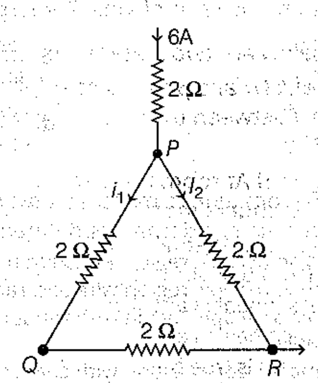
[Given, $g = 10 \text{ ms}^{-2}$]

27. Two identical conducting spheres with negligible volume have 2.1 nC and − 0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is .... $\times 10^{-9}$ N.

[Given, $4\pi\varepsilon_0 = \frac{1}{9 \times 10^9}$ SI unit]

28. The peak electric field produced by the radiation coming from the 8 W bulb at a distance of 10 m is $\frac{x}{10}\sqrt{\frac{\mu_0 c}{\pi}} \frac{\text{V}}{\text{m}}$. The efficiency of the bulb is 10% and it is a point source. The value of x is ....

29. A current of 6 A enters one corner P of an equilateral triangle PQR having three wires of resistance 2 Ω each and leaves by the corner R. The currents i_l in ampere is ....



30. The wavelength of an X-ray beam is 10 Å. The mass of a fictitious particle having the same energy as that of the X-ray photons is $\frac{x}{3}h$ kg. The value of x is ....

(h = Planck's constant)

CHEMISTRY

Section A : Objective Type Questions

1. Which among the following species has unequal bond lengths ?

a. XeF_4 **b.** SiF_4
c. SF_4 **d.** BF_4^-

2. The solubility of $Ca(OH)_2$ in water is

[Given: The. solubility product of $Ca(OH)_2$ in water $= 5.5 \times 10^{-6}$]

a. 1.11×10^{-2} **b.** 1.11×10^{-6}
c. 1.77×10^{-2} **d.** 1.77×10^{-6}

3. Which one of the following statements is false for hydrophilic sols ?

a. They do not require electrolytes for stability.
b. These sols are reversible in nature.
c. Their viscosity is of the order of that of H_2O.
d. The sols cannot be easily coagulated.

4. The correct order of bond dissociation enthalpy of halogens is

a. $F_2 > Cl_2 > Br_2 > I_2$ **b.** $I_2 > Br_2 > Cl_2 > F_2$
c. $Cl_2 > Br_2 > F_2 > I_2$ **d.** $Cl_2 > F_2 > Br_2 > I_2$

5. The method used for the purification of indium is

a. van-Arkel method **b.** liquation
c. zone refining **d.** vapour phase refining

6. Water does not produce CO on reacting with

a. CH_4 **b.** C
c. CO_2 **d.** C_3H_8

7. Given below are two statements.

Statement I α and β-forms of sulphur can change reversibly between themselves with slow heating or slow cooling.

Statement II At room temperature, the stable crystalline form of sulphur is monoclinic sulphur.

In the light of the above statements, choose the correct answer from the options given below.

a. Both statements I and II are true.
b. Both statements I and II are false.
c. Statement I is true but statement II is false.
d. Statement I is false but statement II is true.

8. The major components of German silver are

a. Cu, Zn and Ag **b.** Cu, Zn and Ni
c. Ge, Cu and Ag **d.** Zn, Ni and Ag

9. In which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment?

(i) $[FeF_6]^{3-}$ (ii) $[Co(NH_3)_6]^{3+}$
(iii) $[NiCl_4]^{2-}$ (iv) $[Cu(NH_3)_4]^{2+}$

a. (i) > (iii) > (iv) > (ii) **b.** (ii) > (iii) > (i) > (iv)
c. (iii) > (iv) > (ii) > (i) **d.** (ii) > (i) > (iii) > (iv)

10. Given below are two statements.

Statement I The pH of rain water is normally ~5.6.

Statement II If the pH of rain water drops below 5.6, it is called acid rain.

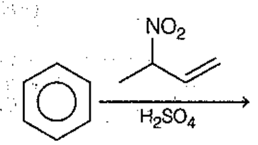
In the light of the above statements, choose the correct answer from the options given below.

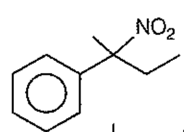
a. Both statements I and II are true.
b. Both statements I and II are false.
c. Statement I is true but statement II is false.
d. Statement I is false but statement II is true.

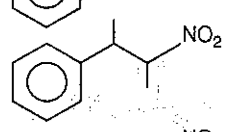
11. Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens?

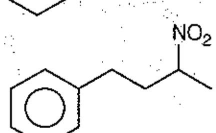
a. Hydrochloric acid
b. Sodium hydroxide
c. Nitric acid
d. Ammonia

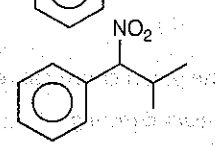
12. The major product of the following reaction is



a.


b.


c.


d.


13. The major product of the following reaction is

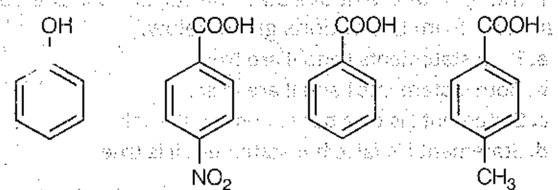
$$CH_3CH_2CH=CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

a. $CH_3CH_2CH_2CH_2CHO$
b. $CH_3CH_2CH_2CHO$
c. $CH_3CH_2CH=CH-CHO$
d. $CH_3CH_2\underset{\substack{|\\CHO}}{C}=CH_2$

14. The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is

a. CH_3COCl / $AlCl_3$, Br_2 / $AlBr_3$, HNO_3 / H_2SO_4, Zn/HCl
b. CH_3COCl / $AlCl_3$, Zn-Hg/HCl, Br_2 / $AlBr_3$, HNO_3 / H_2SO_4
c. Br_2 / $AlBr_3$, CH_3COCl / $AlCl_3$, HNO_3 / H_2SO_4, Zn/HCl
d. HNO_3 / H_2SO_4, Br_2 / $AlCl_3$, CH_3COCl / $AlCl_3$, Zn-Hg/HCl

15. The correct order of acid character of the following compounds is



I II III IV

Options

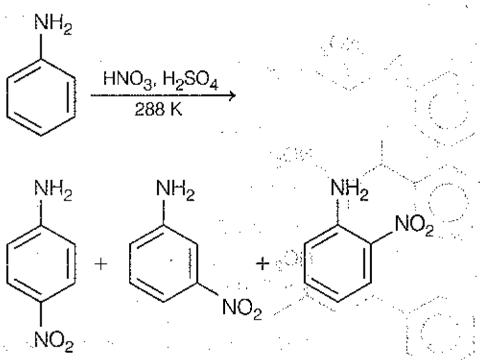
a. I > II > III > IV
b. III > II > I > IV
c. II > III > IV > I
d. IV > III > II > I

16. What is '*X*' in the given reaction?

$$\underset{CH_2OH}{\overset{CH_2OH}{|}} + \text{Oxalic acid} \xrightarrow{210°C} \underset{\text{(Major product)}}{X}$$

a. $\underset{CHO}{\overset{CHO}{|}}$ b. $\underset{CHO}{\overset{CH_2OH}{|}}$ c. $\underset{CH_2}{\overset{CH_2}{\|}}$ d. $\underset{CH_2}{\overset{CH-OH}{\|}}$

17.

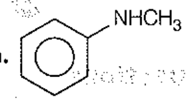
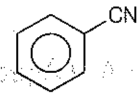
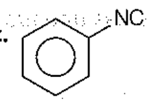
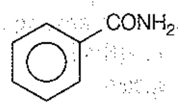


(A) (B) (C)

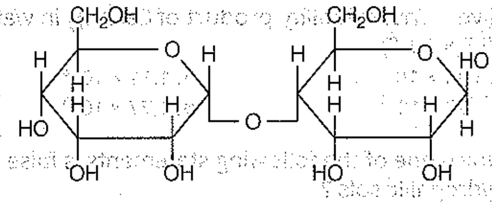
Correct statement about the given chemical reaction is

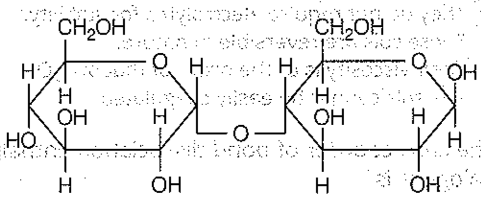
a. $-\ddot{N}H_2$ group is *ortho* and *para* directing, so product (*B*) is not possible.
b. reaction is possible and compound (*B*) will be the major product.
c. the reaction will form sulphonated product instead of nitration.
d. reaction is possible and compound (*A*) will be major product.

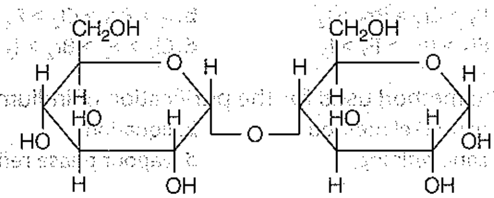
18. Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline?

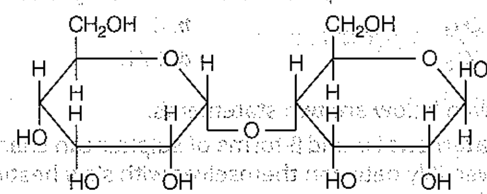
a. 
b. 
c. 
d. 

19. Which of the following is correct structure of α-anomer of maltose?

a.


b.


c.


d.


20. Given below are two statements.

Statement I The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4OH.

Statement II The dimethyl glyoxime is a bidentate neutral ligand.

In the light of the above statements, choose the correct answer from the options given below.

a. Both statements I and II are true.
b. Both statements I and II are false.
c. Statement I is true but statement II is false.
d. Statement I is false but statement II is true.

Section B : Numerical Type Questions

21. Consider titration of NaOH solution *versus* 1.25 M oxalic acid solution. At the end point following burette readings were obtained.

(i) 4.5 mL (ii) 4.5 mL (iii) 4.4 mL (iv) 4.4 mL (v) 4.4 mL

If the volume of oxalic acid taken was 10.0 mL, then the molarity of the NaOH solution is .... M. (Rounded off to the nearest integer)

22. The unit cell of copper corresponds to a face centered cube of edge length 3.596 Å with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is .... [Molar mass of Cu = 63.54 g ; Avogadro number = 6.022×10^{23}]

23. Electromagnetic radiation of wavelength 663 nm is just sufficient to ionise the atom of metal *A*. The ionisation energy of metal *A* in kJ mol^{-1} is .... (Rounded off to the nearest integer)
[$h = 6.63 \times 10^{-34}$ J-s, $c = 3.00 \times 10^8 \, ms^{-1}$, $N_A = 6.02 \times 10^{23} \, mol^{-1}$]

24. Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is .... kJ mol^{-1}. (Rounded off to the nearest integer)
[$R = 8.314 \, J \, mol^{-1}K^{-1}$]

25. If a compound *AB* dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is .... molal. (Rounded off to the nearest integer)
[$K_b = 0.52$ K kg mol^{-1}]

26. Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution. (Assuming fixed $[Cu^{2+}]$ and $p_{NO} = p_{NO_2}$), the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of $2x$ is .... (Rounded off to the nearest integer)
[Given, $E^\circ_{Cu^{2+}/Cu} = 0.34$ V, $E^\circ_{NO_3^-/NO} = 0.96$ V,
$E^\circ_{NO_3^-/NO_2} = 0.79$ V and at 298 K, $\frac{RT}{F}(2.303) = 0.059$]

27. The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C. The value of activation energy in kJ mol^{-1} is .... (Rounded off to the nearest integer)
[$R = 8.314 \, J \, K^{-1} \, mol^{-1}$]

28. Among the following, number of metal(s) which can be used as electrodes in the photoelectric cell is .... (Integer answer)
(i) Li (ii) Na (ii) Rb (iv) Cs

29. The spin only magnetic moment of a divalent ion in aqueous solution (atomic number = 29) is .... BM.

30. The number of compound(s) given below which contain(s) — COOH group is ....
(i) Sulphanilic acid (ii) Picric acid
(iii) Aspirin (iv) Ascorbic acid

MATHEMATICS

Section A : Objective Type Questions

1. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is
a. 4 b. 1
c. 2 d. 3

2. Let *A* be a 3 × 3 matrix with det (*A*) = 4. Let R_i denote the *i*th row of *A*. If a matrix *B* is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on $2A$, then det (*B*) is equal to
a. 16 b. 80
c. 64 d. 128

3. The following system of linear equations
$2x + 3y + 2z = 9$
$3x + 2y + 2z = 9$
$x - y + 4z = 8$
a. does not have any solution
b. has a unique solution
c. has infinitely many solutions
d. has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

4. If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$, then
a. $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in AP
b. $I_2 + I_4, I_3 + I_5, I_4 + I_6$ are in AP
c. $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in GP
d. $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ are in GP

5. A function $f(x)$ is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series
$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \ldots + f\left(\frac{39}{20}\right)$ is equal to
a. $\frac{29}{2}$ b. $\frac{49}{2}$ c. $\frac{39}{2}$ d. $\frac{19}{2}$

6. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is
a. 4 b. 3 c. 2 d. 1

7. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to

a. $a + 1$
b. $a + \frac{1}{a}$
c. $2\sqrt{a}$
d. $2a$

8. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx, x > 0$, is equal to

(where, c is a constant of integration)

a. $\log_e |x^2 + 5x - 7| + c$
b. $4\log_e |x^2 + 5x - 7| + c$
c. $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$
d. $\log_e \sqrt{x^2 + 5x - 7} + c$

9. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to

a. 3
b. -3
c. 7
d. -7

10. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is

a. $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$
b. $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$
c. $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$
d. $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

11. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is

a. $\frac{1}{\sqrt{2}}$
b. $\frac{1}{2\sqrt{2}}$
c. 0
d. $\frac{1}{2}$

12. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is

a. $\frac{x^2}{9} - \frac{y^2}{16} = 1$
b. $\frac{x^2}{9} - \frac{y^2}{4} = 1$
c. $\frac{x^2}{9} - \frac{y^2}{25} = 1$
d. $x^2 - y^2 = 9$

13. A plane passes through the points $A(1, 2, 3)$, $B(2, 3, 1)$ and $C(2, 4, 2)$. If O is the origin and P is $(2, -1, 1)$, then the projection of **OP** on this plane is of length

a. $\sqrt{\frac{2}{3}}$
b. $\sqrt{\frac{2}{11}}$
c. $\sqrt{\frac{2}{7}}$
d. $\sqrt{\frac{2}{5}}$

14. $\lim_{n \to \infty} \left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2}\right]$ is equal to

a. 1
b. $\frac{1}{2}$
c. $\frac{1}{3}$
d. $\frac{1}{4}$

15. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10%, respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is

a. $\frac{7}{45}$
b. $\frac{8}{45}$
c. $\frac{28}{45}$
d. $\frac{14}{45}$

16. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then, the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is

a. $\frac{1}{5}$
b. $\frac{2}{9}$
c. $\frac{97}{297}$
d. $\frac{122}{297}$

17. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to

a. $\frac{1}{2}$
b. $\frac{\sqrt{3}}{2}$
c. $\frac{1 - \sqrt{3}}{2}$
d. $\frac{1 + \sqrt{3}}{2}$

18. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then,

a. $2y = 91x$
b. $2y = 273x$
c. $y = 91x$
d. $y = 273x$

19. $\text{cosec}\left[2\cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to

a. $\frac{56}{33}$
b. $\frac{65}{33}$
c. $\frac{65}{56}$
d. $\frac{75}{56}$

20. The contrapositive of the statement; "If you will work, you will earn money" is

a. to earn money, you need to work
b. you will earn money, if you will not work
c. if you will not earn money, you will not work
d. if you will earn money, you will work

Section B : Numerical Type Questions

21. A function f is defined on [-3, 3] as

$$f(x)=\begin{cases}\min\{|x|, 2-x^2\}, & -2\le x\le 2\\ [|x|] & , 2<|x|\le 3\end{cases}$$

where, $[x]$ denotes the greatest integer $\le x$. The number of points, where f is not differentiable in (–3, 3) is ....

22. If the curve $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)\,dx + x\,dy = 0$, passes through the intersection of the lines $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to ....

23. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ....

24. If $\lim_{x\to 0}\dfrac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of $a - 2b$ is ....

25. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to ....

26. The value of $\int_{-2}^{2}|3x^2-3x-6|\,dx$ is ....

27. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is ....

28. A line 'l' passing through origin is perpendicular to the lines

$l_1 : \mathbf{r} = (3+t)\hat{\mathbf{i}} + (-1+2t)\hat{\mathbf{j}} + (4+2t)\hat{\mathbf{k}}$

$l_2 : \mathbf{r}\,(3+2s)\hat{\mathbf{i}} + (3+2s)\hat{\mathbf{j}} + (2+s)\hat{\mathbf{k}}$

If the coordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a, b, c), then $18(a + b + c)$ is equal to ....

29. A line is a common tangent to the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a + c)$ is equal to ....

30. Let $\mathbf{a} = \hat{\mathbf{i}} + \alpha\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{i}} - \alpha\hat{\mathbf{j}} + \hat{\mathbf{k}}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\mathbf{a}$ and $\mathbf{b}$ is $8\sqrt{3}$ square units, then $\mathbf{a}\cdot\mathbf{b}$ is equal to ....

Answers

Physics

1. (c)	2. (a)	3. (*)	4. (b)	5. (b)	6. (c)	7. (c)	8. (c)	9. (d)	10. (d)
11. (b)	12. (c)	13. (c)	14. (c)	15. (b)	16. (a)	17. (c)	18. (b)	19. (b)	20. (b)
21. (1)	22. (10)	23. (2)	24. (180)	25. (104)	26. (630)	27. (7.56)	28. (2)	29. (2)	30. (10)

Chemistry

1. (c)	2. (a)	3. (c)	4. (c)	5. (c)	6. (c)	7. (c)	8. (b)	9. (a)	10. (a)
11. (c)	12. (b)	13. (a)	14. (b)	15. (c)	16. (c)	17. (d)	18. (c)	19. (c)	20. (a)
21. (6)	22. (9077)	23. (180)	24. (15)	25. (3)	26. (4)	27. (52)	28. (1)	29. (2)	30. (1)

Mathematics

1. (b)	2. (c)	3. (b)	4. (a)	5. (c)	6. (c)	7. (c)	8. (b)	9. (d)	10. (a)
11. (b)	12. (a)	13. (b)	14. (b)	15. (c)	16. (c)	17. (d)	18. (a)	19. (c)	20. (c)
21. (5)	22. (1)	23. (45)	24. (5)	25. (4)	26. (19)	27. (1)	28. (44)	29. (9)	30. (2)

Note *(*) None of the option is correct.*

Solutions

PHYSICS

1. *(c)* Dimensional formula of $[e] = [IT]$

$$[h] = [M^1L^2T^{-1}]$$

$$[c] = [M^0LT^{-1}]$$

$$\left[\frac{1}{4\pi\varepsilon_0}\right] = [M^1L^3T^{-4}I^{-2}]$$

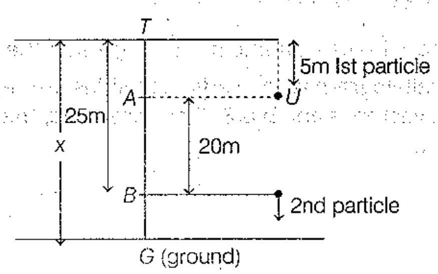
where, $\frac{1}{4\pi\varepsilon_0}$ is Coulomb's constant.

Therefore, dimensional formula of

$$\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc} = [M^1L^3T^{-4}I^{-2}]\cdot\frac{[IT]^2}{[M^1L^2T^{-1}]\,[M^0LT^{-1}]}$$

$$= [M^0L^0T^0I^0] \text{ or } [M^0L^0T^0]$$

2. *(a)* Let the total height of building be x.



$$TA = 5\text{ m}$$

$$TB = 25\text{ m}$$

$\therefore \quad AG = x - 5 \text{ and } BG = x - 25$

For initial conditions, from second equation of motion under gravity,

$$s = ut + 1/2gt^2$$

where, $g = 10\text{ ms}^{-2}$

$\therefore \quad 5 = 0 + 1/2 \times 10t^2$

$\Rightarrow \quad t = 1\text{ s}$

Now, by first equation of motion under gravity,

$$v_A = u + gt$$

$$= 0 + 10 = 10\text{ ms}^{-1}$$

From second equation of motion,

$$x - 5 = v_At + 1/2gt^2 \quad \ldots \text{(i)}$$

Similarly, $x - 25 = 1/2gt^2$

Put the above value in Eq. (i), we get

$$x - 5 = 10t + x - 25$$

$$20 = 10t \Rightarrow t = 2\text{ s}$$

Put the value of t in Eq. (i), we get

$$x - 5 = 10 \times 2 + 1/2 \times 10 \times 4$$

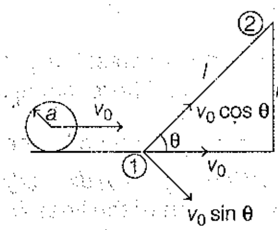
$\Rightarrow \quad x - 5 = 20 + 20$

$\Rightarrow \quad x = 45\text{ m}$

3. *(*)* Given, radius of sphere is a, mass of sphere is m, horizontal speed of sphere is v_0.

Moment of inertia of solid sphere, $I = 2/5\, ma^2$.

Let, h be the height of inclined plane and l be the length of inclined plane.



By using law of conservation of energy,

Total energy at 1 = Total energy at 2

$\therefore \quad PE_1 + KE_1 = PE_2 + KE_2$

$\Rightarrow \quad 0 + 1/2mv_0^2 + 1/2\,I\omega^2 = mgh \quad \ldots \text{(i)}$

As, $\quad v_0 = a\omega \Rightarrow \omega = \frac{v_0}{a}$

$\therefore \quad \frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\frac{v_0^2}{a^2} = mgh$

$\Rightarrow \quad \frac{1}{2}mv_0^2(1 + 2/5) = mgh$

$\Rightarrow \quad \frac{7v_0^2}{10} = gh \quad \ldots \text{(ii)}$

As, $\quad \sin\theta = h/l$

$\Rightarrow \quad h = l\sin\theta$

Put this value in Eq. (ii), we get

$\Rightarrow \quad \frac{7}{10}v_0^2 = gl\sin\theta$

$\Rightarrow \quad l = \frac{7v_0^2}{10g\sin\theta}$

4. *(b)* Given, radius of circle, $R = 0.36$ m

Angular distance, $\theta = 30° = \pi/6$ rad

Let l be the arc length.

$\because \quad l = R\theta$

$\Rightarrow \quad l = \frac{36}{100} \times \frac{\pi}{6} = \frac{6\pi}{100}$ m

As, speed on circular track (v) = Arc length (l) / Time (t)

$\Rightarrow \quad v = \frac{6\pi}{100 \times 0.1} = \frac{6\pi}{10}\text{ ms}^{-1}$

If F be the restoration force and a_r be the radial acceleration $(= v^2/R)$, then

$$F = ma_r$$

$$a_r = \frac{F}{m} = \frac{v^2}{R} = \left(\frac{6\pi}{10}\right)^2 \times \frac{100}{36}$$

$$= \frac{36 \times 9.87}{100} \times \frac{100}{36} = 9.87$$

5. *(b)* Here, p_1 and p_2, T_1 and T_2, V_1 and V_2 are initial and final pressures, temperatures and volumes, respectively.

Given, $\quad V_2 = 2V_1$

$$pV^{1/2} = \text{constant}$$

From graph, γ = adiabatic constant = 1/2

$\Rightarrow \quad p_1V_1^{1/2} = p_2V_2^{1/2} \Rightarrow \frac{p_1}{p_2} = \left(\frac{2V_1}{V_1}\right)^{1/2} = 2^{1/2}$

Also, $p^{1-\gamma}T^{\gamma} = \text{constant}$ (for adiabatic process)

$\Rightarrow \quad p_1^{1-\gamma}T_1^{\gamma} = p_2^{1-\gamma}T_2^{\gamma}$

$\Rightarrow \quad \left(\frac{p_1}{p_2}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma} \Rightarrow \left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} = \frac{T_2}{T_1}$

$\therefore \quad T_2 / T_1 = (2^{1/2})^{\frac{1-0.5}{0.5}} = \sqrt{2}$

6. *(c)* According to **Statement 1** : In diatomic molecule the rotational energy at a given temperature obeys Maxwell's distribution is correct.

But, according to **Statement 2** : In diatomic molecule, the rotational energy at a given temperature equals translational energy for each molecule is false.

Because kinetic energy (KE) of gaseous molecule is $\frac{f}{2}K_BT$.

where, f is the degree of freedom,

and f(diatomic atom) = 5 = [3 (translational) and 2(rotational)]

Therefore, translational KE of gas = $\frac{3}{2}K_BT$ ... (i)

and rotational KE of gas = $2/2\ K_BT = K_BT$ ... (ii)

$\because$ Eq. (i) is not equal to Eq. (ii)

Hence, option (c) is the correct.

7. *(c)* Let spring constants of two springs be k_1 and k_2. Since, two springs are connected in parallel connection and parallel equivalent spring constant, $k_{eq} = k_1 + k_2$

$\Rightarrow \quad k_{eq} = 2k + 2k = 4k$

As, time period, $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{4k}} = \frac{2\pi}{2}\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}$

8. *(c)* Given, displacement-time equation,

$$Y = A\sin(\omega t + \phi_0)$$

Here, A is amplitude, ω is angular frequency, t is time taken and ϕ_0 is the phase constant.

At $t = 0, Y = A/2$

$\therefore \quad Y = A/2 = A\sin(0 + \phi_0) = A\sin\phi_0$

$\Rightarrow \quad \sin\phi_0 = 1/2$

$\Rightarrow \quad \phi_0 = \pi/6$

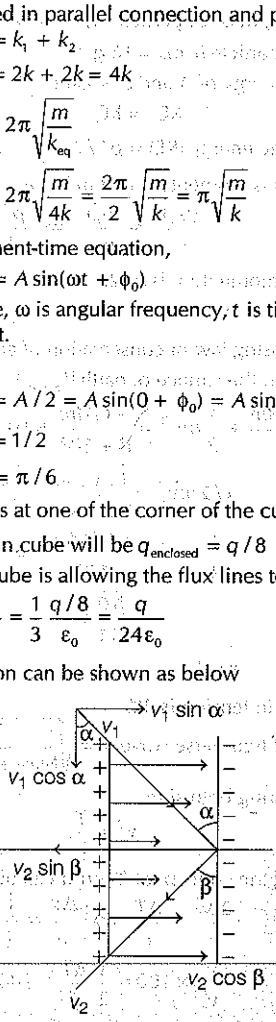
9. *(d)* Given, charge q is at one of the corner of the cube.

$\therefore$ Contribution of q in cube will be $q_{enclosed} = q/8$

As, only 3 faces of cube is allowing the flux lines to pass through it.

$\therefore$ Flux $(\phi) = \frac{q_{enclosed}}{\varepsilon_0} = \frac{1}{3}\frac{q/8}{\varepsilon_0} = \frac{q}{24\varepsilon_0}$

10. *(d)* The given situation can be shown as below



Let, v_1 and v_2 be the incoming and outgoing velocities of electron into the capacitor and out of the capacitor, respectively.

Since, electric field is along X-axis, hence electric force on electron along Y-axis, $(F_y) = 0$

$\therefore$ Change in momentum along Y-axis,

$$\Delta p_y = 0$$

i.e. $\quad p_1 = p_2$ (along Y-axis)

$\Rightarrow \quad m_1v_1\cos\alpha = m_2v_2\cos\beta$

$\Rightarrow \quad v_1/v_2 = \cos\beta/\cos\alpha$

$\because$ Kinetic energy $(K) = 1/2\ mv^2$

If mass is same, $K \propto v^2$

$\therefore \quad \frac{K_1}{K_2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{\cos\beta}{\cos\alpha}\right)^2 = \frac{\cos^2\beta}{\cos^2\alpha}$

11. *(b)* Since, in ferromagnetic material, with increase in temperature susceptibility decreases,

$\therefore$ Ferromagnetic material below Curie temperature will show saturation magnetisation.

Hence, option (b) is the correct i.e. domain is defined as a macroscopic region with saturation magnetisation.

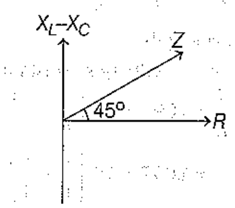
12. *(c)* Given, resistance, $R = 110\,\Omega$

Supply voltage $(V) = 220$ V

and angular frequency $(\omega) = 300$ rads^{-1}

$\because$ Current lag and lead by same angle.

$\therefore$ Circuit is in resonance i.e. $X_L = X_C$



As, $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(110)^2 + (X_L - X_L)^2} = 110 \quad [\because X_L = X_C]$

and $\quad I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{110} = 2$ A

13. *(c)* Given, stopping potential $(V_1) = 0.710$ V

Incident wavelength of electrons $(\lambda_1) = 491$ nm

$= 491 \times 10^{-9}$ m

Let λ_2 will be the new incident wavelength of electrons.

Stopping potential $(V_2) = 1.43$V

As, energy $(E) = \frac{1240}{\lambda_1} = \phi_0 + eV$

where, ϕ_0 is work-function and V is applied potential

$\therefore \quad E_1 = \frac{1240}{491} = \phi_0 + 0.71$ ... (i)

and $\quad E_2 = \frac{1240}{\lambda_2} = \phi_0 + 1.43$ ... (ii)

Now, subtracting Eqs. (i) from (ii), we get

$E_2 - E_1 = 1240\left(\frac{1}{\lambda_2} - \frac{1}{491}\right) = 0.72$

$\Rightarrow \quad \frac{1}{\lambda_2} = 0.00058 + 0.00204 = 0.00262$

$\therefore \quad \lambda_2 = 381.7$ nm

$\lambda_2 \approx 382$ nm

14. *(c)* Given, diameter of pinhole, $a = 0.1\ \mu m = 0.1 \times 10^{-6}$ m

$\because$ Path difference $(\Delta x) = a \sin\phi = n\lambda$ ...(i)

where, ϕ is the phase difference and λ be the wavelength.

As, $I = 4I_0 \cos^2 \phi$

and $\sin\phi = \dfrac{n\lambda}{a}$ [from Eq. (i)]

If a increases $\leftrightarrow$ $\sin\phi$ or ϕ decreases

As ϕ decreases $\leftrightarrow$ $\cos\phi$ increases

$\therefore$ Intensity increases.

Hence, on decreasing diameter of pinhloe, the size of diffraction pattern decreases and intensity increases.

15. *(b)* Given, mass of proton (m_p) is 1836 times the mass of electron (m_e) and velocity of proton (v_p) is equal to velocity of electron (v_e).

As, wavelength $(\lambda) = \dfrac{h}{p} = \dfrac{h}{mv}$

where, h is Planck's constant and p is momentum.

$$\therefore \quad \lambda \propto \frac{1}{m}$$

$$\Rightarrow \quad \frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e} = \frac{1836\ m_e}{m_e} = 1836$$

16. *(a)* Given, electron is moving from $n = 2$ to $n = 1$.

From Bohr's hydrogen spectrum (Rydberg formula)

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

where, λ = wavelength,

$R = 1.097 \times 10^7 \text{m}^{-1}$ (Rydberg's constant)

$$\Rightarrow \quad \frac{1}{\lambda} = 1.097 \times 10^7\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$= 1.097 \times 10^7\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3}{4} \times 1.097 \times 10^7$$

$$\lambda = \frac{4}{3 \times 1.097 \times 10^7} = 1.215 \times 10^{-7}$$

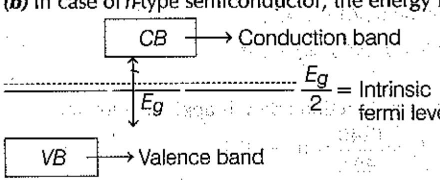
$$= 121.5 \times 10^{-9} = 121.8 \text{ nm}$$

17. *(c)* Let λ be the wavelength of carrier signal.

Since, frequency $(f_c) = \dfrac{\text{speed (c)}}{\text{wavelength } (\lambda)}$

$$\therefore \quad \lambda = \frac{c}{f_c}$$

18. *(b)* In case of n-type semiconductor, the energy level diagram will be



In case of n-type semiconductor $n > p$, so the fermi level will go upward.

Similarly, in case of p-type semiconductor $p > n$, so the fermi level will go downward.

19. *(b)* Rectifier devices are used to convert AC to DC.

Stabiliser is an electronic device which gives constant output even if input changes.

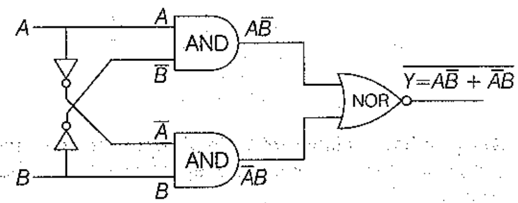
Transformer is used to step up or down the AC voltage.

Filter device which removes ripple in rectified output.

So, the correct match is

$A \to 2, B \to 4, C \to 1$ and $D \to 3$.

20. *(b)* Here A and B be the input and Y be the output.



$$\because Y = \overline{A\overline{B} + \overline{A}B} = \overline{A\overline{B}} \cdot \overline{\overline{A}B} = (\overline{A} + \overline{\overline{B}}) \cdot (\overline{\overline{A}} + \overline{B})$$

$$= (\overline{A} + B)(A + \overline{B}) = \overline{A}A + \overline{A}\,\overline{B} + AB + B\overline{B}$$

$$= 0 + \overline{A}\,\overline{B} + AB + 0 = AB + \overline{A}\,\overline{B}$$

According to the truth table

A	B	$\overline{A}$	$\overline{B}$	AB	$\overline{A}\,\overline{B}$	$Y = AB + \overline{A}\,\overline{B}$
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	1	0	1

21. *(1)* Given, mass of particle $A, m_A = 4$ g

Mass of particle $B, m_B = 16$ g

Kinetic energy of A and B is same

i.e. $KE_A = KE_B$

As, kinetic energy (KE) $= p^2/2m$

where, p is momentum and m is mass.

$$\therefore \quad \frac{(p_A)^2}{m_A} = \frac{(p_B^2)}{m_B} \Rightarrow \frac{p_A^2}{4} = \frac{p_B^2}{16} \Rightarrow \frac{p_A}{p_B} = \frac{1}{2}$$

$\because$ linear momentum is n : 2.

$\therefore \quad n = 1$

22. *(10)* By using law of conservation of energy,

Energy on the surface of earth (E_{Surface}) = Energy at height $(h = 10R)$

$$\Rightarrow \quad \frac{-GMm}{R} + \frac{1}{2}mv_i^2 = \frac{-GMm}{R + 10R} + 0 = \frac{-GMm}{11R}$$

$$\Rightarrow \quad 1/2\,mv_i^2 = \frac{11}{11}\frac{GMm}{R} - \frac{GMm}{11R}$$

$$\Rightarrow \quad \frac{1}{2}mv_i^2 = \frac{10\ GMm}{11R} \Rightarrow v_i^2 = \frac{20GM}{11R}$$

$$\Rightarrow \quad v_i = \sqrt{\frac{10}{11}}\, v_e \quad \left(\because v_e = \sqrt{\frac{2GM}{R}} = \text{escape velocity}\right)$$

Hence, $x = 10$

23. *(2)* Error in tension is 4%.

Speed of transverse wave, $v = \sqrt{\dfrac{\text{Tension } (T)}{\text{Mass per unit length } (\mu)}}$

On squaring both side

$$\Rightarrow \quad v^2 = \frac{T}{\mu}$$

By using the concept of % error calculation

$$\Rightarrow \quad \frac{2\Delta v}{v} = \frac{\Delta T}{T} \Rightarrow \frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\therefore \quad \frac{\Delta v}{v} \times 100 = \frac{1}{2}\frac{\Delta T}{T} \times 100 = \frac{4}{2} = 2\%$$

24. *(180)* Given, $\mathbf{P} \times \mathbf{Q} = \mathbf{Q} \times \mathbf{P}$

$$\Rightarrow \quad \mathbf{P}\times\mathbf{Q} = -\mathbf{P}\times\mathbf{Q} \Rightarrow 2(\mathbf{P}\times\mathbf{Q}) = 0$$

or $\mathbf{P}\times\mathbf{Q} = 0$

It is possible only, if $\mathbf{P} = 0$ or $\mathbf{Q} = 0$,

angle between them is 180°.

$$\therefore \quad \theta = 180^\circ$$

25. *(104)* Given, initial efficiency $(\eta_1) = 1/4$

$$\Rightarrow \quad \eta_1 = 1 - \frac{T_2}{T_1} = \frac{1}{4} \Rightarrow \frac{T_2}{T_1} = 1 - \frac{1}{4} = \frac{3}{4}$$

When temperature of sink is reduced by 50 K,

$$\eta_2 = 2\eta_1 = 1 - \frac{T_2 - 52}{T_1}$$

$$\Rightarrow \quad 2 \times \frac{1}{4} = 1 - \frac{T_2}{T_1} + \frac{52}{T_1}$$

$$\Rightarrow \quad \frac{1}{2} = 1 - \frac{3}{4} + \frac{52}{T_1} \Rightarrow \frac{1}{2} - \frac{1}{4} = \frac{52}{T_1}$$

$$\Rightarrow \quad \frac{2}{4} = \frac{52}{T_1}$$

$$\Rightarrow \quad T_1 = 104 \text{ K}$$

26. *(630)* Given, mass of each spheres, $m = 10$ mg $= 10 \times 10^{-3}$ g

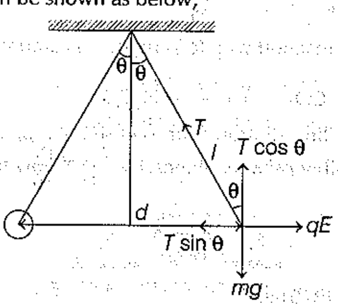
Length of thread $(l) = 0.5$ m

Separation between charges, $d = 0.2$ m

Charge of each sphere, $q = \frac{a}{21} \times 10^{-8}$ C

Acceleration due to gravity $(g) = 10 \text{ ms}^{-2}$

The situation can be shown as below,



Taking component of tension (T)

$$T\cos\theta = mg \quad \text{...(i)}$$

$$T\sin\theta = qE = \frac{kq^2}{d^2} \quad \text{...(ii)}$$

$$\sin\theta = \frac{d/2}{l} = \frac{0.1}{0.5} = \frac{1}{5}$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - 1/25} = \frac{\sqrt{24}}{5}$$

$$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{kq^2}{d^2mg} \quad \text{[using Eqs. (i) and (ii)]}$$

$$\Rightarrow \quad \frac{1/5}{\sqrt{24}/5} = \frac{kq^2}{d^2mg}$$

$$\Rightarrow \quad \frac{1}{\sqrt{24}} = \frac{9\times10^9 \times q^2}{(0.2)^2 \times 10 \times 10^{-3} \times 10}$$

$$\Rightarrow \quad q = \sqrt{\frac{(0.2)^2 \times 10^{-1}}{\sqrt{24} \times 9 \times 10^9}} \Rightarrow q = 3 \times 10^{-7} \text{ C}$$

$$\Rightarrow \quad \frac{a}{21} \times 10^{-8} = 30 \times 10^{-8}$$

$$a = 630$$

27. *(7.56)* Given, $q_1 = 2.1$ nC $= 2.1 \times 10^{-9}$ C, $q_2 = -0.1$ nC $= -0.1 \times 10^{-9}$C

Separation $(d) = 0.5$ m

By Coulomb's law,

$$\text{Force } (F) = \frac{kq_1q_2}{d^2}$$

where, $k = 9 \times 10^9 \text{N-m}^2\text{C}^{-2}$ = Coulomb's constant

$$\therefore \quad F = \frac{9\times10^9 \times 2.1\times10^{-9} \times (-0.1\times10^{-9})}{(0.5)^2} = -7.56 \times 10^{-9} \text{ N}$$

28. *(2)* Given, radiation power $(P) = 8$W

Distance $(d) = 10$ m

$$\because \text{Intensity } (I) = \frac{1}{2} c\,\varepsilon_0 E^2 = \frac{\text{Power } (P)}{\text{Area } (A)} \quad \text{... (i)}$$

$$\text{where, } c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \text{speed of light in vacuum} \quad \text{... (ii)}$$

and E = electric field

From Eq. (ii), we get

$$\varepsilon_0 = \frac{1}{\mu_0 c^2}$$

Put this value of ε_0 in Eq. (i), we get

$$I = \frac{1}{2} c \frac{1}{\mu_0 c^2} E^2 = \frac{P}{A} \Rightarrow \frac{1}{2}\frac{1}{\mu_0 c} E^2 = \frac{P}{A}$$

$$\Rightarrow \quad E = \sqrt{\frac{2P\,\mu_0 c}{A}} = \sqrt{\frac{2P\,\mu_0 c}{4\pi d^2}} = \sqrt{\frac{2P}{4d^2}}\sqrt{\frac{\mu_0 c}{\pi}}$$

$$= \sqrt{\frac{2\times8}{4\times100}} = \sqrt{\frac{\mu_0 c}{\pi}} = \frac{2}{10}\sqrt{\frac{\mu_0 c}{\pi}} \text{ V/m}$$

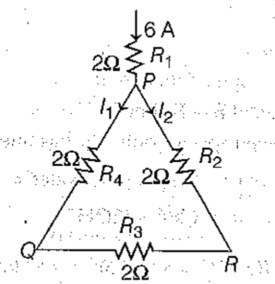
$$\therefore \quad x = 2$$

29. *(2)* Let resistances be R_1, R_2, R_3 and R_4 and I_1 current is passing through R_4 as shown in figure

$\therefore I_2 = (6 - I_1)$ is passing through R_2

As, same current is flowing through R_4 and R_3.

$\therefore R_4$ and R_3 are in series.



and series equivalent resistance, $R_{eq} = R_4 + R_3$

$$\therefore \quad R_{eq} = 2 + 2 = 4\,\Omega$$

Voltage through R_{eq} and R_2 will be same.

$$\Rightarrow \quad I_1 R_{eq} = I_2 R_2 \Rightarrow I_1 4 = (6 - I_1)2$$

$$\Rightarrow \quad 2I_1 = 6 - I_1 \Rightarrow I_1 = 2\text{A}$$

30. *(10)* Given, wavelength of X-rays,

$\lambda = 10\text{Å} = 10 \times 10^{-10}$ m

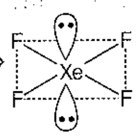
Speed of light in free space, $c = 3 \times 10^8$m/s

$$\text{Since, energy } (E) = \frac{hc}{\lambda} = mc^2 \quad \text{... (i)}$$

$$\Rightarrow \quad m = \frac{h}{c\lambda} = \frac{h}{3\times10^8 \times 10 \times 10^{-10}} = \frac{h}{3\times10^{-1}} = \frac{10h}{3} \text{ kg}$$

$$\therefore \quad x = 10$$

CHEMISTRY

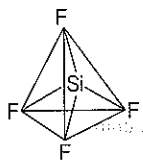
1. (c) (a) $:\ddot{X}eF_4 \Rightarrow$ 

Xe (sp^3d^2-hybridised)

Geometry : Octahedral

Shape : Square planar

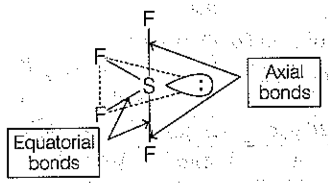
It has four equivalent Xe—F bonds in the same plane.

(b) $SiF_4 \Rightarrow$ 

Si (sp^3-hybridised)

Geometry and shape : Tetrahedral

It has four equivalent Si—F bonds.

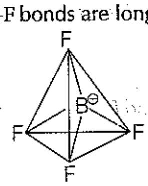
(c) $:SF_4 \Rightarrow$ 

S (sp^3d-hybridised)

Geometry : Trigonal bipyramidal

Shape : *see-saw*

Axial S—F bonds are longer than equatorial bonds.

(d) $BF_4^- \Rightarrow$ 

B (sp^3-hybridised)

Geometry and shape : Tetrahedral

It has four equivalent B—F bonds.

Hence, among given species only SF_4 has unequal bond lengths.

2. (a) Let, solubility of $Ca(OH)_2$ in pure water = S mol/L

$$Ca(OH)_2 \rightleftharpoons \underset{S\,\text{mol/L}}{Ca^{2+}} + \underset{2\times S\,\text{(mol/L)}}{2OH^-}$$

$$K_{sp} = [Ca^{2+}][OH^-]^2 = S \times (2S)^2 = 4S^3 \text{ (mol/L)}$$

The expression of K_{sp} can also be written as,

$$K_{sp} = x^x \cdot y^y \cdot S^{x+y}$$
$$= 1^1 \cdot 2^2 \cdot S^{1+2}$$
$$= 4S^3 \qquad [\because \text{For } Ca(OH)_2 : x = 1, y = 2]$$

x and y are the coefficients of cations and anions respectively

$$S = \left(\frac{K_{sp}}{4}\right)^{1/3} = \left(\frac{5.5\times10^{-6}}{4}\right)^{1/3}$$
$$= 1.11 \times 10^{-2} \text{ mol/L}$$

3. (c) Statement (c) is false whereas other statements are true. Corrected statement is as follows :

Viscosity of hydrophilic sols is higher than that of the dispersion medium, i.e. H_2O, because there is a high concentration of dispersed phase in water.

4. (c) Among halogens (F_2, Cl_2, Br_2 and I_2), bond dissociation enthalpy ($\Delta_{diss}H°$) of I_2, is minimum because of larger size of I-atom there is a steric repulsion between bonded I-atoms, which makes I—I bond weakest.

Whereas, smaller size and highest electronegativity of F-atom cause highest electron density on F-atom of F_2 molecule. As a result, F—F bond becomes weaker due to electrostatic repulsion between bonded F-atoms.

Thus, the order of $\Delta_{diss}H°$ (in kJ mol^{-1}) is

$$\underset{(242.6)}{Cl{-}Cl} > \underset{(192.3)}{Br{-}Br} > \underset{\substack{(158.8)\\ \text{Electrostatic}\\ \text{repulsion}}}{F{-}F} > \underset{\substack{(151.1)\\ \text{Steric}\\ \text{repulsion}}}{I{-}I}$$

5. (c) Zone refining method is very helpful for producing semiconductor and other metals of high purity, e.g., Si, Ge, B, Ga, In etc.

6. (c) Water (steam) can produce CO on reacting with CH_4, C and C_3H_8 as,

$$CH_4 + H_2O \xrightarrow[Ni]{1270\,K} CO + 3H_2$$
$$C + H_2O \xrightarrow{1270\,K} CO + H_2$$
$$C_3H_8 + 3H_2O \xrightarrow[Ni]{1270\,K} 3CO + 7H_2$$

But, water on reaction with CO_2 produces carbonic acid (H_2CO_3), not CO.

$$CO_2 + H_2O \longrightarrow H_2CO_3$$

7. (c) Two crystalline allotropic forms of sulphur (S_α and S_β) can change reversibly between themselves with slow heating (above 369 K) or cooling.

$$\underset{\text{(Rhombic)}}{S_\alpha} \underset{\text{Cooling}}{\overset{>369\,K}{\rightleftharpoons}} \underset{\text{(Monoclinic)}}{S_\beta}$$

So, statement I is true.

At room temperature or at standard conditions of pressure and temperature, rhombic sulphur is the thermodynamically most stable crystalline allotrope of sulphur ($\Delta_f H° = 0$).

So, statement II is false.

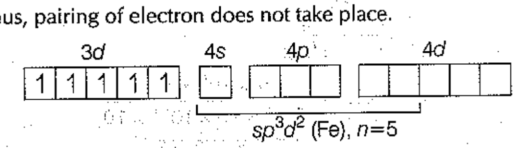
8. (b) The major components of German silver are Cu : 25-30%, Zn : 25-30% and Ni : 40-30%.

9. (a) Spin only magnetic moment, $\mu = \sqrt{n(n+2)}$ BM

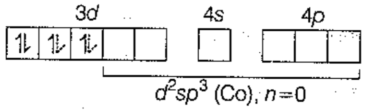
where, n = number of unpaired electrons and $\mu \propto n$.

(i) $[FeF_6]^{3-} \Rightarrow Fe^{3+} = (3d^5) + 6 \times F^-$ (weak field ligand).

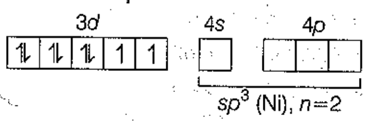
Thus, pairing of electron does not take place.



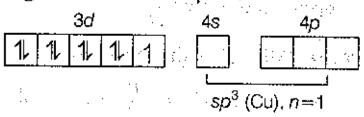
(ii) $[Co(NH_3)_6]^{3+} \Rightarrow Co^{3+} = (3d^6) + 6 \times NH_3$ (strong field ligand). Thus, pairing of electron takes place.



(iii) $[NiCl_4]^{2-} \Rightarrow Ni^{2+} = (3d^8) + 4 \times Cl^-$ (weak field ligand). Thus, pairing of electron takes place.



(iv) $[Cu(NH_3)_4]^{2+} \Rightarrow Cu^{2+} (3d^9) + 4 \times NH_3$(strong field ligand). Thus, pairing of electron takes place.



So, the decreasing order of μ is

$$\underset{(n=5)}{\mu_i} > \underset{(n=2)}{\mu_{iii}} > \underset{(n=1)}{\mu_{iv}} > \underset{(n=0)}{\mu_{ii}}$$

10. *(a)* The pH of rain water is normally 5.6. If it pH drop below 5.6, it is called acid rain.

∴ Both statements are correct.

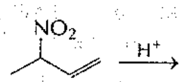
11. *(c)* Nitric acid is added to sodium extract before addition of silver nitrate for testing halogens. Because it decomposes NaCN and Na_2S or else they interfere in the test.

The reaction are as follows :

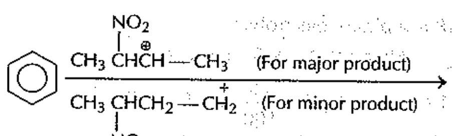
$$NaCN + HNO_3 \longrightarrow NaNO_3 + HCN\uparrow$$

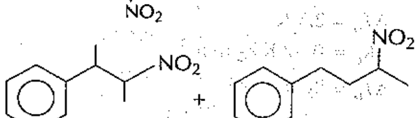
$$\underset{\text{Sodium extract}}{Na_2S} + \underset{\text{Nitric acid}}{HNO_3} \longrightarrow NaNO_3 + H_2S\uparrow$$

12. *(b)* The reaction take place as follows :

Step I 

$$\underset{\substack{\text{(More stable intermediate)}\\ 2^\circ\text{ carbocation}}}{CH_3-\underset{NO_2}{\overset{|}{C}}H-\overset{+}{C}H-CH_3} + \underset{\substack{\text{(Less stable intermediate)}\\ 1^\circ\text{ carbocation}}}{CH_3-\underset{NO_2}{\overset{|}{C}}H-CH_2-\overset{+}{C}H_2}$$

Step II 



(Major product)

13. *(a)* The major product of the reaction is $CH_3CH_2CH_2CH_2CHO$.

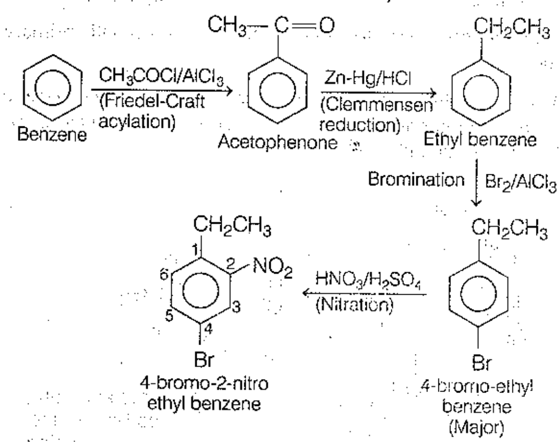
Here, electrophilic addition of $H-\overset{+}{C}=O$ (formylation) take place to the alkene through Markownikoff addition.

(i) $H_2 + \underset{\text{Water gas}}{:C{=}\ddot{O}:} \xrightarrow[\text{(Catalyst)}]{Rh} [H-\overset{\oplus}{C}=O]H^{\ominus}$

(ii) $\underset{\text{But-1-ene}}{CH_3CH_2-CH=CH_2} \xrightarrow[\text{Slow}]{H-\overset{\oplus}{C}=O} CH_3CH_2-\overset{\oplus}{C}H-CH_2-CHO$

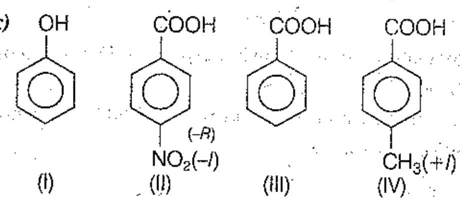
$$\xrightarrow[\text{}]{\text{Fast } H^-} \underset{\text{Pentanal}}{CH_3CH_2CH_2CH_2CHO} \text{ (major)}$$

14. *(b)* Benzene on reaction with $CH_3COCl/AlCl_3$ gives acetophenone which on reduction with Zn-Hg/HCl gives ethyl benzene. Bromination of ethyl benzene give 4-bromo-ethylbenzene which upon nitration gives 4-bromo-2-nitroethyl benzene.



[$-I$-effect of Br directs NO_2^+ at its *meta*-position]

So, option (b) is the correct sequence of reagents used.

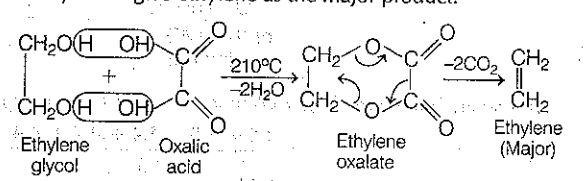
15. *(c)* 

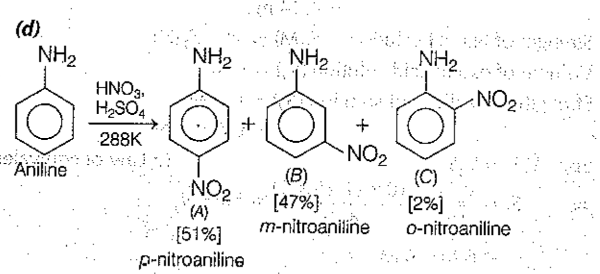
Acidity of phenol (I) is weaker than any carboxylic acid.

Electron withdrawing nature ($-R$, $-I$), of $-NO_2$ group at *para* position increases acidic strength of (II), whereas $+I$ effect of $-CH_3$ group at *para* position decreases acidic strength of (IV).

So, the order of acid character is II > III > IV > I.

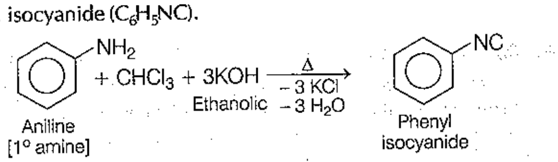
16. *(c)* When ethylene glycol is heated with oxalic acid at 210°C, first we get an unstable cyclic-diester (ethylene oxalate) which readily decarboxylate to give ethylene as the major product.



17. *(d)* 

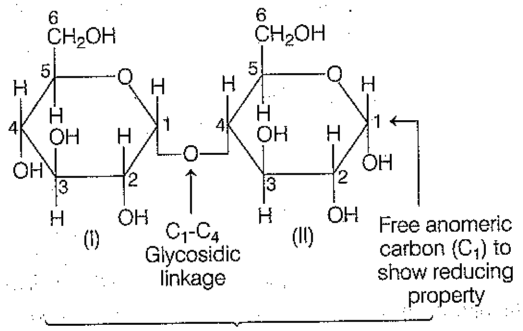
During nitration, in strongly acidic medium, aniline gets protonated to form the anilinium ion i.e, (C_6H_5—$\overset{\oplus}{N}H_3$) which is *meta*-directing as —$\overset{\oplus}{N}H_3$ is an electron withdrawing (−*I*) group. As a result, we get *meta*-nitro aniline (*B*) as the major product.

18. *(c)* Aniline on carbylamine reaction produces a foul smelling gas, phenyl isocyanide (C_6H_5NC).



Carbylamine test is used to detect aliphatic and aromatic primary amines.

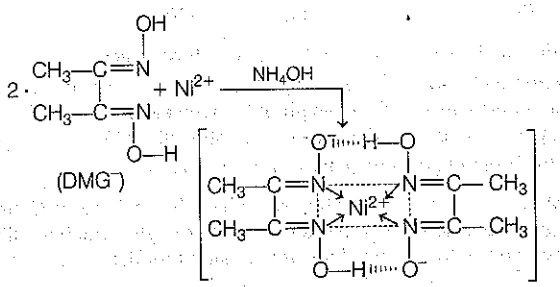
19. *(c)* Maltose is a disaccharide which is made of two α-D-glucose units in which C_1 (anomeric carbon) of one glucose (I) is linked to C_4 of another glucose unit (II).



Maltose

20. *(a)* Both statements are true. Dimethyl glyoxime (DMG) is a neutral bidentate ligand (DMG^-).

Ni^{2+} ion is identified with DMG in presence of NH_4OH to give a red ppt. Its reaction is as follows.



or $[Ni(DMG)_2]$
(Red ppt.)

21. *(6)* Average burette reading = Volume of NaOH solution (V_1)

$$= \frac{4.5 + 4.5 + 4.4 + 4.4 + 4.4}{5}$$

$$= 4.44 \text{ mL}$$

Strength of NaOH solution = S_1(M) (say) = S_1(N)

Volume of oxalic acid solution (V_2) = 10 mL

Strength of oxalic acid solution (S_2) = 1.25 M

$$= 1.25 \times 2 \text{ N}$$

So, $V_1S_1 = V_2S_2$ (∵ Law of equivalence)

$$\Rightarrow S_1 = \frac{V_2S_2}{V_1} = \frac{10 \times (1.25 \times 2)}{4.44} = 5.63 \text{ N}$$

$$\simeq 6 \text{ M} = 6 \text{ M}$$

Note *n*-factor of NaOH = 1

n-factor of $H_2C_2O_4$ = 2 (oxalic acid)

$$N = M \times n$$

22. *(9077)* Density of copper, $d = \dfrac{Z \times M}{a^3 \times N_A}$

Given, $Z = 4$, for fcc lattice,

$$M = 63.54 \text{ g mol}^{-1}$$
$$= 63.54 \times 10^{-3} \text{ kg mol}^{-1},$$
$$a = 3.596 \text{ Å} = 3.596 \times 10^{-10} \text{m},$$
$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

On putting given values, we get

$$\Rightarrow d = \frac{4 \times (63.54 \times 10^{-3})}{(3.596 \times 10^{-10})^3 \times (6.022 \times 10^{23})} \text{ kg/m}^3$$
$$= 9076.26 \simeq 9077 \text{ kg/m}^3$$

23. *(180)* Energy of EMR = IE of the metal (*A*)

$$= h\nu = \frac{hc}{\lambda} \text{ atom}^{-1} = \frac{hc}{\lambda} \times N_A \text{ mol}^{-1}$$

$$= \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8) \times (6.02 \times 10^{23})}{(663 \times 10^{-9})} \text{ J mol}^{-1}$$

[∵ $\lambda = 663$ nm $= 663 \times 10^{-9}$m]

$$= 180600 \text{ J mol}^{-1} = 180.6 \text{ kJ mol}^{-1} \simeq 180 \text{ kJ mol}^{-1}$$

24. *(15)* The gas performs isothermal irreversible work (*W*).

where, $\Delta U = 0$ (change in internal energy)

From, 1st law of thermodynamics,

$$\Rightarrow \Delta U = \Delta Q + W$$
$$\Rightarrow 0 = \Delta Q + W$$
$$\Rightarrow \Delta Q = -W$$

Now, $W = -p_{ext}(V_2 - V_1)$

$$= -p_{ext}\left(\frac{nRT}{p_2} - \frac{nRT}{p_1}\right) = -p_{ext} \times nRT\left(\frac{1}{p_2} - \frac{1}{p_1}\right)$$

Given, $p_{ext} = 4.3$ MPa, $p_1 = 2.1$ MPa, $p_2 = 1.3$ MPa,

$n = 5$ mol, $T = 293$ K and $R = 8.314$ J mol^{-1} K^{-1}

$$= -4.3 \times 5 \times 8.314 \times 293\left(\frac{1}{1.3} - \frac{1}{2.1}\right)$$
$$= -15347.70 \text{ J mol}^{-1}$$
$$= -15.347 \text{ kJ mol}^{-1} \simeq -15 \text{ kJ mol}^{-1}$$
$$\Rightarrow \Delta Q = 15 \text{ kJ mol}^{-1}$$

25. *(3)* As *AB* is a binary electrolyte,

$$\therefore AB \rightleftharpoons A^+ + B^-, n = 2$$

$$i = 1 + \alpha(n-1) = 1 + \frac{75}{100}(2-1) = 1.75$$

Given, $\Delta T_b = 2.5$ K

$$K_b = 0.52 \text{ K kg mol}^{-1}$$

$$\therefore \Delta T_b = K_b \times m \times i$$

$$\Rightarrow m = \frac{\Delta T_b}{K_b \times i} = \frac{2.5}{0.52 \times 1.75}$$

$$= 2.74 \simeq 3 \text{ mol/kg}$$

26. *(4)* Cell-I ($HNO_3 \longrightarrow NO$)

$$3Cu + 2NO_3^- + 8H^+ \longrightarrow 3Cu^{2+} + 2NO + 4H_2O$$

$$Q_1 = \frac{[Cu^{2+}]^3 \times (p_{NO})^2}{[NO_3^-]^2 \times [H^+]^8}$$

$\because \quad E_1^\circ = 0.96 - (-0.34) = 1.3\text{ V}$

$$E_1 = 1.3 - \frac{0.059}{6}\log Q_1$$

Cell-II ($HNO_3 \rightarrow NO_2$)

$$Cu + 2NO_3^- + 4H^+ \longrightarrow Cu^{2+} + 2NO_2 + 2H_2O$$

$$Q_2 = \frac{[Cu^{2+}]\times(p_{NO_2})^2}{[NO_3^-]^2\times[H^+]^4}$$

$\because \quad E_2^\circ = 0.79 - (-0.34)\text{V} = 1.13\text{V}$

$$E_2 = 1.13 - \frac{0.059}{2}\log Q_2$$

Now, $E_1 = E_2$

$$1.3 - \frac{0.059}{6}\log Q_1 = 1.13 - \frac{0.059}{2}\log Q_2$$

$$0.17 = \frac{0.059}{6}[\log Q_1 - 3\log Q_2] = \frac{0.059}{6}\log\frac{Q_1}{Q_2}$$

$$= \frac{0.059}{6}\log\frac{[Cu^{2+}]^3\times(p_{NO})^2}{[NO_3^-]^2\times[H^+]^8}\times\frac{[NO_3^-]^6\times[H^+]^{12}}{[Cu^{2+}]^3\times(p_{NO_2})^6}$$

$$= \frac{0.059}{6}\log\frac{[H^+]^4\times[NO_3^-]^4}{(p_{NO_2})^4} \qquad [\because p_{NO} = p_{NO_2}]$$

$$= \frac{0.059}{6}\log\frac{[HNO_3]^4}{(p_{NO_2})^4}$$

Now, $\quad p_{NO_2} \equiv [HNO_3]$

So, $\quad 0.17 = \frac{0.059}{6}\log[HNO_3]^8$

$$= \frac{0.059}{6}\times 8\log[HNO_3]$$

$\log[HNO_3] = 2.16$

$[HNO_3] = 10^{2.16}\text{M} = 10^x\text{M}$

$\therefore \quad x = 2.16$

$\Rightarrow \quad 2x = 2\times 2.16 = 4.32 \simeq 4$

27. *(52)* $T_1 = (273+27) = 300\text{ K}$, $T_2 = (273+52) = 325\text{ K}$

Given, temperature coefficient of the reaction,

$$\alpha_T = \frac{K_{325}}{K_{300}} = 5$$

$$\log\frac{K_{T_2}}{K_{T_1}} = \frac{E_a}{2.303R}\times\left(\frac{T_2 - T_1}{T_1T_2}\right)$$

$$\log\frac{K_{325}}{K_{300}} = \frac{E_a}{2.303\times 8.314}\left(\frac{325-300}{300\times 325}\right)$$

$$\log 5 = \frac{E_a}{2.303\times 8.319}\times\frac{25}{300\times 325}$$

$E_a = 52194.78\text{ J mol}^-$

$= 52.194\text{ kJ mol}^{-1}$

$\simeq 52\text{ kJ mol}^{-1}$

28. *(1)* Given metals (Li, Na, Rb, Cs) are alkali metals (Group 1). Their ionisation energy (IE) decreases down the group.

Here, Cs has lowest IE, i.e. it has lowest value of threshold energy (E_0) or work function (W). Hence, it can be used as electrode in photoelectric cell.

29. *(2)* $Z = 29\,[Cu] \xrightarrow{-2e^-} Cu^{2+} = [Ar]\,3d^9$

$3d^9 = \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}$;

Number of unpaired electron, $n = 1$

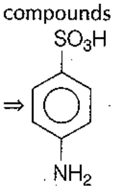
$\therefore$ Spin only magnetic moment,

$$\mu = \sqrt{n(n+2)}\text{BM} = \sqrt{1(1+2)}\text{ BM} = \sqrt{3}\text{ BM}$$

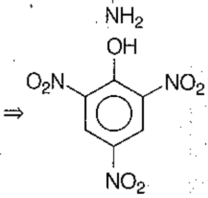
$$= 1.73\text{ BM} \simeq 2\text{ BM}$$

30. *(1)* Structure of given compounds are as follows

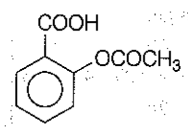
(i) (Sulphanilic acid) $\Rightarrow$



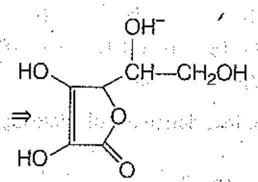
(ii) (Picric acid) $\Rightarrow$



(iii) (Aspirin) $\Rightarrow$



(iv) (Ascorbic acid) $\Rightarrow$



Only one compound (iii) contains —COOH group.

MATHEMATICS

1. *(b)* Given, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$

$\Rightarrow \quad A^T = \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix}$

Given, $AA^T = I_2$ i.e.

$$\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating these matrices,

$1 = \alpha^2 + 1$ gives, $\alpha = 0$

$\alpha(1-\beta) = 0$

$\alpha^2 + \beta^2 = 1$

Put $\alpha = 0$ in $\alpha^2 + \beta^2 = 1$, we get

$0 + \beta^2 = 1$, gives $\beta = \pm 1$

where we take $\beta^4 = 1$

$\therefore \quad \alpha^4 + \beta^4 = 0^4 + 1 = 1$

2. *(c)* Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Then, $2A = \begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{bmatrix}$

Now, perform the operation

$R_2 \to 2R_2 + 5R_3$ on $2A$, we get

$$B = \begin{bmatrix} 2a & 2b & 2c \\ 4d+10g & 4e+10h & 4f+10i \\ 2g & 2h & 2i \end{bmatrix}$$

$$|B| = \begin{vmatrix} 2a & 2b & 2c \\ 4d+10g & 4e+10h & 4f+10i \\ 2g & 2h & 2i \end{vmatrix}$$

Using property of invariance to calculate $|B|$, apply

$R_2 \to R_2 - 5R_3$

$$|B| = \begin{vmatrix} 2a & 2b & 2c \\ 4d & 4e & 4f \\ 2g & 2h & 2i \end{vmatrix} = 2 \times 4 \times 2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$= 16 \times \det(A)$

$= 16 \times 4 = 64 \qquad [\because \det(A) = 4]$

3. *(b)* The given system of equations is non-homogeneous and it can be written as,

$$\begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 8 \end{bmatrix}$$

i.e,. $AX = B$

Now, $|A| = 2(8+2) - 3(12-2) + 2(-3-2)$

$= 20 - 30 - 10 = -20 \neq 0$

$\because |A| \neq 0$, then this system have unique solution.

4. *(a)* $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\,dx = \int_{\pi/4}^{\pi/2} \cot^{n-2} x(\cot^2 x)dx$

$$I_n = \int_{\pi/4}^{\pi/2} \cot^{n-2} x \operatorname{cosec}^2 x\,dx - \int_{\pi/4}^{\pi/2} \cot^{n-2} x\,dx$$

$$I_n + I_{n-2} = \int_{\pi/4}^{\pi/2} \cot^{n-2} x . \operatorname{cosec}^2 x\,dx$$

Now, let $\cot x = t$, then $\operatorname{cosec}^2 x\,dx = -dt$, limit will be

$$I_n + I_{n-2} = \int_1^0 -t^{n-2}dt$$

$$= \left[\frac{-(t)^{n-1}}{n-1}\right]_1^0 = -\left\{\frac{0}{n-1} - \frac{(1)^{n-1}}{n-1}\right\}$$

$$I_n + I_{n-2} = \frac{1}{n-1}$$

Now, put $n = 4$

$\Rightarrow \quad I_2 + I_4 = \frac{1}{3}$, then $\frac{1}{I_2 + I_4} = 3 \qquad$ ...(i)

Put $n = 5$

$\Rightarrow \quad I_5 + I_3 = \frac{1}{4}$, then $\frac{1}{I_3 + I_5} = 4 \qquad$ ...(ii)

Put $n = 6$

$\Rightarrow \quad I_6 + I_4 = \frac{1}{5}$, then $\frac{1}{I_4 + I_6} = 5 \qquad$ ...(iii)

Here, from Eqs. (i), (ii) and (iii), we conclude

$\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}$ and $\frac{1}{I_4 + I_6}$ are in AP with common difference 1.

5. *(c)* Given, $f(x) = \frac{5^x}{5^x + 5}$, then, $f(2-x) = \frac{5^{2-x}}{5^{2-x} + 5}$

$\Rightarrow \quad f(2-x) = \frac{5}{5^x + 5}$

This gives, $f(x) + f(2-x) = \frac{5^x + 5}{5^x + 5} = 1$

$\Rightarrow \quad f\left(\frac{1}{20}\right) + f\left(2 - \frac{1}{20}\right) = f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right) = 1$

Similarly,

$$f\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right) = 1 \text{ and so on,}$$

$\therefore \quad f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + \ldots + f\left(\frac{38}{20}\right) + f\left(\frac{39}{20}\right)$

$= 1 + 1 + \ldots + 1 + f\left(\frac{20}{20}\right) = 19 + f(1) = 19 + \frac{1}{2} = \frac{39}{2}$

6. *(c)* We have, $x^2 - 6x - 2 = 0$

Given, α and β are roots of above quadratic equation, then

$\alpha^2 - 6\alpha - 2 = 0 \qquad$ ...(i)

$\beta^2 - 6\beta - 2 = 0 \qquad$ ...(ii)

Also, given $a_n = \alpha^n - \beta^n$, then $\dfrac{a_{10} - 2a_8}{3a_9}$

$$= \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)} = \frac{\alpha^{10} - 2\alpha^8 - \beta^{10} + 2\beta^8}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)}$$

[from Eqs. (i) and (ii) $\alpha^2 - 2 = 6\alpha, \beta^2 - 2 = 6\beta$]

$$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)} = \frac{6\alpha^9 - 6\beta^9}{3(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)}$$

$$= 2$$

7. *(c)* We already know, Arithmetic mean ≥ Geometric mean,

Let us take AM and GM of two terms a^{a^x} and a^{1-a^x},

$\Rightarrow \text{AM} = \dfrac{a^{a^x} + a^{1-a^x}}{2}$ and $\text{GM} = \sqrt{a^{a^x} \cdot a^{1-a^x}}$

$\because \text{AM} \geq \text{GM} \Rightarrow \dfrac{a^{a^x} + a^{1-a^x}}{2} \geq \sqrt{a^{a^x} \cdot a^{1-a^x}} \Rightarrow a^{a^x} + a^{1-a^x} \geq 2\sqrt{a^1}$

$\therefore$ Minimum value of $f(x) = a^{a^x} + a^{1-a^x}$ is $2\sqrt{a}$.

8. *(b)* $I = \displaystyle\int \frac{e^{3\log_e(2x)} + 5e^{2\log_e(2x)}}{e^{4\log_e(x)} + 5e^{3\log_e(x)} - 7e^{2\log_e(x)}}\,dx$

$$= \int \frac{e^{\log_e(2x)^3} + 5e^{\log_e(2x)^2}}{e^{\log_e x^4} + 5e^{\log_e(x)^3} - 7e^{\log_e(x)^2}}\,dx$$

[using property $a \log x = \log x^a$]

$$= \int \frac{8x^3 + 5(2x)^2}{x^4 + 5(x)^3 - 7x^2}\,dx \qquad \text{[using property } a^{\log_a x} = x]$$

$$= \int \frac{8x^3 + 20x^2}{x^4 + 5x^3 - 7x^2}\,dx = \int \frac{4x^2(2x + 5)}{x^2(x^2 + 5x - 7)}\,dx$$

$$= \int \frac{4(2x + 5)}{x^2 + 5x + 7}\,dx$$

Let $x^2 + 5x - 7 = t$, then $(2x + 5)dx = dt$

$$I = \int \frac{4dt}{t} = 4\log_e t + c$$

Put $t = x^2 + 5x - 7$

$$I = 4\log_e|x^2 + 5x - 7| + c$$

9. *(d)* Given, root of $z^2 + \alpha z + \beta = 0$ is $1 - 2i$.

Since, it is quadratic equation and one root is complex in nature, its another root is complex conjugate.

$\therefore$ Two roots are $1-2i$ and $1 + 2i$.

Now, sum of roots $= -\dfrac{\alpha}{1} = -\alpha$

$$= (1 - 2i) + (1 + 2i) = 2$$

Gives, $\alpha = -2$

Product of roots $= \dfrac{\beta}{1} = \beta$

$$= (1 - 2i)(1 + 2i) = 1 + 4 = 5$$

Gives, $\beta = 5$

$\therefore\ \alpha - \beta = -2 - 5 = -7$

10. *(a)* Curve $x^2 + 2y^2 = 2$ intersect the line $x + y = 1$ at points P and Q.

First we have to find any common relation between these two curves. Use substitution for the same as follows,

$$x^2 + 2y^2 = 2 \qquad \text{...(i)}$$

$x + y = 1$, then $(x + y)^2 = 1^2$

$$\Rightarrow \quad x^2 + y^2 + 2xy = 1 \qquad \text{...(i)}$$

We can write Eq. (i) as,

$$x^2 + 2y^2 - 2(1)^2 = 0$$

$$\Rightarrow \quad x^2 + 2y^2 - 2(x + y)^2 = 0 \qquad \text{[using Eq. (ii) in Eq. (i)]}$$

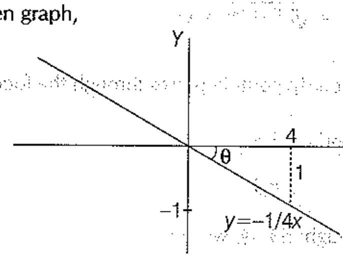
$$\Rightarrow x^2 + 2y^2 - 2x^2 - 2y^2 - 4xy = 0$$

$$\Rightarrow \quad -x^2 - 4xy = 0 \Rightarrow -x(x + 4y) = 0$$

Gives, $x = 0$ and $x + 4y = 0$ or $y = \dfrac{-1}{4}x$

Draw the line $y = \dfrac{-1}{4}x$ on graph and take arbitrary point (any one) as follows,

From given graph,



$$\tan\theta = \frac{1}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{4}\right)$$

We have two lines, $y = -\dfrac{1}{4}x$ and $x = 0$ (i.e. Y - axis). Thus, any line joining these two curves makes an angle $\dfrac{\pi}{2} + \theta$ at origin.

$\therefore$ Answer is $\dfrac{\pi}{2} + \tan^{-1}\left(\dfrac{1}{4}\right)$.

11. *(b)* Let (x, y) be any arbitrary point on curve $x^2 = 2y$ and find the tangent line equation at this point, such that tangent line at (x, y) is parallel to line $x - y = 1$.

To find tangent equation, differentiate the following equation so that we can find slope,

$$x^2 - 2y = 0 \qquad \text{...(i)}$$

$$2x - 2\frac{dy}{dx} = 0 \text{ gives } \frac{dy}{dx} = x$$

Slope (say m_1) = x

Also, slope of line $x - y = 1$ or $y = x - 1$ is 1 (say m_2). Since, $x - y = 1$ and tangent line is parallel,

therefore, their slope be equal.

Hence, $m_1 = m_2$ gives, $x = 1$

Put $x = 1$ in Eq. (i), we get $y = 1/2$

Thus, $(x, y) = \left(1, \dfrac{1}{2}\right)$

Perpendicular distance between line $x - y = 1$ and point $\left(1, \dfrac{1}{2}\right)$ is given as,

$$P = \left|\frac{(1)(1) + \left(\frac{1}{2}\right)(-1) - 1}{\sqrt{(1)^2 + (-1)^2}}\right|$$

$$= \left|\frac{-1}{2\sqrt{2}}\right|$$

$$= \frac{1}{2\sqrt{2}} \qquad (\because \text{using perpendicular distance formula})$$

12. *(a)* Given, ellipse is $\frac{x^2}{25}+\frac{y^2}{16}=1$

or $\frac{x^2}{(5)^2}+\frac{y^2}{(4)^2}=1$

Compare it with standard equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

we get $a = 5$, $b = 4$

Now, focus of ellipse $= (\pm c, 0)$ where

$$c=\sqrt{a^2-b^2}$$

Put the values of a and b, we get

$$c=\sqrt{5^2-4^2}=\sqrt{25-16}=\sqrt{9}$$

$\therefore$ Focus $= (\pm 3, 0)$

According to question, hyperbola passes through the focus of ellipse.

Let equation of hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Since, it passes through $(\pm 3, 0)$, we get

$\frac{(\pm 3)^2}{a^2}-\frac{0}{b^2}=1$, gives $a=\pm 3$ or $a^2=9$

Also, given that product of eccentricities is 1.

Now, (Eccentricity of ellipse) (Eccentricity of hyperbola) = 1

$$\Rightarrow \left(\sqrt{1-\frac{16}{25}}\right)\left(\sqrt{1+\frac{b^2}{9}}\right)=1$$

(using formula of eccentricity of ellipse and hyperbola)

$$\Rightarrow \left(\sqrt{\frac{9}{25}}\right)\left(\sqrt{1+\frac{b^2}{9}}\right)=1$$

Squaring on both sides,

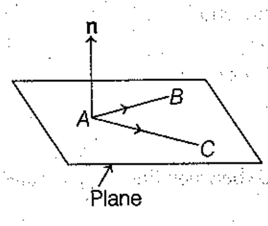
$$1+\frac{b^2}{9}=\frac{25}{9}$$

$$\Rightarrow b^2=16$$

Thus, equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$.

13. *(b)* Refer diagram, the normal vector be **n** and it is perpendicular to both **AB** and **AC**.

$\mathbf{AB}\times\mathbf{AC}=\mathbf{n}$



Now, $A(1, 2, 3)$, $B(2, 3, 1)$ and $C(2, 4, 2)$

Then, $\mathbf{AB}=(2-1)\hat{i}+(3-2)\hat{j}+(1-3)\hat{k}$

$=\hat{i}+\hat{j}-2\hat{k}$

$\mathbf{AC}=(2-1)\hat{i}+(4-2)\hat{j}+(2-3)\hat{k}$

$=\hat{i}+2\hat{j}-\hat{k}$

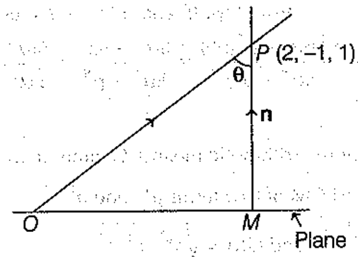
Now, $\mathbf{AB}\times\mathbf{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -2\\ 1 & 2 & -1\end{vmatrix}$

$=\hat{i}(-1+4)-\hat{j}(-1+2)+\hat{k}(2-1)$

$=3\hat{i}-\hat{j}+\hat{k}$

$\mathbf{n}=3\hat{i}-\hat{j}+\hat{k}$

Let P be any point on normal vector and O be origin. Then refer the diagram, projection of OP on plane have length OM.



$\mathbf{OP}=2\hat{i}-\hat{j}+\hat{k}$ and $\mathbf{n}=3\hat{i}-\hat{j}+\hat{k}$

$\mathbf{OP}.\ \mathbf{n}=|\mathbf{OP}|\,|\mathbf{n}|\cos\theta$

$(6+1+1)=\sqrt{4+1+1}(\sqrt{9+1+1})\cos\theta$

$8=\sqrt{6}\sqrt{11}\cos\theta \Rightarrow \cos\theta=\frac{8}{\sqrt{66}}$

Again, $\sin\theta=\frac{|OM|}{|OP|}$, gives $|OM|=\sin\theta.|OP|$

$\Rightarrow \quad |OM|=\sqrt{1-\cos^2\theta}|OP|$

$=\sqrt{1-\frac{64}{66}}\sqrt{4+1+1}$ (use $\cos\theta=8\sqrt{66}$)

$=\sqrt{\frac{2}{66}}.\sqrt{6}$

$\therefore \quad |OM|=\sqrt{\frac{2}{11}}$

14. *(b)* Let $L=\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\ldots+\frac{n}{(2n-1)^2}\right]$

or $L=\lim\limits_{n\to\infty}\left[\frac{n}{(n+0)^2}+\frac{n}{(n+1)^2}+\ldots+\frac{n}{(n+n-1)^2}+\frac{n}{(n+n)^2}-\frac{n}{(n+n)^2}\right]$

$=\lim\limits_{n\to\infty}\left[\frac{n}{(n+0)^2}+\frac{n}{(n+1)^2}+\ldots+\frac{n}{(n+n)^2}\right]$

$-\lim\limits_{n\to\infty}\left[\frac{n}{(n+n)^2}\right]$

$=\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n}\frac{n}{(n+r)^2}-\lim\limits_{n\to\infty}\frac{1}{4n}$

$=\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n}\frac{n}{(n+r)^2}-0\left(\text{since, }\lim\limits_{n\to\infty}\frac{1}{n}=0\right)$

Now, for solving limit summation, we integrate it using some replacement.

$$L=\lim_{n\to\infty}\sum_{r=0}^{n}\frac{1}{n(1+r/n)}$$

Take $\frac{r}{n}$ as x and $\frac{1}{n}$ as dx.

Lower limit is obtained by putting $r=0$ in $\frac{r}{n}$, we get Lower limit = 0

Upper limit is obtained by putting $r = n$ in $\frac{r}{n}$, we get

Upper limit = 1

$\therefore \quad L = \int_0^1 \frac{1}{(1+x)^2}dx = \left[\frac{-1}{(1+x)}\right]_0^1 = -\left(\frac{1}{2}-1\right) = \frac{1}{2}$

$\therefore \quad L = \frac{1}{2}$

15. *(c)* Let 'A' be the event of smokers and non-vegetarian.
Let 'B' be the event of smokers and vegetarian.
Let 'C' be the event of non-smokers and vegetarian.
Let 'E' be the event of chest disorders.
According to question,

$P(A) = \frac{160}{400} = \frac{2}{5}, P(B) = \frac{100}{400} = \frac{1}{4}, P(C) = \frac{140}{400} = \frac{7}{20}$

$P\left(\frac{E}{A}\right) = 35\% = \frac{35}{100}, P\left(\frac{E}{B}\right) = \frac{20}{100}, P\left(\frac{E}{C}\right) = \frac{10}{100}$

Thus, $P\left(\frac{A}{E}\right) = \dfrac{P(A).P\left(\frac{E}{A}\right)}{P(A).P\left(\frac{E}{A}\right) + P(B)P\left(\frac{E}{B}\right) + P(C)P\left(\frac{E}{C}\right)}$

$= \dfrac{\frac{2}{5}.\frac{35}{100}}{\frac{2}{5}.\frac{35}{100} + \frac{1}{4}.\frac{20}{100} + \frac{7}{20}.\frac{10}{100}}$

$= \dfrac{\frac{14}{100}}{\frac{14}{100} + \frac{5}{100} + \frac{\frac{7}{2}}{100}} = \dfrac{14}{14 + 5 + \frac{7}{2}} = \frac{28}{45}$

16. *(c)* Here, we have four digit natural numbers, then total cases will exclude those number which contain zero at thousands place.
Hence, total cases will be

$= ({}^4C_1 \times 9 \times 9 \times 9) - ({}^3C_1 \times 9 \times 9) = 2673$

Again, only those numbers will have remainder 2 when divided by 5 either they have 2 at its unit place or 7 at its unit place.
When unit digit is 2, then total number of four digit numbers will be $= ({}^3C_1 \times 9 \times 9) - ({}^2C_1 \times 9) = 225$

When unit digit is 7, then total number of four digit numbers will be $= 8 \times 9 \times 9 = 648$

Now, total favourable cases $= 225 + 648 = 873$

Required probability $= \dfrac{\text{Total favourable cases}}{\text{Total number of cases}}$

$= \frac{873}{2673} = \frac{97}{297}$

17. *(d)* Given, $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - \left[2\cos^2\left(\frac{x+y}{2}\right) - 1\right] = \frac{3}{2}$

[Use formula,

$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$,

$\cos 2x = 2\cos^2 x - 1$]

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right)$

$= \frac{3}{2} - 1 = \frac{1}{2}$

$\Rightarrow 4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 4\cos^2\left(\frac{x+y}{2}\right)$

$= \frac{1}{2} \times 2 = 1 = \cos^2\frac{(x-y)}{2} + \sin^2\frac{(x-y)}{2}$

$\Rightarrow \left[\cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right)\right]^2 + \sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \sin\left(\frac{x-y}{2}\right) = 0$ and $\cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right) = 0$

$\Rightarrow x = y$ and $\cos 0 - 2\cos x = 0$ (use $x = y$)

Gives, $\cos x = \frac{1}{2} = \cos y$

$\therefore \sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

$\therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1+\sqrt{3}}{2}$

18. *(a)* $x = \{f : A \to B, f \text{ is one - one}\}$

$y = \{g : A \to A \times B, g \text{ is one one}\}$

Number of elements in $A = 3$ i.e. $|A| = 3$

Similarly, $|B| = 5$

Then, $|A \times B| = |A| \times |B| = 3 \times 5 = 15$

Now, number of one-one function from A to B will be

${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$

$\therefore \quad x = 60$

Now, number of one-one function from A to $A \times B$ will be

$= {}^{15}P_3 = \frac{15!}{(15-3)!} = \frac{15!}{12!}$

$= 15 \times 14 \times 13 = 2730$

$\therefore \quad y = 2730$

Thus, $2 \times (2730) = 91 \times (60)$

$2y = 91x$

19. *(c)* $\operatorname{cosec}[2\cot^{-1}(5) + \cos^{-1}(4/5)]$

$= \operatorname{cosec}\left[2\tan^{-1}\left(\frac{1}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right)\right] \quad \left[\because \tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right)\right]$

$= \operatorname{cosec}\left[\tan^{-1}\left(\dfrac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}\right) + \cos^{-1}\left(\frac{4}{5}\right)\right] \left[\therefore 2\tan^{-1}\theta = \tan^{-1}\left(\frac{2\theta}{1-\theta^2}\right)\right]$

$= \operatorname{cosec}\left(\tan^{-1}\frac{5}{12} + \cos^{-1}\frac{4}{5}\right)$

Let $\tan^{-1}\left(\frac{5}{12}\right) = x$, then $\tan x = \frac{5}{12}$ gives $\sin x = \frac{5}{13}$, $\cos x = \frac{12}{13}$

Let $\cos^{-1}\left(\frac{4}{5}\right) = y$, then $\cos y = \frac{4}{5}$ gives, $\sin y = \frac{3}{5}$

Now, $\operatorname{cosec}(x+y) = \dfrac{1}{\sin(x+y)} = \dfrac{1}{\sin x \cos y + \cos x \sin y}$

$= \dfrac{1}{\left(\frac{5}{13}\right)\left(\frac{4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{3}{5}\right)} = \frac{65}{56}$

20. *(c)* Contrapositive of $p \to q$ is $\sim q \to \sim p$

p : you will work

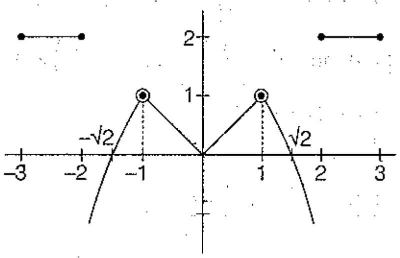
$\Rightarrow \sim p$: you will not work

q : you will earn money

$\Rightarrow \sim q$: you will not earn money

Then, $\sim q \to \sim p$: if you will not earn money, you will not work

21. *(5)* For this particular problem, try to draw graph in the region $(-3,3)$, it will be as follows,



Thus, points of discontinuity will be at $-2, 2$ because the curve breaks at these points and at $-1, 0, 1$ because curve has sharp points.

$\therefore$ Point of discontinuity are $-2, -1, 0, 1, 2$ i.e. 5 points.

22. *(1)* Given, $(2xy^2 - y)dx + xdy = 0$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = -2y^2$$

$$\Rightarrow \quad \frac{-1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = 2 \qquad \text{...(i) [divide by } y^2]$$

Let $\frac{1}{y} = v$, then $-\frac{1}{y^2}\cdot\frac{dy}{dx} = \frac{dv}{dx}$, putting in Eq. (i)

$$\frac{dv}{dx} + v\left(\frac{1}{x}\right) = 2 \qquad \text{(this is a linear form)}$$

Now, integrating factor (IF) $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$$\therefore \quad (IF)v = \int 2.(IF)dx = \int 2x dx = 2\frac{x^2}{2} + C$$

$$\therefore \quad (IF)v = x^2 + C$$

Put $v = \frac{1}{y}$, this gives

$$x^2 + c = \frac{x}{y}$$

Now, first find point of intersection of lines $2x - 3y = 1$ and $3x = -2y + 8$ by elimination method, we get $x = 2$, $y = 1$

$\therefore$ The curve $x^2 + c = \frac{1}{y}$ passes through $(2, 1)$.

Put $x = 2, y = 1$, we get $c = -2$

$$\frac{x}{y} = x^2 - 2$$

or
$$y = \frac{x}{x^2 - 2}$$

Put $x = 1$, we get $y(1) = \frac{1}{1-2} = -1$

$$\therefore \quad |y(1)| = 1$$

23. *(45)* We may write, $7^n = (10 - 3)^n$ or $7^n = 10k + (-3)^n$ (using expansion)

$$\therefore 7^n + 3^n = 10k + (-3)^n + 3^n = \begin{cases} 10k, & n = \text{odd} \\ 10k + 2.3^n, & n = \text{even} \end{cases}$$

Let $n = \text{even} = 2t, t \in N$

Then, $3^n = 3^{2t} = 9^t = (10 - 1)^t$

$$= 10p + (-1)^t = 10p \pm 1$$

If n = even, then $7^n + 3^n$ will never be multiple of 10.

This implies n = odd

$$n = 11, 13, 15, \ldots 99 \quad \text{(since, } n \text{ is two digit)}$$

$$\Rightarrow \quad 10 < n < 100$$

Total possible 'n' are 45.

24. *(5)* $\lim\limits_{x\to 0}\frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)} = L$ (say) $\qquad \left[\frac{0}{0}\text{ form}\right]$

Apply L - Hospital rule,

$$L = \lim_{x\to 0}\frac{a - 4e^{4x}}{a(e^{4x} - 1) + ax(4e^{4x})}$$

[Limit exist everywhere except $a = 4$]

Again, apply L-Hospital rule,

$$L = \lim_{x\to 0}\frac{-16e^{4x}}{a(4e^{4x}) + a(4e^{4x}) + ax(16e^{4x})}$$

$$= \frac{-16}{4a + 4a} = \frac{-2}{a}$$

$$= \frac{-2}{4} = \frac{-1}{2} \qquad \text{(use } a = 4)$$

Given, $\quad L = b \Rightarrow \frac{-2}{a} = \frac{-1}{2} = b$

Then, $a - 2b = 4 - 2\left(\frac{-1}{2}\right) = 4 + 1 = 5$

25. *(4)* If the curves cut at right angle, then product of slopes will be -1.

First curve $x = y^4$

Differentiate it, we get

$$1 = 4y^3\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{4y^3}$$

Slope of first curve $(m_1) = \frac{1}{4y_1^3}$ $\qquad$ [at point (x_1, y_1)]

Second curve $xy = k$

Differentiate it, $0 = x\frac{dy}{dx} + y$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x}$$

Slope of second curve $(m_2) = \frac{-y_1}{x_1}$ $\qquad$ [at (x_1, y_1)]

$$\Rightarrow \quad m_1.m_2 = -1$$

$$\Rightarrow \quad \frac{1}{4y_1^3}\left(\frac{-y_1}{x_1}\right) = -1 \Rightarrow \frac{-1}{4y_1^2x_1} = -1$$

$$\Rightarrow \quad \frac{-1}{4(y_1)^6} = -1 \qquad \text{[using } x_1 = y_1^4]$$

$$\Rightarrow \quad y_1^6 = \frac{1}{4}$$

Also, $x_1y_1 = k$, using $x_1 = y_1^4$, we get $k = y_1^5$ or $k^6 = (y_1)^{30}$

$\because \quad y_1^6 = \frac{1}{4}$, then $y_1^{30} = \left(\frac{1}{4}\right)^5$

$\therefore \quad (4k)^6 = 4^6.k^6 = 4^6(y_1)^{30} = 4^6\left(\frac{1}{4}\right)^5 = 4$

$\therefore \quad (4k)^6 = 4$

26. *(19)* $\int_{-2}^{2}|3x^2-3x-6|dx = I$ (say)

$$I = 3\int_{-2}^{2}|x^2-x-2|dx$$

$$= 3\left[\int_{-2}^{-1}(x^2-x-2)dx + \int_{-1}^{2}(-x^2+x+2)dx\right]$$

$$= 3\left[\left(\frac{x^3}{3}-\frac{x^2}{2}-2x\right)_{-2}^{-1} - \left(\frac{x^3}{3}-\frac{x^2}{2}-2x\right)_{-1}^{2}\right]$$

$$= 19$$

27. *(1)* Given, when x is divided by 4, the remainder is 3.

Let $x = 4p+3$, then

$$(2020+x)^{2022} = (2020+4p+3)^{2022}$$

$$= (2024+4p-1)^{2022}$$

$$= (4k-1)^{2022} \quad (\because 2024 \text{ is divisible by } 4)$$

$$= {}^{2022}C_0(4K)^{2022}(-1)^0 + {}^{2022}C_1(4K)^{2021}(-1)^1 + \dots + {}^{2022}C_{2022}(4A)^0(-1)^{2022}$$

On expansion $(2020+x)^{2022}$, we get the form of $8\lambda+1$. Since, each terms have 2022 and $4k$, so if we take 2 common from 2022 we get 8. Thus, each term have 8 in common.

Hence, remainder is 1.

28. *(44)* Let $L_1 \Rightarrow \frac{x-3}{1} = \frac{y-3}{2} = \frac{z-4}{2} = u$ (say)

$\Rightarrow$ Direction ratios of $L_1 = 1, 2, 2$

$$L_2 \Rightarrow \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = v \text{ (say)}$$

Direction ratios of $L_2 = 2, 2, 1$

Line L passing through origin is perpendicular to L_1 and L_2.

Hence, direction ratios of L is parallel to $(L_1 \times L_2)$.

$\Rightarrow \quad (-2, 3, -2)$

Equation of $L \Rightarrow \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \lambda$ (say)

Solve L and L_1, we get

$(2\lambda, -3\lambda, 2\lambda) = (\mu+3, 2\mu-1, 2\mu+4)$

Gives, $\lambda = 1, \mu = -1$

So, intersection point $P(2, -3, 2)$.

Let $Q(2v+3, 2v+3, v+2)$ be required point on L_2.

Now, $PQ = \sqrt{17}$ (given)

$$PQ = \sqrt{(2v+1)^2+(2v+6)^2+(v)^2} = \sqrt{17}$$

$\Rightarrow (2v+1)^2+(2v+6)^2+v^2 = 17$ (squaring on both sides)

$\Rightarrow 9v^2+28v+20 = 0$

On solving, we get $v = -2$ (rejected), $\frac{-10}{9}$ (accepted)

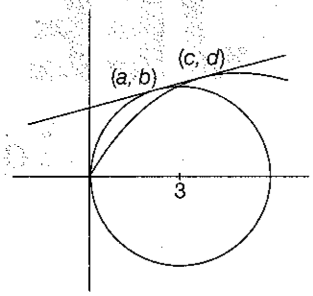
$\therefore$ Q is $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

$$\therefore 18(a+b+c) = 18\left(\frac{7}{9}+\frac{7}{9}+\frac{8}{9}\right) = 44$$

29. *(9)* Given, circle $\Rightarrow (x-3)^2+y^2 = 9$

Parabola $\Rightarrow y^2 = 4x$

Let equation of common tangent be



$$y = mx + \frac{a}{m}$$

$$\Rightarrow \quad y = mx + \frac{1}{m} \quad (\because y^2 = 4x)$$

$$\Rightarrow m^2x - my + 1 = 0$$

The above line is tangent to circle.

$\therefore$ Perpendicular from (3, 0) to line = 3

$$\Rightarrow \left|\frac{(3m^2-0+1)}{\sqrt{m^2+m^4}}\right| = 3$$

$$\Rightarrow (3m^2+1)^2 = 9(m^2+m^4) \Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

Tangent is $y = \frac{1}{\sqrt{3}}x + \sqrt{3}$

$$\Rightarrow \quad m = \frac{1}{\sqrt{3}}$$

or $\quad y = \frac{(-1)}{\sqrt{3}} \times (-\sqrt{3})$ (rejected)

For parabola, point of contact is $(c, d) = \left(\frac{a}{m^2}, \frac{2a}{n}\right)$

$\therefore \quad (c, d) = (3, 2\sqrt{3})$

Again, solve circle and line equation, we get

$$(x-3)^2 + \left[\left(\frac{1}{\sqrt{3}}x+\sqrt{3}\right)\right]^2 = 9$$

$$\Rightarrow \quad x^2+9-6x+\frac{1}{3}x^2+3+2x = 9$$

$$\Rightarrow \quad \frac{4}{3}x^2 - 4x + 3 = 0$$

$$\Rightarrow \quad x = \frac{3}{2} = a$$

$$\therefore \quad 2(a+c) = 2\left(\frac{3}{2}+3\right) = 9$$

30. *(2)* Area of parallelogram $= |\mathbf{a} \times \mathbf{b}|$

$$= |(\hat{\mathbf{i}}+\alpha\hat{\mathbf{j}}+3\hat{\mathbf{k}}) \times (3\hat{\mathbf{i}}-\alpha\hat{\mathbf{j}}+\hat{\mathbf{k}})|$$

$(64)(3) = 16\alpha^2 + 64 + 16\alpha^2$ (given, area $= 8\sqrt{3}$)

(squaring on both sides)

$\Rightarrow \quad \alpha^2 = 4$

Now, $\mathbf{a}.\mathbf{b} = 3 - \alpha^2 + 3$

$$= 6 - \alpha^2 = 6 - 4 = 2$$

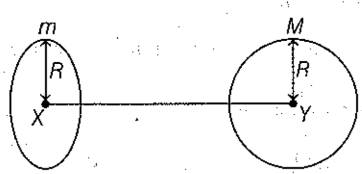
JEE Main 2021

26 FEBRUARY SHIFT I

PHYSICS

Section A : Objective Type Questions

1. Find the gravitational force of attraction between the ring and sphere as shown in the figure, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}\,R$ is the distance between the centres of a ring (of mass m) and a sphere (of mass M), where both have equal radius R.



a. $\frac{\sqrt{8}}{9} \cdot \frac{GmM}{R}$ **b.** $\frac{2\sqrt{2}}{3} \cdot \frac{GMm}{R^2}$

c. $\frac{1}{3\sqrt{8}} \cdot \frac{GMm}{R^2}$ **d.** $\frac{\sqrt{8}}{27} \cdot \frac{GmM}{R^2}$

2. Consider the combination of two capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is 15/4 time the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors $\frac{C_2}{C_1}$.

a. $\frac{15}{11}$ **b.** $\frac{111}{80}$

c. $\frac{29}{15}$ **d.** $\frac{15}{4}$

3. In a typical combustion engine, the work done by a gas molecule is given $W = \alpha^2 \beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be

a. $[MLT^{-2}]$ **b.** $[M^0LT^0]$

c. $[M^2LT^{-2}]$ **d.** $[MLT^{-1}]$

4. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of $\lambda_1 : \lambda_2$ is

a. 1 : 9 **b.** 7 : 108

c. 7 : 135 **d.** 1 : 3

5. A short straight object of height 100 cm lies before the central axis of a spherical mirror, whose focal length has absolute value $f = 40$ cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information.

a. Image is real, same side of concave mirror

b. Image is virtual, opposite side of concave mirror

c. Image is real, same side of convex mirror

d. Image is virtual, opposite side of convex mirror

6. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance $\left(\frac{R}{2}\right)$ from the earth's centre, where R is the radius of the earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period?

a. $\frac{2\pi R}{g}$ **b.** $\frac{g}{2\pi R}$

c. $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ **d.** $2\pi\sqrt{\frac{R}{g}}$

7. An alternating current is given by the equation $i = i_1 \sin \omega t + i_2 \cos \omega t$. The rms current will be

a. $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$ **b.** $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$

c. $\frac{1}{2}(i_1^2 + i_2^2)^{1/2}$ **d.** $\frac{1}{\sqrt{2}}(i_1 + i_2)$

8. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure p is applied uniformly on all sides, will be

a. $\frac{\rho K}{p}$ **b.** $\frac{\rho p}{K}$

c. $\frac{K}{\rho p}$ **d.** $\frac{pK}{\rho}$

9. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by

a. $T \propto R^2$ **b.** $T \propto R^{3/2}$

c. $T \propto R^{5/2}$ **d.** $T \propto R^{4/3}$

10. A planet revolving in elliptical orbit has

I. a constant velocity of revolution

II. has the least velocity when it is nearest to the Sun

III. its areal velocity is directly proportional to its velocity

IV. areal velocity is inversely proportional to its velocity.

V. to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below.

a. Only I **b.** Only IV **c.** Only III **d.** Only V

11. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A Body P having mass M moving with speed u has head-on collision elastically with another body Q having mass m initially at rest. If $m << M$, body Q will have a maximum speed equal to $2u$ after collision.

Reason R During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below.

a. A is not correct but R is correct.

b. Both A and R are correct but R is not the correct explanation of A.

c. Both A and R are correct and R is the correct explanation of A.

d. A is correct but R is not correct.

12. Four identical solid spheres each of mass m and radius a are placed with their centres on the four corners of a square of side b. The moment of inertia of the system about one side of square, where the axis of rotation is parallel to the plane of the square is

a. $\frac{4}{5}ma^2 + 2mb^2$ **b.** $\frac{8}{5}ma^2 + mb^2$

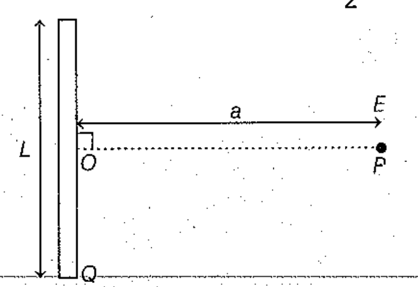
c. $\frac{8}{5}ma^2 + 2mb^2$ **d.** $\frac{4}{5}ma^2$

13. In a Young's double slit experiment, two slits are separated by 2 mm and the screen is placed one metre away. When a light of wavelength 500 nm is used, the fringe separation will be

a. 0.25 mm **b.** 0.50 mm

c. 0.75 mm **d.** 1 mm

14. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a = \frac{\sqrt{3}}{2}L$.



a. $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$ **b.** $\frac{Q}{3\pi\varepsilon_0 L^2}$

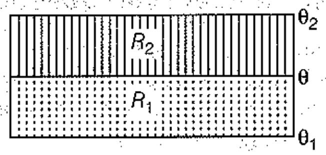
c. $\frac{Q}{2\sqrt{3}\pi\,\varepsilon_0 L^2}$ **d.** $\frac{Q}{4\,\pi\varepsilon_0 L^2}$

15. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor

a. $\frac{1}{2}, \sqrt{2}$ **b.** $\frac{1}{4}, \sqrt{2}$

c. $\frac{1}{4}, 2\sqrt{2}$ **d.** $\frac{1}{2}, 2\sqrt{2}$

16. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by



a. $\frac{\theta_2 R_2 - \theta_1 R_1}{R_2 - R_1}$ **b.** $\frac{\theta_1 R_2 - \theta_2 R_1}{R_2 - R_1}$

c. $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$ **d.** $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$

17. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A An electron microscope can achieve better resolving power than an optical microscope.

Reason R The de-Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below.

a. A is true but R is false.

b. Both A and R are true and R is the correct explanation of A.

c. Both A and R are true but R is not the correct explanation of A.

d. A is false but R is true.

18. LED is constructed from GaAsP semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour. [$h = 6.63 \times 10^{-34}$ Js and $c = 3 \times 10^8$ ms^{-1}]

a. 1046 nm and red colour **b.** 654 nm and orange colour

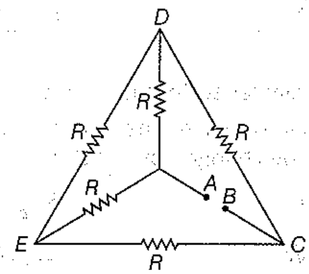
c. 1046 nm and blue colour **d.** 654 nm and red colour

19. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be

a. $\frac{2T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$ **b.** $\frac{2T}{rJ}$

c. $\frac{3T}{rJ}$ **d.** $\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

20. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is



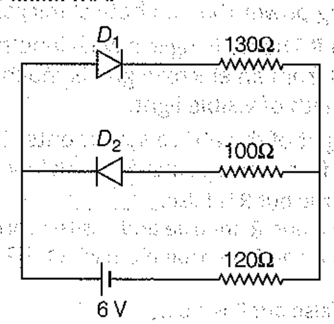
a. $2R$
b. $\frac{R}{2}$
c. $\frac{3R}{2}$
d. R

Section B : Numerical Type Questions

21. A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person, if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be .... N. [g = 10 m/s^2]

22. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is .... J.

23. The circuit contains two diodes each with a forward resistance of 50 Ω and with infinite reverse resistance. If the battery voltage is 6V, the current through the 120 Ω resistance is .... mA.



24. A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x \times 10^{-1}$ V/m. Value of x is ....

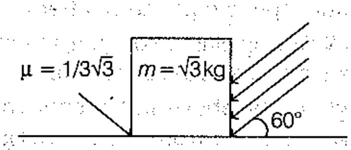
(Rounded-off to the nearest integer)

[Take, $\varepsilon_0 = 8.85 \times 10^{-12}$ C^{2}N^{-1}m^{-2}, $c = 3 \times 10^8$ ms^{-1}]

25. A boy pushes a box of mass 2 kg with a force $\mathbf{F} = (20\hat{\mathbf{i}} + 10\hat{\mathbf{j}})$ N on a frictionless surface. If the box was initially at rest, then .... m is displacement along the X-axis after 10 s.

26. As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $1/3\sqrt{3}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be $3x$. The value of x will be ....

[$g = 10$ ms^{-2}; $\sin 60° = \frac{\sqrt{3}}{2}$; $\cos 60° = \frac{1}{2}$]



27. A container is divided into two chambers by a partition. The volume of first chamber is 4.5 L and second chamber is 5.5 L. The first chamber contain 3.0 mol of gas at pressure 2.0 atm and second chamber contain 4.0 mol of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then the common equilibrium pressure existing in the mixture is $x \times 10^{-1}$ atm. Value of x is ....

28. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form $y = -0.21 \sin(x + 30t)$ is produced in it, where x is in metre and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is .... (Round-off to the nearest integer)

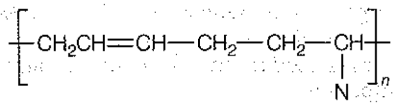
29. In a series L-C-R resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be ....

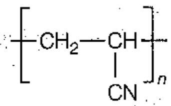
30. The maximum and minimum amplitude of an amplitude modulated wave is 16 V and 8 V, respectively. The modulation index for this amplitude modulated wave is $x \times 10^{-2}$. The value of x is ....

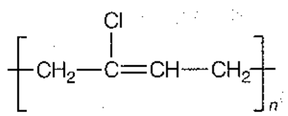
CHEMISTRY

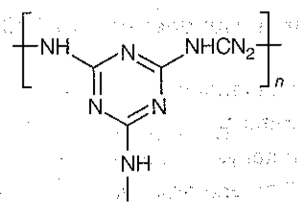
Section A : Objective Type Questions

1. The structure of neoprene is

a. 

b. 

c. 

d. 

2. Find *A*, *B* and *C* in the following reactions:

$NH_3 + A + CO_2 \longrightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + B \longrightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + C$

a. $A - O_2$, $B - CO_2$, $C - Na_2CO_3$

b. $A - H_2O$, $B - O_2$, $C - Na_2CO_3$

c. $A - H_2O$, $B - O_2$, $C - NaHCO_3$

d. $A - H_2O$, $B - CO_2$, $C - NaHCO_3$

3. The presence of ozone in troposphere

a. protects us from the UV radiation

b. protects us from the X-ray radiation

c. protects us from green house effect

d. generates photochemical smog

4. Match List-I with List-II.

	List-I (Electronic configuration of elements)		List-II (Δ_i in kJ mol^{-1})
A.	$1s^2 2s^2$	(i)	801
B.	$1s^2 2s^2 2p^4$	(ii)	899
C.	$1s^2 2s^2 2p^3$	(iii)	1314
D.	$1s^2 2s^2 2p^1$	(iv)	1402

Choose the most appropriate answer from the options given below.

	A	B	C	D
a.	(ii)	(iii)	(iv)	(i)
b.	(i)	(iv)	(iii)	(ii)
c.	(i)	(iii)	(iv)	(ii)
d.	(iv)	(i)	(ii)	(iii)

5. Given below are two statements: One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation.

Reason (R) Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical.

In the light of the above statements, choose the most appropriate answer from the options given below.

a. A is false but R is true.

b. Both A and R are true and R is the correct explanation of A.

c. A is true R is false.

d. Both A and R are true but R is not the correct explanation of A.

6. Statements about heavy water are given below.

A. Heavy water is used in exchange reactions for the study of reaction mechanisms.

B. Heavy water is prepared by exhaustive electrolysis of water.

C. Heavy water has higher boiling point than ordinary water.

D. Viscosity of H_2O is greater than D_2O.

Whic oc the given statement are correct.

a. A, B and C **b.** Only A and B

c. Only A and D **d.** Only A and C

7. The orbital having two radial as well as two angular nodes is

a. 3*p* **b.** 4*f* **c.** 4*d* **d.** 5*d*

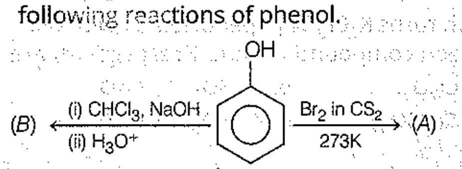
8. Match List-I with List-II.

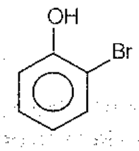
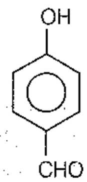
	List-I (Ore)		List-II (Element present)
A.	Kernite	(i)	Tin
B.	Cassiterite	(ii)	Boron
C.	Calamine	(iii)	Fluorine
D.	Cryolite	(iv)	Zinc

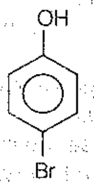
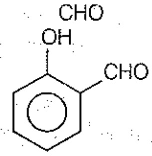
Choose the most appropriate answer from the options given below.

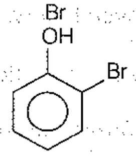
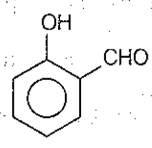
	A	B	C	D
a.	(i)	(iii)	(iv)	(ii)
b.	(ii)	(i)	(iv)	(iii)
c.	(ii)	(iv)	(i)	(iii)
d.	(iii)	(i)	(ii)	(iv)

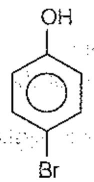
9. Identify the major products *A* and *B* respectively in the following reactions of phenol.



a.  and 

b.  and 

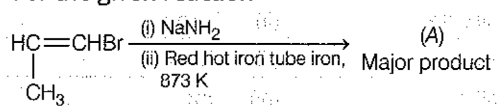
c.  and 

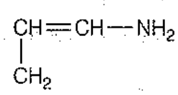
d.  and 

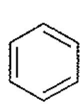
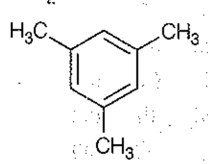
10. Given below are two statements :
Statement I A mixture of chloroform and aniline can be separated by simple distillation.
Statement II When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point. In the light of the above statements, choose the most appropriate answer from the options given below.

a. Statement I is false but statement II is true
b. Both statement I and statement II are false
c. Statement I is true but statement II is false
d. Both statement I and statement II are true

11. For the given reaction



a. $CH_3CH_2CH_2NH_2$ b. 

c.  d. 

12. On treating a compound with warm dil. H_2SO_4, gas X is evolved, which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y. X and Y respectively are

a. $X = SO_2, Y = Cr_2O_3$ b. $X = SO_3, Y = Cr_2O_3$
c. $X = SO_2, Y = Cr_2(SO_4)_3$ d. $X = SO_3, Y = Cr_2(SO_4)_3$

13. Which of the following is a false statement?

a. Carius tube is used in the estimation of sulphur in an organic compound
b. Carius method is used for the estimation of nitrogen in an organic compound
c. Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture
d. Kjeldahl's method is used for the estimation of nitrogen in an organic compound

14. Which of the following vitamin is helpful in delaying the blood clotting ?

a. Vitamin C b. Vitamin B
c. Vitamin E d. Vitamin K

15. $\underset{(C_4H_8Cl_2)}{A} \xrightarrow[373\text{ K}]{\text{Hydrolysis}} \underset{(C_4H_8O)}{B}$

B reacts with hydroxyl amine but does not give Tollen's test. Identify A and B.

a. 1,1-dichlorobutane and 2-butanone
b. 2,2-dichlorobutane and butanal
c. 1,1-dichlorobutane and butanal
d. 2,2-dichlorobutane and 2-butan-one

16. Compound A used as a strong oxidising agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is

a. PbO_2 b. PbO
c. $PbSO_4$ d. Pb_3O_4

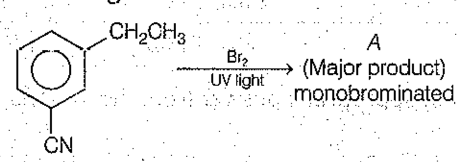
17. Which one of the following lanthanoids does not form MO_2? [M is lanthanoid metal]

a. Pr b. Dy
c. Nd d. Yb

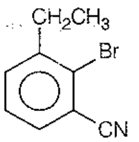
18. Given below are two statements:
Statement I *o*-nitrophenol is steam volatile due to intramolecular hydrogen bonding.
Statement II *o*-nitrophenol has high melting due to hydrogen bonding.
In the light of the above statements, choose the most appropriate answer from the options given below.

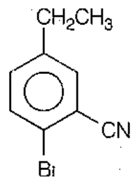
a. Statement I is false but statement II is true
b. Both statement I and statement II are true
c. Both statement I and statement II are false
d. Statement I is true but statement II is false

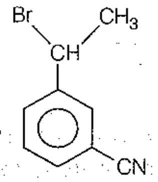
19. For the given reaction,

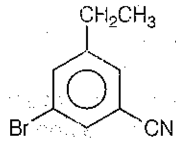


What is A?

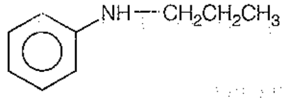
a. 

b. 

c. 

d. 

20. An amine on reaction with benzene sulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is

a. 

b. $CH_3CH_2NH_2$

c. $CH_3CH_2CH_2NHCH_3$

d. $CH_3CH_2CH_2\overset{H}{N}—CH_2CH_3$

Section B : Numerical Type Questions

21. For a chemical reaction, $A + B \rightleftharpoons C + D\,(\Delta_r H° = 80$ kJ mol^{-1}) the entropy change $\Delta_r S°$ depends on the temperature T (in K) as $\Delta_r S° = 2T$ (JK^{-1} mol^{-1}). Minimum temperature at which it will become spontaneous is .... K.

22. The number of significant figures in 50000.020×10^{-3} is ....

23. An exothermic reaction $X \rightarrow Y$ has an activation energy 30 kJ mol^{-1}. If energy change ΔE during the reaction is –20 kJ mol^{-1}, then the activation energy for the reverse reaction in kJ is ....

24. Consider the following reaction,

$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O, \quad E° = 1.51$ V.

The quantity of electricity required in Faraday to reduce five moles of MnO_4^- is ....

25. A certain gas obeys $p(V_m - b) = RT$. The value of $\left(\frac{\partial Z}{\partial p}\right)_T$ is xb/RT. The value of x is ....
(Z = compressibility factor)

26. A homogeneous ideal gaseous reaction $AB_2(g) \rightleftharpoons A(g) + 2B(g)$ is carried out in a 25 L flask at 27°C. The initial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm. The value of K_p is $x \times 10^{-2}$. The value of x is ....

27. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is ....

28. 224 mL of $SO_2(g)$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour pressure of solution (assuming the solution is dilute), ($p_{(H_2O)}$ = 24 mm of Hg) is $x \times 10^{-2}$ mm of Hg, the value of x is ....

29. 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The volume of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is ....
[R = 0.0821 L atm K^{-1} mol^{-1}]

30. Number of bridging CO ligands in $[Mn_2(CO)_{10}]$ is ....

MATHEMATICS

Section A : Objective Type Questions

1. If **a** and **b** are perpendicular, then $\mathbf{a} \times (\mathbf{a} \times (\mathbf{a} \times (\mathbf{a} \times \mathbf{b})))$ is equal to

a. **0**
b. $\frac{1}{2}|\mathbf{a}|^4\,\mathbf{b}$
c. $\mathbf{a} \times \mathbf{b}$
d. $|\mathbf{a}|^4\,\mathbf{b}$

2. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

a. $\frac{15}{2^{13}}$
b. $\frac{15}{2^{12}}$
c. $\frac{15}{2^{8}}$
d. $\frac{15}{2^{14}}$

3. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

a. 4
b. 1
c. 6
d. 12

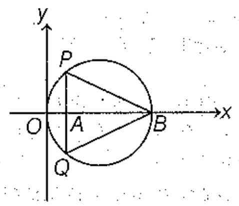
4. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4th, 6th and 8th terms is equal to

a. 30
b. 26
c. 35
d. 32

5. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}\, dx$, where $[x]$ is the greatest integer $\le x$, is

a. $100(e-1)$ b. $100(1-e)$
c. $100e$ d. $100(1+e)$

6. In the circle given below, let $OA = 1$ unit, $OB = 13$ unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is



a. $24\sqrt{2}$ b. $24\sqrt{3}$
c. $26\sqrt{3}$ d. $26\sqrt{2}$

7. The sum of the infinite series $1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots$ is equal to

a. $\frac{13}{4}$ b. $\frac{9}{4}$
c. $\frac{15}{4}$ d. $\frac{11}{4}$

8. The value of $\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cos h-\sin h)}\right\}$ is

a. $\frac{4}{3}$ b. $\frac{2}{\sqrt{3}}$ c. $\frac{3}{4}$ d. $\frac{2}{3}$

9. The maximum value of the term independent of 't' in the expansion of $\left(tx^{1/5}+\frac{(1-x)^{1/10}}{t}\right)^{10}$, where $x \in (0, 1)$ is

a. $\frac{10!}{\sqrt{3}(5!)^2}$ b. $\frac{2\cdot 10!}{3\sqrt{3}(5!)^2}$
c. $\frac{2\cdot 10!}{3(5!)^2}$ d. $\frac{10!}{3(5!)^2}$

10. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 h. If the population of bacteria is 2000 after $\frac{k}{\log_e(6/5)}$ h, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

a. 4 b. 8
c. 2 d. 16

11. If $(1, 5, 35)$, $(7, 5, 5)$, $(1, \lambda, 7)$ and $(2\lambda, 1, 2)$ are coplanar, then the sum of all possible values of λ is

a. $\frac{39}{5}$ b. $-\frac{39}{5}$ c. $\frac{44}{5}$ d. $-\frac{44}{5}$

12. If $\frac{\sin^{-1}(x)}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}$, $0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

a. $\frac{1-y^2}{y\sqrt{y}}$ b. $1-y^2$
c. $\frac{1-y^2}{1+y^2}$ d. $\frac{1-y^2}{2y}$

13. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is

a. 42 b. 82
c. 77 d. 35

14. Let f be any function defined on R and let it satisfy the condition $|f(x)-f(y)| \le |(x-y)^2|, \forall (x, y) \in R$
If $f(0) = 1$, then

a. $f(x)$ can take any value in R
b. $f(x) < 0, \forall x \in R$
c. $f(x) = 0, \forall x \in R$
d. $f(x) > 0, \forall x \in R$

15. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point

a. $(2, 2)$ b. $(0, 0)$
c. $(2, 9)$ d. $\left(3, \frac{21}{2}\right)$

16. The intersection of three lines $x-y=0$, $x+2y=3$ and $2x+y=6$ is a

a. right angled triangle b. equilateral triangle
c. isosceles triangle d. None of these

17. Consider the three planes
$P_1 : 3x+15y+21z=9$,
$P_2 : x-3y-z=5$ and
$P_3 : 2x+10y+14z=5$
Then, which one of the following is true ?

a. P_1 and P_2 are parallel b. P_1 and P_3 are parallel
c. P_2 and P_3 are parallel d. P_1, P_2 and P_3 all are parallel

18. The value of $\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ (a+2)(a+3) & a+3 & 1\\ (a+3)(a+4) & a+4 & 1\end{vmatrix}$ is

a. $(a+2)(a+3)(a+4)$ b. -2
c. $(a+1)(a+2)(a+3)$ d. 0

19. The value of $\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}\,dx$ is

a. $\frac{\pi}{4}$ b. 4π c. $\frac{\pi}{2}$ d. 2π

20. Let $R = \{(P, Q) \mid, P$ and Q are at the same distance from the origin$\}$ be a relation, then the equivalence class of (1, –1) is the set

a. $S = \{(x, y) \mid x^2 + y^2 = 4\}$
b. $S = \{(x, y) \mid x^2 + y^2 = 1\}$
c. $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$
d. $S = \{(x, y) \mid x^2 + y^2 = 2\}$

Section B : Numerical Type Questions

21. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is ....

22. The number of integral values of k for which the equation $3 \sin x + 4 \cos x = k + 1$ has a solution, $k \in R$ is ....

23. The number of solutions of the equation $\log_4 (x - 1) = \log_2 (x - 3)$ is ....

24. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

25. Let $m, n \in N$ and $gcd(2, n) = 1$. If $30\binom{30}{0} + 29\binom{30}{1} + \ldots + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m$, then $n + m$ is .... (Here, $\binom{n}{k} = {}^nC_k$)

26. If $y = y(x)$ is the solution of the equation $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$, $y(0) = 0$, then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2} y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}} y\left(\frac{\pi}{4}\right)$ is ....

27. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point (4, –2, 2). If the plane is perpendicular to the line joining the points (–2, –21, 29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is ....

28. The area bounded by the lines $y = \| x - 1| - 2 |$ is ....

29. The value of the integral $\int_0^{\pi} | \sin 2x \,| dx$ is ....

30. If $\sqrt{3} (\cos^2 x) = (\sqrt{3} - 1) \cos x + 1$, the number of solutions of the given equation when $x \in [0, \pi/2]$ is ....

Answers

Physics

1. *(d)*	2. *(*)*	3. *(b)*	4. *(c)*	5. *(d)*	6. *(d)*	7. *(a)*	8. *(b)*	9. *(a)*	10. *(d)*
11. *(c)*	12. *(c)*	13. *(a)*	14. *(c)*	15. *(a)*	16. *(c)*	17. *(b)*	18. *(d)*	19. *(d)*	20. *(d)*
21. *(492)*	22. *(300)*	23. *(20)*	24. *(137)*	25. *(500)*	26. *(3.33)*	27. *(25.5)*	28. *(1215)*	29. *(400)*	30. *(33.33)*

Chemistry

1. *(c)*	2. *(d)*	3. *(d)*	4. *(a)*	5. *(a)*	6. *(a)*	7. *(d)*	8. *(b)*	9. *(b)*	10. *(d)*
11. *(d)*	12. *(c)*	13. *(b)*	14. *(d)*	15. *(d)*	16. *(a)*	17. *(d)*	18. *(d)*	19. *(c)*	20. *(d)*
21. *(200)*	22. *(7)*	23. *(50)*	24. *(25)*	25. *(1)*	26. *(73)*	27. *(6)*	28. *(18)*	29. *(2)*	30. *(0)*

Mathematics

1. *(d)*	2. *(a)*	3. *(a)*	4. *(c)*	5. *(a)*	6. *(b)*	7. *(a)*	8. *(a)*	9. *(b)*	10. *(a)*
11. *(c)*	12. *(c)*	13. *(c)*	14. *(d)*	15. *(a)*	16. *(c)*	17. *(b)*	18. *(b)*	19. *(a)*	20. *(d)*
21. *(2)*	22. *(11)*	23. *(1)*	24. *(3)*	25. *(45)*	26. *(1)*	27. *(8)*	28. *(4)*	29. *(2)*	30. *(1)*

Note *(*) None of the option is correct.*

Solutions

PHYSICS

1. *(d)* Given, distance between centre of ring and sphere, $d = \sqrt{8}R$

Since, gravitational field at the axis of ring, $E = \dfrac{Gmd}{(d^2 + R^2)^{3/2}}$

Here, G is the gravitational constant.

$$\Rightarrow \quad E = \frac{GmR\sqrt{8}}{(8R^2 + R^2)^{3/2}} = \frac{GmR\sqrt{8}}{(3R)^3}$$

$$\Rightarrow \quad E = \frac{GmR\sqrt{8}}{27R^3} = \frac{Gm\sqrt{8}}{27R^2}$$

Force between ring and sphere, $F = ME$ ...(i)

Substituting the value of E in Eq. (i), we get

$$F = \frac{\sqrt{8}}{27}\frac{GmM}{R^2}$$

2. (*) Let, equivalent capacitance of two capacitors C_1 and C_2 connected in parallel be C_a and equivalent capacitance of same, when connected in series be C_b.

According to given data,

$$C_a = \frac{15}{4}C_b \quad \text{...(i)}$$

Since, equivalent capacitance in parallel combination,

$$C_{eq} = C_1 + C_2$$

$$\therefore \quad C_a = C_1 + C_2 \quad \text{...(ii)}$$

and equivalent capacitance in series combination,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore \quad \frac{1}{C_b} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1C_2}$$

$$C_b = \frac{C_1C_2}{C_1 + C_2} \quad \text{...(iii)}$$

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$C_1 + C_2 = \frac{15}{4}\frac{C_1C_2}{C_1 + C_2}$$

$$\Rightarrow \quad 4(C_1 + C_2)^2 = 15\,C_1C_2$$

$$\Rightarrow \quad 4\,C_1^2 + 4\,C_2^2 + 8\,C_1C_2 = 15\,C_1C_2$$

$$\Rightarrow \quad 4\,C_1^2 + 4\,C_2^2 - 7C_1C_2 = 0$$

On dividing both sides by C_1^2, we get

$$4 + 4\left(\frac{C_2}{C_1}\right)^2 - 7\frac{C_2}{C_1} = 0$$

or $$4\left(\frac{C_2}{C_1}\right)^2 - 7\left(\frac{C_2}{C_1}\right) + 4 = 0$$

If $\dfrac{C_2}{C_1} = x$, then $4x^2 - 7x + 4 = 0$

By using the concept of quadratic equation,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{7 \pm \sqrt{49 - 64}}{8}$$

$$\Rightarrow \quad x = \frac{C_2}{C_1} = \frac{7 \pm \sqrt{-15}}{8} = \frac{7 \pm \sqrt{15}i}{8}$$

3. *(b)* Given, work done by gas molecule,

$$W = \alpha^2\beta e^{-\beta x^2/kT}$$

Here, x is displacement, k is Boltzmann constant, α and β are constants and T is temperature.

Dimensional formula of $[W] = [ML^2T^{-2}]$

$\therefore$ Dimensions of $[\alpha^2\beta] = [ML^2T^{-2}]$

$$\Rightarrow \quad \alpha = \left[\frac{ML^2T^{-2}}{\beta}\right]^{1/2} \quad \text{...(i)}$$

The term $[e^{-\beta x^2/kT}]$ should be dimensionless, i.e. $[M^0L^0T^0]$.

$$\Rightarrow \quad \left[\frac{\beta x^2}{kT}\right] = [M^0L^0T^0]$$

$$\Rightarrow \quad [\beta] = \frac{[k][T]}{[x^2]} \quad \text{...(ii)}$$

Energy of gaseous molecule $(E) = \dfrac{7}{2}kT$

$$\Rightarrow \quad [k] = [E]/[T] = [ML^2T^{-2}K^{-1}]$$

Substituting the value of k in Eq. (ii), we get

$$[\beta] = \frac{[ML^2T^{-2}K^{-1}][K]}{[L^2]} = [MT^{-2}]$$

Substituting the value of β in Eq. (i), we get

$$[\alpha] = \left\{\frac{[ML^2T^{-2}]}{[MT^{-2}]}\right\}^{1/2} = [M^0L^1T^0]$$

4. *(c)* By using Rydberg's formula, $\dfrac{1}{\lambda} = R\left[\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right]$

where, R is Rydberg constant.

For wavelength of third member of Lyman series $n_2 = 1$ and $n_1 = 4$.

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{4^2}\right] = R\left[\frac{1}{1} - \frac{1}{16}\right] \quad \text{...(i)}$$

For wavelength of first member of Paschen series, $n_2 = 3$ and $n_1 = 4$

$$\therefore \quad \frac{1}{\lambda_2} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = R\left[\frac{1}{9} - \frac{1}{16}\right] \quad \text{...(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{\left[\frac{1}{9} - \frac{1}{16}\right]}{\left[\frac{1}{1} - \frac{1}{16}\right]} = \frac{7}{9 \times 15} = \frac{7}{135}$$

5. *(d)* Given, height of object, $h_o = 100$ cm

Focal length of mirror, $f = 40$ cm

Height of image, $h_i = 25$ cm

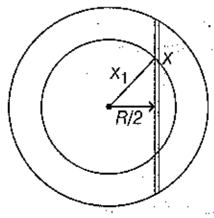
Nature of image is erect means virtual.

As, h_i is less than h_o, so mirror used is convex mirror.

Hence, (d) is correct option, i.e. image is virtual, opposite and mirror is convex.

6. *(d)* Given, radius of earth = R

Distance of chord from centre of earth = $\frac{R}{2}$



Let x_1 be the radius of inner circle and M be the mass of earth.

$\therefore m'$ (effective mass of earth) = $\frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi x_1^3$

$$\Rightarrow \quad m' = \frac{M}{R^3} x_1^3$$

If F is the gravitational force exerted by earth on particle at position x and ω be the angular velocity in time period T, then

$$F = \frac{Gm'm}{x_1^2} = \frac{Gm}{x_1^2} \cdot \frac{M}{R^3} x_1^3$$

$$\Rightarrow \quad m\omega^2 x_1 = \frac{GMmx_1}{R^3}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{GM}{R^3}} \quad \text{...(i)}$$

Since, $\omega = \frac{2\pi}{T}$ and $GM = gR^2$

Substituting the above values in Eq. (i), we get

$$\Rightarrow \quad \frac{2\pi}{T} = \sqrt{\frac{gR^2}{R^3}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

7. *(a)* Given, $i = i_1 \sin\omega t + i_2 \cos\omega t$

Let I_{rms} be the rms current.

$$\therefore \quad I_{rms} = \left(\frac{i_1^2 + i_2^2}{2}\right)^{1/2}$$

$$\Rightarrow \quad I_{rms} = \frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$$

8. *(b)* Given, density of material = ρ

Bulk modulus of elasticity = K

and applied pressure = p

Let change in volume and density be ΔV and $\Delta\rho$ respectively and initial volume and density be V and ρ.

$$\text{Since, } K = \frac{p}{-\frac{\Delta V}{V}} \quad \text{...(i)}$$

$$\text{and density } (\rho) = \frac{\text{mass}(m)}{\text{volume }(V)}$$

$$\therefore \quad \frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V}$$

Substituting it in Eq. (i), we get

$$\frac{-\Delta V}{V} = \frac{p}{K} = \frac{\Delta\rho}{\rho}$$

$$\therefore \quad \Delta\rho = \frac{p\rho}{K}$$

9. *(a)* Given, radius of circle = R

Central fictitious force is, $F \propto \frac{1}{R^3}$.

Let T be the time period of revolution, m, ω be the mass and angular velocity of Earth.

$$\because \quad F = m\omega^2 R \propto \frac{1}{R^3}$$

$$\Rightarrow \quad \omega^2 \propto \frac{1}{R^4} \Rightarrow \omega = \frac{2\pi}{T} \propto \frac{1}{R^2}$$

$$\Rightarrow \quad T \propto R^2$$

10. *(d)* According to Kepler's second law of planetary motion, areal velocity of every planet moving around the sun should remain constant in elliptical orbit.

11. *(c)* Let v_1 and v_2 are the speed of P and Q after collision.

By using law of conservation of mementum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad Mu + m \cdot 0 = Mv_1 + mv_2$$

$$\Rightarrow \quad \frac{M(u - v_1)}{m} = v_2 \quad \text{...(i)}$$

and by using law of conservation of energy,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad Mu^2 + 0 = Mv_1^2 + mv_2^2$$

$$\Rightarrow \quad M(u^2 - v_1^2) = mv_2^2$$

$$\Rightarrow \quad M(u - v_1)(u + v_1)/m = v_2^2 \quad \text{...(ii)}$$

Substituting the value of $M\frac{(u - v_1)}{m}$ from Eq. (i) in Eq. (ii), we get

$$v_2(u + v_1) = v_2^2$$

$$\Rightarrow \quad u + v_1 = v_2$$

$$\because \quad M >> m$$

$$\therefore \quad v_1 = u$$

$$\text{and} \quad v_2 = 2u$$

Hence, option (c) is the correct.

12. *(c)* Given, mass of solid sphere = m

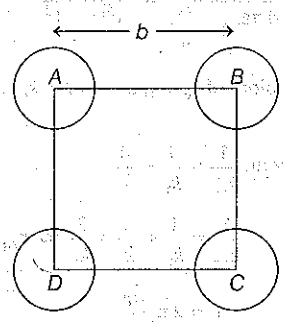
Radius of solid sphere = a

Side of square = b

Let I_{net} be the net moment of inertia of system.

Moment of inertia of sphere, $I_s = \frac{2}{5}ma^2$

Axis of rotation is BC.



$\therefore$ Moment of inertia of any body at distance,

$$d = md^2 = mb^2 \quad (\because d = b)$$

$$\therefore \quad I_{net} = 4\left(\frac{2}{5}ma^2\right) + 2(mb^2)$$

$$\Rightarrow \quad I_{net} = \frac{8}{5}ma^2 + 2mb^2$$

13. *(a)* Given, in YDSE, the separation between two slits, $d = 2$ mm $= 2 \times 10^{-3}$ m

Distance between slit and screen, $D = 1$ m

Wavelength of light, $\lambda = 500$ nm $= 500 \times 10^{-9}$ m

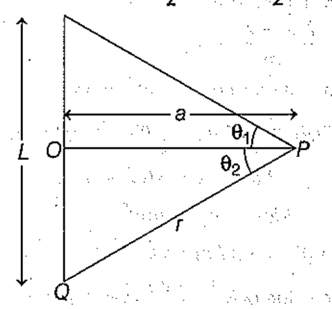
Let B will be the fringe width

$$\therefore \quad B = \frac{\lambda D}{d} = \frac{500 \times 10^{-9} \times 1}{2 \times 10^{-3}} = 250 \times 10^{-6}$$
$$= 0.25 \text{ mm}$$

14. *(c)* Given, length of conductor $= L$

Charge on conductor $= Q$

According to figure, $OP = a = \frac{\sqrt{3}}{2}L$, $OQ = \frac{L}{2}$



Let $\quad PQ = r = \sqrt{OP^2 + OQ^2}$

$$\Rightarrow \quad PQ = \sqrt{\left(\frac{\sqrt{3}}{2}L\right)^2 + \left(\frac{L}{2}\right)^2} = \sqrt{\frac{3}{4}L^2 + \frac{L}{4}} = L$$

and E be the electric field at point P.

Since, E (due to finite wire) $= \frac{k\lambda}{a}(\sin\phi_1 + \sin\phi_2)$ ...(i)

where, $k =$ Coulomb's constant $= \frac{1}{4\pi\varepsilon_0}$

$\lambda =$ linear charge density $= \frac{Q}{L}$

and $\sin\phi_1 = \sin\phi_2 = \frac{L/2}{L} = \frac{1}{2}$

Substituting the above value in Eq. (i), we get

$$E = \frac{k\lambda}{a} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{L \cdot \frac{\sqrt{3}L}{2}} = \frac{1}{2\sqrt{3}\pi\varepsilon_0} \frac{Q}{L^2}$$

15. *(a)* Let series equivalent of spring constant $= k_{eq}$ and T be the time period.

In series arrangement, $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$

$$\Rightarrow \quad \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_1} = \frac{2}{k_1} \Rightarrow k_{eq} = \frac{k_1}{2}$$

As, $\quad T = 2\pi\sqrt{\frac{m}{k_1}}$

where, m is mass of body connected with spring.

$$\Rightarrow \quad T \propto \sqrt{\frac{1}{k_1}}$$

and $\quad T' \propto \sqrt{\frac{2}{k_1}} \Rightarrow T' = \sqrt{2}\,T$

16. *(c)* Let, $Q =$ heat current,

$k =$ thermal conductivity,

$A =$ area,

$l =$ length of capacitor

and $\Delta\theta =$ change in temperature.

$$\therefore \quad Q = \frac{kA\Delta\theta}{l} = \frac{\Delta\theta}{R} \qquad \left(\because R = \frac{l}{kA}\right)$$

$$\Rightarrow \quad \frac{\theta_2 - \theta}{R_2} = \frac{\theta - \theta_1}{R_1}$$

$$\Rightarrow \quad R_1\theta_2 - R_1\theta = R_2\theta - R_2\theta_1$$

$$\Rightarrow \quad \theta = \frac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$$

17. *(b)* As we know that,

Resolving power of microscope $= \frac{1}{\Delta\theta} = \frac{2u\sin\theta}{\lambda}$

i.e. resolving power $\propto \frac{1}{\lambda}$

and since, wavelength of electron emitted $(\lambda_e) <$ wavelength of visible light (λ_v)

$\therefore$ Resolving power of electron microscope $>$ Resolving power of optical microscope.

Hence, option (b) is the correct.

18. *(d)* Given, energy gap of LED, $E = 1.9$ eV

Speed of light in free space, $c = 3 \times 10^8 \text{ms}^{-1}$

Planck's constant, $h = 6.63 \times 10^{-34}$ J-s

As we know that, $E = \frac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{hc}{E}$$

$$\Rightarrow \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.9 \times 1.6 \times 10^{-19}}$$
$$= 654 \times 10^{-9} \text{ m}$$
$$= 654 \text{ nm}$$

As, wavelength of red light is 600 nm.

$\therefore$ Required wavelength will be of red colour.

19. *(d)* Given, radius of small drop $= r$

Radius of big drop $= R$

Surface tension $= T$

and mechanical equivalent of heat $= J$

As, small drops combine to form big drop.

$\therefore$ Volume of big drop $(V_B) = n \times$ Volume of small drop (V_S)

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = n . \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad nr^3 = R^3$$

$$\Rightarrow \quad r = \frac{R}{n^{1/3}} \quad \text{...(i)}$$

Surface energy of small drop $(E_S) =$ Surface tension $(T) \times$ Area (A)

$$\Rightarrow \quad E_S = n \times 4\pi r^2 T$$

and $\quad E_B = 4\pi R^2 T$

Now, change in energy will be

$$\Delta E = E_B - E_S = 4\pi T(nr^2 - R^2)$$

$\therefore$ Heat energy per unit volume $= \frac{\Delta E}{V} = \frac{4\pi T(nr^2 - R^2)}{J \times \frac{4}{3}\pi R^3}$

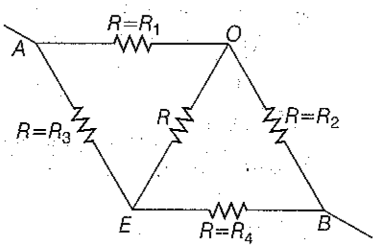
$$= \frac{3T}{J}\left(\frac{nr^2}{R^3} - \frac{1}{R}\right) = \frac{3T}{J}\left(n\frac{R^2}{n^{2/3}R^3} - \frac{1}{R}\right)$$

$$= \frac{3T}{J}\left[\frac{n^{1/3}}{R} - \frac{1}{R}\right] \quad \text{[from Eq. (i)]}$$

$$= \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

20. *(d)* Given all resistances have same resistance R.

Now, we can redraw the circuit as below



Let resistances be R_1, R_2, R_3 and R_4.

$\because \quad \frac{R_1}{R_3} = \frac{R_2}{R_4}$

So, circuit will behave as a Wheatstone bridge and no current will flow through middle resistor.

$$\therefore \quad R_{eq} = \frac{(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2) + (R_3 + R_4)}$$

$$= \frac{(R + R)(R + R)}{(R + R) + (R + R)}$$

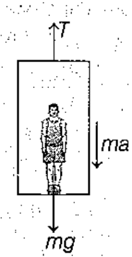
$$= R$$

21. *(492)* Given, mass of man $(m) = 60$ kg

Downward acceleration of lift, $a = 1.8\ \text{ms}^{-2}$

Let T be the tension in the rope connected with lift, g be the acceleration due to gravity $(10\ \text{ms}^{-2})$.

As, lift is moving in downward direction



$\therefore \quad mg - T = ma$

$\Rightarrow \quad T = mg - ma = m(g - a)$

$= 60(10 - 1.8)$

$= 60 \times 8.2$

$= 492$ N

Hence, the weight of the man during downward acceleration is 492 N.

22. *(300)* Given, charge passing through circuit, $q = 20$ C

Potential difference between two plates,

$V = 15$ V

Let W be the amount of work done by battery.

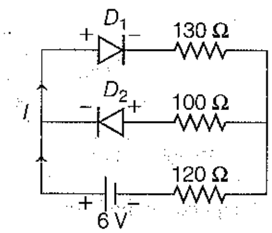
$\therefore \quad W = qV = 20 \times 15 = 300$ J

23. *(20)* Given, forward resistance, $R_1 = 50\ \Omega$

Reverse resistance, R_2 = infinity

Battery voltage = 6 V

According to circuit diagram,



In this case, diode D_1 is forward biased, whereas diode D_2 is reverse biased.

So, D_2 will act as open circuit.

By using Kirchhoff's voltage law,

$6 - 50I - 130I - 120I = 0$

$\Rightarrow \quad 6 = 300I$

$\Rightarrow \quad I = \frac{6}{300} = \frac{1}{50}$

$= \frac{2}{100} = 0.02\ \text{A} = 20$ mA

Hence, current through $120\Omega = 20$ mA

24. *(137)* Given , power of bulb, $P = 1000$ W

Distance $d = 2$ m

Efficiency of bulb = 1.25%,

$\varepsilon_0 = 8.854 \times 10^{-12} \text{C}^2\text{N}^{-1}\text{m}^{-1}$,

$c = 3 \times 10^8\ \text{ms}^{-1}$

$\therefore$ Intensity, $(I) = \frac{\text{Power}(P)}{\text{Area}(A)} = \frac{1}{2}\varepsilon_0 E^2 c$

$\Rightarrow \quad \frac{1.25}{100} \times \frac{1000}{4\pi(2)^2} = \frac{1}{2} \times 8.854 \times 10^{-12} \times E^2 \times 3 \times 10^8$

$\Rightarrow \quad E = \sqrt{\frac{12.5}{16\pi} \times \frac{2}{8.854 \times 10^{-4} \times 3}}$

$= 13.7\ \text{NC}^{-1}$

$= 137 \times 10^{-1}\ \text{NC}^{-1}$ or Vm^{-1}

$\therefore \quad x = 137$

25. *(500)* Given, mass of box, $m = 2$ kg

Force, $F = 20\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$ N

Initial speed of box, $u = 0\ \text{ms}^{-1}$

Time, $t = 10$ s

Let acceleration of box is a and displacement along X-axis after 10 s is s_x.

As, $\quad F = ma$

$\Rightarrow \quad a = \frac{F}{m} = \frac{20\hat{\mathbf{i}} + 10\hat{\mathbf{j}}}{2} = (10\hat{\mathbf{i}} + 5\hat{\mathbf{j}})\ \text{ms}^{-2}$

By second equation of motion along X - axis,

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

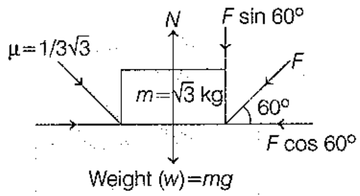
$$s_x = 0 + \frac{1}{2} \times 10 \times (10)^2 = 500\ \text{m}$$

Hence, displacement along X-axis after 10 s is 500 m.

26. *(3.33)* Given, mass of block, $m = \sqrt{3}$ kg

Coefficient of friction, $\mu = 1/3\sqrt{3}$

According to diagram,



Let F be the force applied on the body,

w be the weight $(= mg)$,

N be the normal reaction.

Friction force $f = \mu N$

For no movement of body along X-axis, net force along X-axis should be zero.

If, F_y be the net force along y-axis then it will also be zero because body is not accelerating at all.

$\therefore \quad N = F\sin 60° + mg$

$\Rightarrow \quad N = \dfrac{\sqrt{3}}{2}F + 10\sqrt{3} \quad \ldots(i)$

Similarly, $F_x = F\cos 60° - \mu N = 0$

From Eq. (i), we get

$\Rightarrow \quad \dfrac{F}{2} - \dfrac{1}{3\sqrt{3}}\left(\dfrac{\sqrt{3}}{2}F + 10\sqrt{3}\right) = 0$

$\Rightarrow \quad \dfrac{F}{2} = \dfrac{F}{6} + \dfrac{10}{3}$

$\Rightarrow \quad \dfrac{F}{2} - \dfrac{F}{6} = \dfrac{10}{3}$

$\Rightarrow \quad F = 10\text{ N}$

Given, $\quad F = 3x$

$\Rightarrow \quad x = \dfrac{10}{3} = 3.33$

27. *(25.5)* Given, volume of 1st chamber, $V_1 = 4.5$ L

Volume of 2nd chamber, $V_2 = 5.5$ L

$n_1 = 3$ mol

$p_1 = 2$ atm,

$n_2 = 4$ mol

$p_2 = 3$ atm

By using ideal gas equation, $pV = nRT$

$\therefore \quad p_1V_1 + p_2V_2 = p(V_1 + V_2)$

$\Rightarrow \quad 2 \times 4.5 + 3 \times 5.5 = p \times 10$

$\Rightarrow \quad 9 + 16.5 = 10\,p$

$\Rightarrow \quad \dfrac{25.5}{10} = p$

$\therefore \quad p = 25.5 \times 10^{-1}$ atm

Hence, $x = 25.5$ atm

28. *(1215)* Given, mass per unit length, $\mu = 0.135$ g/cm

Transverse wave equation, $y = -0.21\sin(x + 30t)$

From given equation, $\omega = 30$ rad/s, $k = 1$

Speed of wave, $v = \dfrac{\omega}{k} = \dfrac{30}{1} = 30\text{ ms}^{-1}$

Also, $\quad v = \sqrt{\dfrac{T}{\mu}}$

$\Rightarrow \quad T = v^2\mu$

$T = (30)^2 \times \dfrac{0.135 \times 10^{-3}}{10^{-2}}$

$= 900 \times 0.0135$

$= 12.15$ N

$= 1215 \times 10^{-2}$ N

Hence, $x = 1215$

29. *(400)* Given, initial quality factor $(Q_i) = 100$

Let initial inductance $(x_{L_i}) = x$

Final inductance $(x_{L_f}) = 2x$

and initial resistance $(R_i) = R$

Final resistance $(R_f) = \dfrac{R}{2}$

Final quality factor $= Q_f$

Since, $Q_i = \dfrac{X_L}{R}$

and $\quad Q_f = \dfrac{2X_L}{R/2}$

$\Rightarrow \quad Q_f = \dfrac{4X_L}{R} = 4Q_i = 4 \times 100$

$= 400$

Hence, final quality factor will be 400.

30. *(33.33)* Given, maximum amplitude $A_{\max} = 16$

Minimum amplitude, $A_{\min} = 8$

Since, modulation index $= \dfrac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$

$= \dfrac{16 - 8}{16 + 8} = \dfrac{8}{24}$

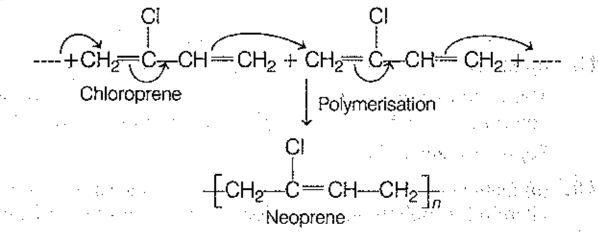
$= \dfrac{1}{3} = 0.3333$

$= 33.33 \times 10^{-2} = x \times 10^{-2}$

Hence, $x = 33.33$

CHEMISTRY

1. (c) Neoprene is an addition homopolymer of chloroprene.



2. (d) The given reaction take place as follows

(i) Ammonia (NH_3) reacts with H_2O and CO_2 to give ammonium carbonate [$(NH_4)_2CO_3$].

$$2NH_3 + \underset{(A)}{H_2O} + CO_2 \longrightarrow (NH_4)_2CO_3$$

(ii) Ammonium carbonate react with water (H_2O) and CO_2 to give ammonium hydrogen carbonate (NH_4HCO_3).

$$(NH_4)_2CO_3 + H_2O + \underset{(B)}{CO_2} \longrightarrow 2NH_4HCO_3$$

(iii) Ammonium hydrogen carbonate reacts with sodium chloride (NaCl) to give ammonium chloride (NH_4Cl) along with sodium bicarbonate ($NaHCO_3$).

$$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + \underset{(C)}{NaHCO_3}$$

So, A - H_2O, B - CO_2 ; C - $NaHCO_3$.

3. (d) Ozone is a strong oxidising agent like NO_2. In troposphere both O_3 and NO_2 react with the unburnt hydrocarbons to produce the chemical components of photochemical smog, like formaldehyde (HCHO), acrolein ($CH_2{=}CH{-}CHO$) and PAN or peroxyacetyl nitrate ($CH_3{-}\underset{\underset{O}{\|}}{C}{-}O{-}O{-}NO_2$).

Hence, in troposphere, the presence of ozone generates photochemical smog.

4. (a) Here, (B), (C) and (D) are p-block elements of the 2nd period.

(D) $\longrightarrow p^1$ configuration $\rightarrow$ B of group 13

(C) $\longrightarrow p^3$ configuration $\rightarrow$ N of group 15

(B) $\longrightarrow p^4$ configuration $\rightarrow$ O of group 16

Stability order : $\underbrace{p^3}_{\text{Half-filled}} > \underbrace{p^4 >> p^1}_{\text{Partially filled}}$

We know, ionisation enthalpy ($\Delta_i H$) $\propto$ stability of the subshell concerned.

Therefore, half-filled subshell is more stable than partially filled.

$\therefore \Delta_i H$ order is $c > b >> d$

(A) is a s-block element (group 2) of 2nd period with s^2-configuration $\rightarrow$ Be of group 2 [fully-filled; stable]

So, the correct order of IE, or $\Delta_i H_1$ (in kJ mol^{-1}) of the 2nd period elements will be

$$\begin{array}{ccccccc} \text{B} & & \text{Be} & & \text{O} & & \text{N} \\ \text{(d)} & < & \text{(a)} & \ll & \text{(b)} & < & \text{(c)} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{(i)} & & \text{(ii)} & & \text{(iii)} & & \text{(iv)} \\ [801] & & [899] & & [1319] & & [1902] \end{array}$$

The correct matching is (A)-(ii), (B)-(iii), (C)-(iv), (D)-(i).

Note The order of IE, or $\Delta_i H_1$ of 2nd period elements is

Li < B < Be < C < O < N < F < Ne

5. (a) Assertion is false but Reason is true.

Corrected Assertion Diple-dipole interactions are purely covalent interactions of dipolar covalent bonds.

Ends of dipoles of a polar covalent bonds possess partial charges (δ^+ and δ^-) which are less than unit electronic charge, $e = 1.6 \times 10^{-19}$ C

6. (a) Statement A is true. Heavy water (D_2O) is used in exchange reactions as a tracer compound for the study of reaction mechanisms.

e.g. $NaOH + D_2O \longrightarrow NaOD + HDO$

Statement B is true. Heavy water is prepared by prolonged exhaustive (multi-stage) electrolysis of ordinary water (H_2O) containing NaOH.

Statement C is true. Heavy water has higher boiling point (374.4K) than ordinary water (373K).

Statement D is false. Because viscosity (in centipoise) of D_2O is greater (1.107) than H_2O (0.8903).

So, statements A, B and C are correct (option-a).

7. (d) Number of radial nodes = $(n - l - 1)$

[n = principal quantum number, l = azimuthal quantum number]

Number of angular nodes = l

	$(n-l-1)$	l
(a) $3p(n=3, l=1)$ $\Rightarrow$	1	1
(b) $4f(n=4, l=3)$ $\Rightarrow$	0	3
(c) $4d(n=4, l=2)$ $\Rightarrow$	1	2
(d) $5d(n=5, l=2)$ $\Rightarrow$	2	2

So, $5d$-orbital has two radial as well as two angular nodes (option-d).

Note $l = 0$ for s-orbital, $l = 1$ for p-orbital, $l = 2$ for d-orbital and $l = 3$ for f-orbital.

8. (b) (A) Kernite ($Na_2B_4O_7 \cdot 4H_2O$) is sodium tetraborate tetrahydrate. It is also called rasovite. Kernite is also written as $Na_2B_4O_6(OH)_2 \cdot 3H_2O$

It is an ore of boron (ii).

(B) Cassiterite (SnO_2) is the oxide.

It is an ore of tin (i).

(C) Calamine ($ZnCO_3$) is zinc carbonate. It is also called smith sonite.

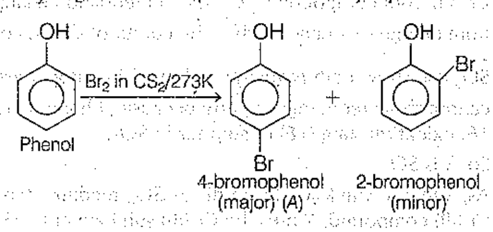
It is an ore of zinc (iv).

(D) Cryolite (Na_3AlF_6) is sodium hexafluoroaluminate.

It is an ore of fluorine (iii).

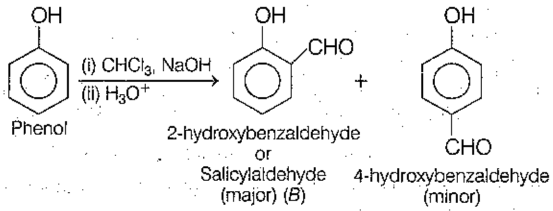
So, correct match is (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii).

9. (b) Phenol on reaction with Br_2 in CS_2 / 273 K undergoes an electrophilic substitution reaction by Br^+ (electrophile) in aprotic solvent CS_2 to give 4-bromophenol as the major product.



Phenol on reaction with $CHCl_3$, NaOH followed by hydrolysis gives salicylaldehyde as a major product.

It is Reimer-Tiemann reaction. It is also an electrophilic substitution reaction of phenol by dichlorocarbene CCl_2 (electrophilic).



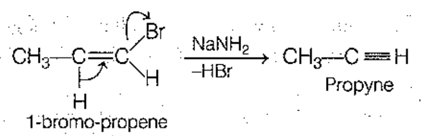
10. (*d*) Statement I is true, i.e. a mixture of chloroform and aniline can be separated by simple distillation. Boiling points of chloroform (3⁻ K) and aniline (457 K) differ largely. So, on boiling the mixture, vapours of $CHCl_3$ are formed first which is then condensed to pure liquid $CHCl_3$. Whereas, the vapours of aniline will form later and liquid aniline can be collected separately.

Statement II is also true, i.e. aniline and water can be separated by steam distillation technique. Aniline is steam volatile but immiscible with water. So, a mixture of aniline and water will boil close to but below 373 K. After distillation, the mixture of aniline (bottom layer) and water (top layer) can be separated by separating funnel.

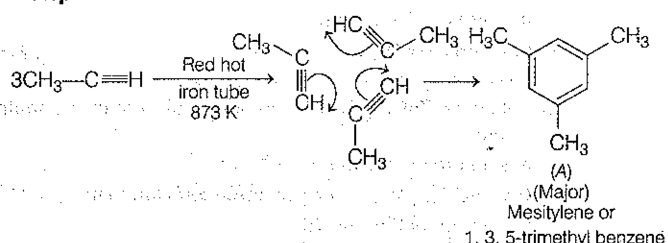
So, both statements I and II are true (option-d).

11. (*d*) When 1-bromo-prop-1-ene reacts with $NaNH_2$ (a strong base), β-elimination or dehydrobromination, (–HBr) takes place to give propyne in next step. When propyne is passed through red hot iron tube, cyclic trimerisation (aromatisation also) occurs to produce mesitylene (*A*) as major product.

Step I



Step II



12. (c) Gas, *X* turns acidified dichromate ($K_2Cr_2O_7$) green that means it is a reducing agent.

Cr (VI) from compound $K_2\overset{+6}{Cr_2}O_7$ on reduction changes its colour from orange to green which is the colour of Cr (III) compound *Y*.

$\overset{+4}{SO_2}$ can show both reducing and oxidising properties whereas $\overset{+6}{SO_3}$ cannot show reducing property because of highest group number 16, oxidation state (+6) of sulphur in SO_3.

So, *X* is SO_2.

As, *X* reacts with $K_2Cr_2O_7$ in dil. H_2SO_4 medium, the green coloured Cr (III) compound, *Y* must be Cr (III) sulphate or $Cr_2(SO_4)_3$.

So, *Y* is $Cr_2(SO_4)_3$.

The overall reactions can be shown as :

$$\underset{\text{Water soluble } SO_3^{2-} \text{ salt}}{\text{(Compound)}} \xrightarrow[H_2SO_4]{\text{Dil.}} \underset{(X)}{SO_2(g)} \xrightarrow[\text{dil. } H_2SO_4]{K_2Cr_2O_7} \underset{\substack{(Y)\\ \text{(Green)}}}{Cr_2(SO_4)_3}$$

(i) $SO_3^{2-} + H_2SO_4 \longrightarrow \underset{(X)}{SO_2\uparrow} + H_2O + SO_4^{2-}$

(ii) $\underset{\text{(Orange)}}{K_2\overset{+6}{Cr_2}O_7} + \underset{(X)}{\overset{+4}{SO_2}} + H_2SO_4 \longrightarrow \underset{\substack{(Y)\\ \text{(Green)}}}{\overset{+3}{Cr_2}(SO_4)_3} + K_2\overset{+6}{SO_4} + H_2O$

13. (*b*) Statement (b) is false whereas all other statements are true.

There are two methods for estimation of nitrogen in an organic compound which are Duma's method and Kjeldahl's method.

So, the statement in option (b) is false.

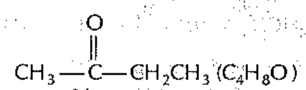
14. (*d*) Deficiency of vitamin K increases blood clotting time. So, vitamin K is helpful in blood clotting. It is a fat-soluble vitamin.

Vitamin C is used to prevent and treat scurvy. It is a water soluble vitamin.

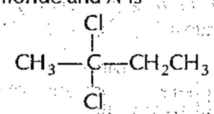
Vitamin B are also water soluble and play significant roles in cell metabolism and synthesis of RBC.

Vitamin E is a fat-soluble vitamin. Deficiency of it may cause increased fragility of RBCs, nerve problems and mascular weakness. Vitamin E is a fat-soluble anti-oxidant which protects cell membranes.

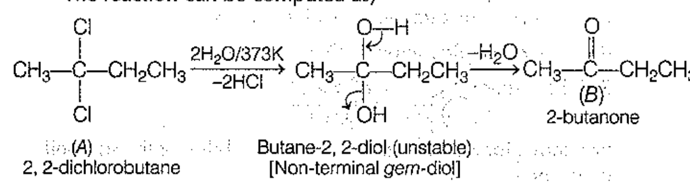
15. (*d*) Compound $B(C_4H_8O)$ reacts with hydroxylamine (NH_2OH). So, compound *B* is an aldehyde or a ketone. Again, *B* does not give Tollen's test which indicates that *B* is a ketone but not an aldehyde. So, *B* is



Compound $A\,(C_4H_8Cl_2)$ is a dihalide which undergoes hot hydrolysis (H_2O / 373 K) to give *B*, a ketone. So, *A* is a non-terminal *geminal* or *gem* dichloride and *A* is



The reaction can be computed as,



16. (*a*) In the set of four lead compounds Pb (II) compounds are PbO and $PbSO_4$. PbO_2 is a Pb (IV) compound whereas Pb_3O_4 is a mixed oxide of Pb (II) and Pb (IV) i.e. $2\overset{II}{Pb}O\cdot\overset{(IV)}{Pb}O_2$.

Pb is a member of group 14 and it shows +2 and +4 oxidation states. But due to inert pair effect, Pb^{2+} is more stable than Pb^{4+}. So, Pb (IV) compounds are strong oxidising agents as Pb^{4+} gets easily reduced to more stable Pb^{2+}.

$$Pb^{+4} + 2e^- \longrightarrow Pb^{2+},\ \Delta G^\circ < 0 \text{ (spontaneous)}$$

So, PbO_2 or Pb_3O_4 can be the compound *A*. But out of these two compounds only PbO_2 is used in lead storage batteries where a grid of lead packed with PbO_2 acts as cathode and also it is amphoteric in nature. It reacts with both acids and alkali.

(i) $\overset{+4}{Pb}O_2 + 2H\overset{-1}{Cl} \longrightarrow \overset{+2}{Pb}Cl_2 + \overset{0}{Cl_2} + H_2O$

Here, PbO_2 acts as a basic oxide as well as an oxidising agent.

(ii) $PbO_2 + 2NaOH \longrightarrow Na_2PbO_2 + H_2O$

Here, PbO_2 acts as an acidic oxide.

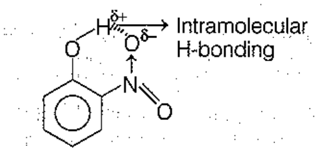
So, the compound *A* is PbO_2 (option-a).

17. (*d*) In oxides, MO_2 (M is lanthanoid metal) only four lanthanoids exhibit + 4 oxidation state. These lanthanoids are praseodymium (Pr, Z = 59), neodymium (Nd, Z = 60) terbium (Tb, Z = 65) dysprosium (Dy, Z = 66).

So, Yb (ytterbium) option (d) does not form MO_2 type of oxide.

Note The common and predominant oxidation state of lanthanoids is +3. Consequently M^{4+} compounds are strong oxidising agents which changes to the common +3 state. Similarly, lanthanoid compounds of +2 state have a tendency to show reducing property as they get changed to +3 state easily.

18. (*d*) Statement I is true. Because of closer proximity (1, 2-positions) of —OH and —NO_2 groups, *o*-nitrophenol shows intramolecular hydrogen bonding.

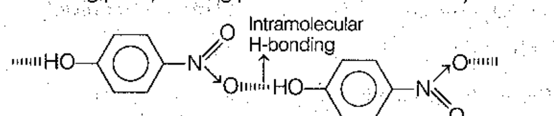


o-nitrophenol

So, *o*-nitrophenol exists in monomeric state and becomes steam volatile.

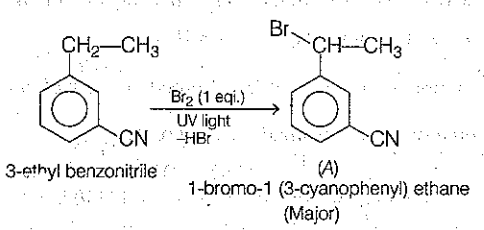
Statement II is false, because, due to the presence of intramolecular hydrogen bonding, boiling point and melting point of *o*-nitrophenol will be lower.

Note *p*-nitrophenol is the positional isomer of *o*-nitrophenol. *p*-nitrophenol shows intermolecular hydrogen bonding and so, it has higher boiling point, melting point and water solubility.



p-nitrophenol

19. (*c*) When 3-ethyl benzonitrile undergoes photochemical reaction (UV) with bromine (1 equivalent), we get a monobrominated product 1 bromo -1 (3 - cyanophenyl) ethane (*A*) as the major product.

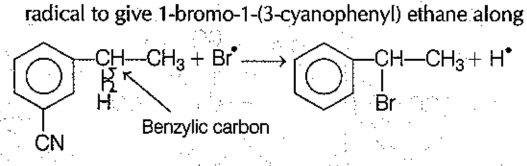


The reaction follows benzylic free radical substitution mechanism which has a 2° benzylic free radical intermediate (stable due to resonance with the benzene ring).

(i) **Initiation step** In this step radicals are generated *via* homolytic fission of covalent bond.

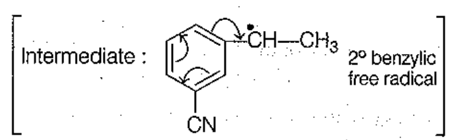
$$Br—Br \xrightarrow{UV} Br^{\bullet} + Br^{\bullet}$$

(ii) **Propagation step** 3-methyl benzonitrile reacts with bromine radical to give 1-bromo-1-(3-cyanophenyl) ethane along with $H^{\bullet}$.



(iii) **Termination step** $H^{\bullet}$ and $Br^{\bullet}$ reacts to form HBr.

$$H^{\bullet} + Br^{\bullet} \longrightarrow HBr$$



20. (*d*) The amine on reaction with benzene sulphonyl chloride (Heisenberg reagent) produces a compound insoluble in alkali. It indicates the amine is a 2° amine. i.e. all options are possible except option (b) which is a 1° amine ($CH_3CH_2NH_2$).

As this 2° amine can be prepared by ammonolysis of ethyl chloride, the 2°-amine should have at least one ethyl (C_2H_5) group.

(a) Ph—NH—$CH_2CH_2CH_3$ does not have ethyl group.

(c) $CH_3CH_2CH_2$—NH —CH_3 does not have ethyl group.

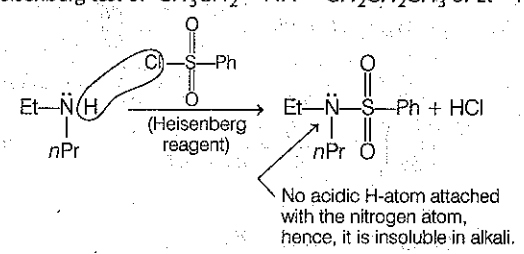
(d) $CH_3CH_2CH_2$—NH—CH_2CH_3 has one ethyl group.

So, option (d) is the correct answer.

Preparation $CH_3CH_2CH_2$—NH—CH_2CH_3 by ammonolysis of CH_3CH_2Cl

$$\underset{\text{Ethyl chloride}}{CH_3CH_2Cl} \xrightarrow[-HCl]{NH_3} \underset{\substack{\text{Ethanamine}\\(1^\circ)}}{CH_3CH_2—NH_2} \xrightarrow[(n\text{-propyl chloride})]{CH_3CH_2Cl} \underset{\substack{\text{1-(N-ethyl)-propanamine}\\(2^\circ)}}{CH_3CH_2—NH—CH_2CH_2CH_3}$$

Heisenberg test of CH_3CH_2—NH—$CH_2CH_2CH_3$ or Et—NH—*n*Pr



21. (**200**) For the reaction, $A + B \rightleftharpoons C + D$

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$

The reaction will be spontaneous when, $\Delta G^\circ < 0$,

i.e. $|T\Delta S^\circ| > |\Delta H^\circ|$

[Given, $\Delta H^\circ = 80$ kJ mol^{-1}; $\Delta S^\circ = 2T$ J mol^{-1}K^{-1}]

$$\Rightarrow \quad T > \frac{|\Delta H^\circ|}{|\Delta S^\circ|} \Rightarrow T > \frac{80 \times 1000 \text{ J mol}^{-1}}{2T \text{ J mol}^{-1}\text{K}^{-1}}$$

$$T^2 > 4 \times 10^4 \text{ K} \Rightarrow T > 200 \text{ K}$$

So, the minimum temperature (T) at which the reaction will be spontaneous is 200 K.

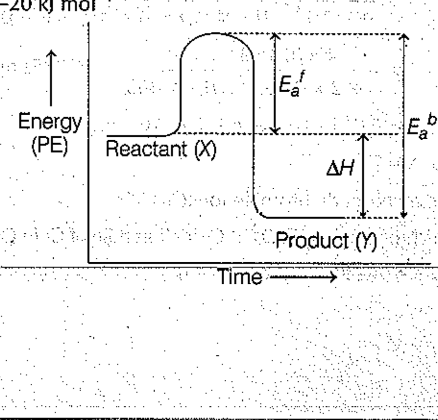
22. (**7**) Non-zero digits are always significant. Any zeros between two significant digits are significant.

∴ Zero's between 5 and 2 are all significant.

$\underbrace{50000.020}_{\uparrow} \times 10^{-3}$

(Number of significant figures = 7)

23. (**50**) $X \longrightarrow Y$, it is an exothermic reaction whose ΔE or $\Delta H = -20$ kJ mol^{-1}



Given, activation energy of the forward reaction ($X \rightarrow Y$),

$$E_a^f = 30 \text{ kJ mol}^{-1}$$

Activation energy of the backward or reverse reaction ($Y \rightarrow X$), E_a^b can be calculated as,

$$\Delta H = E_a^f - E_a^b$$

$$\Rightarrow E_a^b = E_a^f - \Delta H = 30 - (-20) = 50 \text{ kJ mol}^{-1}$$

24. (25) In the reduction half-cell,

$$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$$

1 mol 5F

5 mol 25 F

As one mole of MnO_4^- required 5F of charge,

5 moles of MnO_4^- will require charge,

$Q = 5 \times 5$ Faraday $= 25$ Faraday

25. (1) For 1 mole of a real gas, the van der Waals' equation is,

$$\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

At very high pressure, the equation becomes,

$$p(V_m - b) = RT$$

$$\Rightarrow pV_m = RT + pb \Rightarrow \frac{pV_m}{RT} = 1 + \frac{pb}{RT}$$

$$\Rightarrow Z = 1 + \frac{pb}{RT} \quad \left[\because Z = \frac{pV_m}{RT} = \text{compressibility}\right]$$

$$\therefore \left(\frac{\delta Z}{\delta p}\right)_T = 0 + \frac{b}{RT} = \frac{b}{RT} = \frac{xb}{RT} \quad \text{(Given)}$$

$$\Rightarrow x = 1$$

26. (73)

$$AB_2(g) \rightleftharpoons A(g) + 2B(g)$$

	$AB_2(g)$	$A(g)$	$2B(g)$
$t = 0$	1	0	0
$t = t_{eq}$	$(1-x)$	x	$2x$

$(\Sigma \text{ mole})_{t_{eq}} = 1 - x + x + 2x = (1 + 2x)$

Partial pressure (atm)	$\frac{1-x}{1+2x}p$	$\frac{x}{1+2x}p$	$\frac{2x}{1+2x}p$

[p = Total pressure at equilibrium = 1.9 atm]

Now, at equilibrium $pV = (1 + 2x)RT$

$$\Rightarrow 1 + 2x = \frac{pV}{RT} = \frac{1.9 \times 25}{0.082 \times 300} = 1.93$$

[$V = 25$ L, $R = 0.082$ L atm mol^{-1}K^{-1} $T = 300$ K]

$$\Rightarrow x = \frac{1.93 - 1}{2} = 0.465$$

$$\Rightarrow K_p = \frac{p_A \times p_B^2}{p_{AB_2}} \Rightarrow \frac{\left(\frac{x}{1+2x}p\right) \times \left(\frac{2x}{1+2x}p\right)^2}{\left(\frac{1-x}{1+2x}p\right)}$$

$$= \frac{4x^3 \times p^3}{(1+2x)^3} \times \frac{(1+2x)}{(1-x) \times p} = \frac{4x^3 \times p^2}{(1+2x)^2 \times (1-x)}$$

$$= \frac{4 \times (0.465)^3 \times (1.9)^2}{(1 + 2 \times 0.465)^2 \times (1 - 0.465)} = 0.7285 \text{atm}$$

$$= 72.85 \times 10^{-2} \text{ atm} \simeq 73 \times 10^{-2} = x \times 10^{-2}$$

$$\therefore x = 73$$

27. (6) In basic medium dichromate ion ($Cr_2O_7^{2-}$) changes in chromate ion (CrO_4^{2-}). Oxidation state of Cr in CrO_4^{2-} is +6.

$$\Rightarrow CrO_4^{2-} = x + 4(-2) = -2 \text{ or } x = +6.$$

Dichromate and chromate equilibrium depends on pH of the medium as

$$\underset{\substack{(aq)\\(\text{Orange})}}{\overset{+6}{Cr_2O_7^{2-}}} \underset{H^+(pH<7)}{\overset{OH^-(pH>7)}{\rightleftharpoons}} \underset{\substack{(aq)\\(\text{Yellow})}}{CrO_4^{2-}}$$

28. (18) Number of moles of SO_2 passed

$$n_1 = \frac{pV}{RT} = \frac{1 \times (224 \times 10^{-3})}{0.082 \times 298} = 9.16 \times 10^{-3} \text{mol}$$

$$= 0.00916 \text{ mol}$$

Number of moles of NaOH in the solution,

$$n_2 = \frac{100 \times 0.1}{100} = 1 \times 10^{-2} \text{ mol} = 0.01 \text{ mol}$$

$$\underset{\substack{2 \text{ mol}\\0.01 \text{ mol}}}{2NaOH} + \underset{\substack{1 \text{ mol}\\0.015 \text{ mol}}}{SO_2} \longrightarrow \underset{\substack{1 \text{ mol}\\0.05 \text{ mol}}}{Na_2SO_3} + H_2O$$

Here, NaOH is the limiting reagent as it will leave (0.00916 − 0.005) mole of SO_2 after the reaction.

So, number of moles of Na_2SO_3 (solute) produced in the solution,

$$n_2' = \frac{1}{2} \times 0.01 = 0.005 \text{ mol}$$

It is added into 36 g of water to observe the colligative property, Relative Lowering of Vapour Pressure (RLVP).

No. of moles of solute (Na_2SO_3) = 0.005mol = n_2'

No. of moles of solvent (H_2O) = 36/18 = 2 mol = n_1'

For Na_2SO_3 van't Hoff factor, $i = 3$, as

$Na_2SO_3 \longrightarrow 2Na^+ + SO_3^{2-}$ [$\alpha = 1$, strong electrolyte]

$i = [1 + \alpha(n - 1)] = [1 + 1 \times (3 - 1)] = 3$

The RLVP equation is $\frac{\Delta p}{p^\circ} = x'_{\text{solute}} \times i = \frac{n_2'}{n_1' + n_2'} \times i$

Lowering of VP (of the solution)

$$= \frac{n_2'}{n_1' + n_2'} \times i \times p^\circ = \frac{0.005}{2 + 0.005} \times 3 \times 24 \text{ mm of Hg}$$

$$= 0.1795 \simeq 0.18 \text{ mm of Hg} = 18 \times 10^{-2} \text{ mm of Hg}$$

$$= x \times 10^{-2} \text{ mm of Hg} \Rightarrow x = 18$$

29. (2) Number of moles (n) of O_2 adsorbed on 1.2 g of Pt = $\frac{3.12}{32}$ mol

Volume (V) of O_2 adsorbed on 1.2 g of Pt

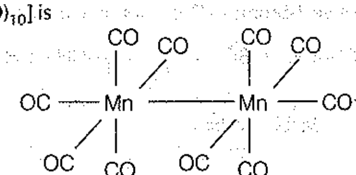
$$V = \frac{nRT}{p} = \frac{\frac{3.12}{32} \times 0.082 \times 300}{1} = 2.398 \text{ L}$$

[$R = 0.082$ L atm mol^{-1} K^{-1}, $p = 1$ atm, $T = 300$ K]

$\Rightarrow$ Volume of O_2 adsorbed per gram of adsorbent (Pt) in

$$= \frac{2.398}{1.2} = 1.998 \text{ L} \simeq 2 \text{ L}$$

30. (0) [$Mn_2(CO)_{10}$] is a polynuclear metal carbonyl compound. It is decarboxyldimanganese (0) which is made up of two square pyramidal $Mn(CO)_5$ units joined by a Mn—Mn bond. The structure of [$Mn_2(CO)_{10}$] is



Here, no CO ligand is a bridging ligand. So, number of bridging CO ligands in [$Mn_2(CO)_{10}$] is 0 (zero).

MATHEMATICS

1. (*d*) $\mathbf{a} \times [\mathbf{a} \times \{\mathbf{a} \times (\mathbf{a} \times \mathbf{b})\}]$

$= \mathbf{a} \times (\mathbf{a} \times [(\mathbf{a} \cdot \mathbf{b})\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\mathbf{b}])$

[Using, $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$]

$= \mathbf{a} \times [\mathbf{a} \times ((\mathbf{a} \cdot \mathbf{b})\mathbf{a} - |\mathbf{a}|^2 \mathbf{b})]$

$= \mathbf{a} \times [(\mathbf{a} \times (\mathbf{a} \cdot \mathbf{b})\mathbf{a}) - |\mathbf{a}|^2 (\mathbf{a} \times \mathbf{b})]$

$= \mathbf{a} \times [0 - |\mathbf{a}|^2 (\mathbf{a} \times \mathbf{b})]$ $\quad [\because \mathbf{a} \cdot \mathbf{b} = 0]$

$= -|\mathbf{a}|^2 [\mathbf{a} \times (\mathbf{a} \times \mathbf{b})]$

$= -|\mathbf{a}|^2 [(\mathbf{a} \cdot \mathbf{b})\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\mathbf{b}]$

$= -(\mathbf{a} \cdot \mathbf{b})\mathbf{a} |\mathbf{a}|^2 + |\mathbf{a}|^4 \mathbf{b}$ $\quad [\because (\mathbf{a} \cdot \mathbf{a}) = |\mathbf{a}|^2]$

$= 0 + |\mathbf{a}|^4 \mathbf{b}$

$= |\mathbf{a}|^4 \mathbf{b}$

2. (*a*) Let the coin is tossed 'n' times.

Also, let x denote number of times head occurs.

According to the question,

$$P(X = 7) = P(X = 9)$$

Using formula $P(X = r) = {}^nC_r p^r q^{n-r}$

where, p = probability of getting head in tossing a coin $= 1/2$

and $q = 1 - p = 1 - 1/2 = 1/2$

$\therefore P(X = 7) = P(X = 9)$

$$\Rightarrow \quad {}^nC_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{n-7} = {}^nC_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{n-9}$$

$$\Rightarrow \quad {}^nC_7 \left(\frac{1}{2}\right)^n = {}^nC_9 \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad {}^nC_7 = {}^nC_9$$

$\Rightarrow n = 7 + 9 = 16$ [$\because {}^nC_r = {}^nC_q$ only when $n = r + q$]

$$\therefore \quad P(X = 2) = {}^{16}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{14}$$

$$P(X = 2) = \frac{16!}{2!14!} \left(\frac{1}{2}\right)^{16}$$

$$= \frac{16 \times 15}{2} \left(\frac{1}{2}\right)^{16}$$

$$= \frac{2^3 \times 15}{2^{16}} = \frac{15}{2^{13}}$$

3. (*a*) Let A be the matrix as follows,

$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, since A is symmetric matrix.

Now, $A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & ab + bc \\ ab + bc & b^2 + c^2 \end{bmatrix}$

Given that, diagonal entries of A^2 is 1.

i.e. $a^2 + b^2 + b^2 + c^2 = 1$ or $a^2 + 2b^2 + c^2 = 1$

***Case* 1.** $a = 0$

Then, $2b^2 + c^2 = 1$

(a) $c = 0$ gives, $b^2 = \frac{1}{2}$ or $b = \pm \frac{1}{\sqrt{2}}$

$\therefore a = 0, b = 1/\sqrt{2}, c = 0$ (2 matrices)

$a = 0, b = -1/\sqrt{2}, c = 0$

(b) $b = 0$, gives $c^2 = 1$ or $c = \pm 1$

$\therefore a = 0, b = 0, c = 1$

and $a = 0, b = 0, c = -1$ (2 matrices)

***Case* 2.** $b = 0$, then $a^2 + c^2 = 1$

(a) $a = 0$, then $c = \pm 1$

$a = 0, b = 0, c = 1$ and $a = 0, b = 0, c = -1$

This is repeated case.

(b) $c = 0$, then $a = \pm 1$

$a = 1, b = 0, c = 0$ and $a = -1, b = 0, c = 0$

Again 2 matrices.

Thus, only acceptable matrices are as follows

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

Then possible number of such matrices are 4.

4. (*c*) Let the first term of geometric series be 'a' and common ratio be 'r'.

Then, nth term of given series is given as

$$T_n = ar^{n-1}$$

Now, given that sum of second and sixth term is $\frac{25}{2}$.

i.e. $\quad T_2 + T_6 = \frac{25}{2}$

$$\Rightarrow \quad ar + ar^5 = \frac{25}{2}$$

$$\Rightarrow \quad ar(1 + r^4) = \frac{25}{2} \quad \text{... (i)}$$

Also, given that product of third and fifth term is 25.

i.e. $\quad (T_3)(T_5) = 25$

$$\Rightarrow \quad (ar^2)(ar^4) = 25$$

$$\Rightarrow \quad a^2r^6 = 25 \quad \text{...(ii)}$$

Squaring Eq. (i), we get

$$a^2r^2(1 + r^4)^2 = \left(\frac{25}{2}\right)^2 \quad \text{...(iii)}$$

Divide Eq. (iii) by Eq. (ii),

$$\frac{a^2r^2(1 + r^4)^2}{a^2r^6} = \frac{(25)^2}{4(25)}$$

$$\Rightarrow \quad \frac{(1 + r^4)^2}{r^4} = \frac{25}{4} \Rightarrow 4(1 + r^4)^2 = 25r^4$$

$$\Rightarrow \quad 4(1 + r^8 + 2r^4) = 25r^4 \Rightarrow 4r^8 - 17r^4 + 4 = 0$$

$$\Rightarrow \quad 4r^8 - 16r^4 - r^4 + 4 = 0$$

$$\Rightarrow \quad 4r^4(r^4 - 4) - 1(r^4 + (-4)) = 0$$

$$\Rightarrow \quad (r^4 - 4)(4r^4 - 1) = 0$$

Given, $r^4 = 4$ or $r^4 = 1/4$

We have to find sum of 4th, 6th and 8th term, i.e.

$$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$$

$$= ar(r^2 + r^4 + r^6)$$

$$= ar^3(1 + r^2 + r^4) \quad \text{... (iv)}$$

Using Eq. (ii),

$$(ar^3)^2 = 25$$

$$\Rightarrow \quad ar^3 = 5$$

Also, we take $r^4 = 4$ because given series is increasing and $r^2 = 2$.

$$\therefore \quad T_4 + T_6 + T_8 = 5(1 + 2 + 4)$$

$$= 5(7) = 35$$

5. (*a*) Let '*x*' be any real number, then $x = [x] + \{x\}$, where $[x]$ is integer part of x and $\{x\}$ is fractional part of x.

Then, $x - [x] = \{x\}$, Also period of $\{x\} = 1$

Now, $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx = \sum_{n=1}^{100} \int_{n-1}^{n} e^{\{x\}}dx$

[Difference between upper and lower limit is 1 unit]

$$= \int_0^1 e^{\{x\}}dx + \int_1^2 e^{\{x\}}dx + \ldots + \int_{99}^{100} e^{\{x\}}dx$$

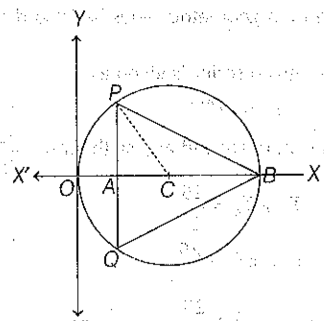
$$= e^x]_0^1 + e^{(x-1)}]_1^2 + \ldots + e^{(x-99)}]_{99}^{100}$$

$$= (e-1) + (e-1) + \ldots + (e-1) = 100(e-1)$$

6. (*b*) Given, $OA = 1$ unit, $OB = 13$ unit

Since, OB is diameter of circle.

Then, radius $(r) = 13/2 = 6.5$ units



Draw a line joining points P and C, where C is the centre of the given circle.

Then, PC = radius of circle = 6.5 units

OC = radius of circle = 6.5 units

Now, $AC = OC - OA = 6.5 - 1 = 5.5$ unit

Then, using Pythagoras theorem,

$$(PA)^2 = (PC)^2 - (AC)^2$$
$$= (6.5)^2 - (5.5)^2$$
$$= (6.5 - 5.5)(6.5 + 5.5)$$
$$= (1)(12) = 12$$

$\therefore \quad PA = \sqrt{12}$

Then, $PQ = 2PA = 2\sqrt{12}$

Hence, area of $\Delta PQB = \frac{1}{2} \times$ Base $\times$ (Height)

$$= \frac{1}{2} \times (PQ) \times (AB)$$
$$= \frac{1}{2} \times (PQ) \times (OB - OA)$$
$$= \frac{1}{2} \times (2\sqrt{12}$$
$$= 12\sqrt{12} = 24\sqrt{3} \text{ sq units}$$

7. (*a*) Given, $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots$

Let, $S_1 = \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots \quad \ldots \text{(i)}$

Multiply 1/3 in series Eq. (i),

$$\frac{S_1}{3} = \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \ldots \quad \ldots \text{(ii)}$$

Subtract Eq. (ii) from Eq. (i), we get

$$S_1 - \frac{S_1}{3} = \frac{2}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \ldots$$

$$\Rightarrow \frac{2S_1}{3} = \frac{2}{3} + \left[\frac{5}{3^2} + \frac{5}{3^3} + \ldots\right]$$

$$= \frac{2}{3} + \left[\frac{5/3^2}{1 - 1/3}\right]$$

$\left[\because \frac{5}{3^2} + \frac{5}{3^3} + \ldots\right.$ is a geometric series with $r = 1/3$, sum upto infinity of this series is $= \frac{a}{1-r}$, where a = first term]

$$= \frac{2}{3} + \left[\frac{5}{6}\right] = \frac{9}{6} = \frac{3}{2}$$

$$\Rightarrow S_1 = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$$

$$\therefore \quad S = 1 + S_1 = 1 + \frac{9}{4} = \frac{13}{4}$$

8. (*a*) $\lim_{h \to 0} 2 \left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cosh - \sinh)}\right\}$

$$= \lim_{h \to 0} 2 \left\{\frac{2\left(\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right) - \frac{1}{2}\cos\left(\frac{\pi}{6}+h\right)\right)}{2 \times \sqrt{3}h\left(\frac{\sqrt{3}}{2}\cosh - \frac{1}{2}\sinh\right)}\right\}$$

$$= \lim_{h \to 0} 2 \left\{\frac{\cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}+h\right) - \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\cos\frac{\pi}{6}\cosh - \sin\frac{\pi}{6}\sinh\right)}\right\}$$

$$= \lim_{h \to 0} 2 \left\{\frac{\sin\left(\frac{\pi}{6}+h-\frac{\pi}{6}\right)}{\sqrt{3}h\cos\left(h+\frac{\pi}{6}\right)}\right\} = \lim_{h \to 0} \frac{2}{\sqrt{3}}\left\{\frac{\sinh}{h\cos(h+\pi/6)}\right\}$$

$$= \frac{2}{\sqrt{3}} \cdot \lim_{h \to 0}\frac{\sinh}{h} \cdot \lim_{h \to 0}\frac{1}{\cos(h+\pi/6)}$$

$$= \frac{2}{\sqrt{3}} \cdot (1) \cdot \frac{1}{\cos(\pi/6)}$$

$$= \frac{2}{\sqrt{3}} \cdot 1 \cdot \frac{2}{\sqrt{3}} = \frac{4}{3}$$

9. (*b*) Using Binomial expansion, its $(r+1)$ th term be,

$$T_{r+1} = {}^{10}C_r(tx^{1/5})^{10-r}\left\{\frac{(1-x)^{1/10}}{t}\right\}^r$$

$$= {}^{10}C_r\frac{(t)^{10-r}}{(t)^r}(x^{1/5})^{10-r}(1-x)^{r/10}$$

$$= {}^{10}C_r(t)^{10-2r}(x)^{\frac{10-r}{5}}(1-x)^{r/10}$$

If this term is independent of 't', then we have $10 - 2r = 0$ gives, $r = 5$

$\therefore \quad T_6 = {}^{10}C_5(x)^1(1-x)^{1/2}$

Let $f(x) = x(1-x)^{1/2}$, to obtain its maximum value, we have to differentiate it and equate it to 0.

i.e. $f'(x) = 0 \Rightarrow \frac{x}{2\sqrt{1-x}}(-1) + \sqrt{1-x} = 0$

$\Rightarrow -x + 2(1-x) = 0 \Rightarrow -3x + 2 = 0$

$\Rightarrow \quad x = \frac{2}{3}$ and $f''\left(\frac{2}{3}\right) < 0$ (Maximum value)

Thus, greatest term will be

$$T_6 = {}^{10}C_5\left(\frac{2}{3}\right)\left(1-\frac{2}{3}\right)^{1/2} = {}^{10}C_5\frac{2}{3\sqrt{3}} = \frac{10! \cdot 2}{(5!)^2(3\sqrt{3})}$$

10. (*a*) Let x be the number of bacteria at any time t.

Given that, $\frac{dx}{dt} \propto x$ $\left(\because \text{Rate of growth} = \frac{dx}{dt}\right)$

$$\Rightarrow \frac{dx}{dt} = \lambda x \Rightarrow \frac{dx}{x} = \lambda dt$$

After integrating it, we get

$$\log x = \lambda t + c \quad \text{... (i)}$$

Given, when $t = 0$, $x = 1000$ which gives

$$\log 1000 = 0 + c \Rightarrow c = \log 1000$$

From Eq. (i), we have

$$\log x - \log 1000 = \lambda t \text{ or } \log\left(\frac{x}{1000}\right) = \lambda t \quad \text{... (ii)}$$

Given that in 2h, number of bacteria increased by 20% i.e. when $t = 2\text{h}$, $x = 1200$

Put, $t = 2$ and $x = 1200$ in Eq. (ii),

$$\log\left(\frac{1200}{1000}\right) = 2\lambda \text{ gives, } \lambda = \frac{1}{2}\log\left(\frac{6}{5}\right)$$

Again, from Eq. (ii),

$$\log\left(\frac{x}{1000}\right) = \frac{1}{2}\log\left(\frac{6}{5}\right)\cdot t$$

or $$\frac{x}{1000} = e^{\frac{t}{2}\log\left(\frac{6}{5}\right)} \quad \text{... (iii)}$$

Given, $x = 2000$ at $t = k/\log_e(6/5)$, put in Eq. (iii),

$$\frac{2000}{1000} = e^{\frac{k}{2}\log\left(\frac{6}{5}\right)/\log\left(\frac{6}{5}\right)}$$

$$2 = e^{k/2} \text{ or } \log 2 = k/2$$

$$\Rightarrow k/\log 2 = 2$$

$$\therefore (k/\log_e 2)^2 = (2)^2 = 4$$

11. (*c*) Let $P(1, 5, 35)$, $Q(7, 5, 5)$, $R(1, \lambda, 7)$, $S(2\lambda, 1, 2)$

Given P, Q, R, S are coplanar. Then, **PQ**, **PR**, **PS** lie on the same plane.

$$\mathbf{PQ} = (7-1)\hat{i} + (5-5)\hat{j} + (5-35)\hat{k} = 6\hat{i} - 30\hat{k}$$

$$\mathbf{PR} = (1-1)\hat{i} + (\lambda-5)\hat{j} + (7-35)\hat{k} = (\lambda-5)\hat{j} - 28\hat{k}$$

$$\mathbf{PS} = (2\lambda-1)\hat{i} + (1-5)\hat{j} + (2-35)\hat{k} = (2\lambda-1)\hat{i} - 4\hat{j} - 33\hat{k}$$

$\because$ **PQ**, **PR** and **PS** lie on same plane, then

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

Expand along first row,

$$6[-33(\lambda-5) - 112] + 30[(2\lambda-1)(\lambda-5)] = 0$$

$$\Rightarrow 6(-33\lambda + 53) + 30(2\lambda^2 - 11\lambda + 5) = 0$$

$$\Rightarrow 60\lambda^2 - 528\lambda + 468 = 0$$

$$\Rightarrow 10\lambda^2 - 88\lambda + 78 = 0$$

$$\Rightarrow 5\lambda^2 - 44\lambda + 39 = 0 \quad \text{... (i)}$$

Possible value of λ are roots of Eq. (i).

Then, sum of all possible values of λ = Sum of roots of Eq. (i)

$$= \frac{-(-44)}{5} = \frac{44}{5}$$

$$[\because ax^2 + bx + c = 0, \text{ sum of roots} = -b/a]$$

12. (*c*) $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$ ...(i)

Take first two terms of Eq. (i)

$$\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b}$$

$$\Rightarrow \frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\sin^{-1} x + \cos^{-1} x}{a+b}$$

$$\Rightarrow \frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\pi/2}{a+b} \quad \left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right]$$

$$\Rightarrow \frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\pi/2}{a+b} = \frac{\tan^{-1} y}{c}$$

Using last two terms,

$$\frac{\tan^{-1} y}{c} = \frac{\pi/2}{a+b}$$

$$\Rightarrow \tan^{-1} y = \frac{\pi c}{2(a+b)}$$

$$\Rightarrow 2\tan^{-1} y = \frac{\pi c}{(a+b)}$$

$$\Rightarrow \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) = \frac{\pi c}{a+b} \quad \left[\because 2\tan^{-1} y = \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$$

$$\Rightarrow \frac{1-y^2}{1+y^2} = \cos\left(\frac{\pi c}{a+b}\right)$$

$$\therefore \cos\left(\frac{\pi c}{a+b}\right) = \frac{1-y^2}{1+y^2}$$

13. (*c*) To form a seven digit number with sum of digits 10, all the digits can't be 1, 2 or 3.

Hence, seven digit number must have the following cases,

Case 1. Using 1, 1, 1, 1, 1, 2, 3

Possible seven digit numbers will be $= \frac{7!}{5!} = 7 \times 6 = 42$

Case 2. Using 2, 2, 2, 1, 1, 1, 1

Possible numbers will be $= \frac{7!}{3!4!} = \frac{7 \times 6 \times 5}{3 \times 2} = 35$

No more cases will be formed.

Hence, total number of seven digit numbers possible = 42 + 35 = 77

14. (*d*) Given, $|f(x) - f(y)| \le |x-y|^2$

$$\Rightarrow \frac{|f(x) - f(y)|}{|x-y|} \le |x-y|$$

Now, taking the limit,

$$\lim_{x \to y}\left|\frac{f(x) - f(y)}{x-y}\right| \le \lim_{x \to y} |x-y|$$

$$\Rightarrow |f'(y)| \le 0 \quad \text{[using the definition of } f'(y)]$$

$$\Rightarrow f'(y) = 0$$

[since, modulus value can never be less than 0]

On integrating it, we get

$$f(y) = c \text{ (constant)}$$

Given, $f(0) = 1$ gives $c = 1$

$\therefore$ $f(y) = 1 \, \forall \, y \in R$

From given options, $f(x) > 0 \, \forall \, x \in R$ is satisfied only.

Hence, answer will be option (d).

15. (*a*) Given, curve is $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ ...(i)

First, find the slope of given curve i.e. dy/dx,

Differentiate Eq. (i),

$$\frac{dy}{dx} = \frac{1}{2}(4x^3) - 5(3x^2) + 18(2x) - 19$$

$$= 2x^3 - 15x^2 + 36x - 19$$

Now, let $f(x) = 2x^3 - 15x^2 + 36x - 19$ is slope of the curve and find its maximum value as follows,

$$f'(x) = 2(3x^2) - 15(2x) + 36 = 6x^2 - 30x + 36$$

Equate $f'(x) = 0$ and solve for 'x',

$$6x^2 - 30x + 36 = 0$$

$$\Rightarrow \quad x^2 - 5x + 6 = 0$$

$$\Rightarrow \quad x^2 - 3x - 2x + 6 = 0$$

$$\Rightarrow \quad (x-3)(x-2) = 0$$

$$\Rightarrow \quad x = 2 \text{ and } 3$$

Now, $f''(x) = \dfrac{d}{dx}(6x^2 - 30x + 36)$

$= 12x - 30$

Then, $f''(2) = 12(2) - 30 = 24 - 30$

$= -6 < 0$

and $f''(3) = 12(3) - 30 = 6 > 0$

$\because f''(2) < 0$, this implies '2' is point of maxima.

$\therefore$ At $x = 2$, slope will be maximum.

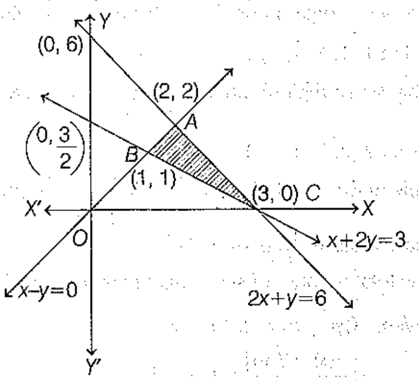
Since, at $x = 2$, slope will be maximum, then y-coordinate will be,

$$y = \frac{1}{2}(2)^4 - 5(2)^3 + 18(2)^2 - 19(2)$$

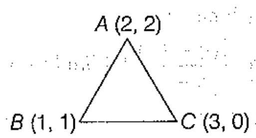
$$= 8 - 40 + 72 - 38 = 72 - 70 = 2$$

$\therefore$ Maximum slope occurs at point (2, 2).

16. (c) Given lines, $x - y = 0$, $x + 2y = 3$, $2x + y = 6$



The only triangle which include all three lines is ΔABC.



Now, $AB = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{2}$

$AC = \sqrt{(2-3)^2 + (2-0)^2} = \sqrt{5}$

$BC = \sqrt{(3-1)^2 + (0-1)^2} = \sqrt{5}$

$\Rightarrow \quad AC = BC$ (two sides are equal)

$\Rightarrow \Delta ABC$ is isosceles triangle.

17. (b) Given, $P_1 \Rightarrow 3x + 15y + 21z = 9$

$P_2 \Rightarrow x - 3y - z = 5$

$P_3 \Rightarrow 2x + 10y + 14z = 5$

Consider plane P_1, it can be written as

$3x + 15y + 21z = 9$ or $x + 5y + 7z = 3$

Again, consider plane P_3, it can be written as,

$2x + 10y + 14z = 5$ or $x + 5y + 7z = 5/2$

Hence, P_1 and P_3 are parallel.

18. (b) Given, $A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$

Apply $R_2 \to R_2 - R_1$

$$A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(2) & 1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Apply $R_3 \to R_3 - R_1$

$$A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

Now, expanding along third column,

$$A = 1[4(a+2) - (4a+10)] = 4a + 8 - 4a - 10$$

$$= -2$$

19. (a) Let $I = \displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\,dx$ ... (i)

Using the property, $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2(\pi/2 - \pi/2 - x)}{1 + 3^{\pi/2 - \pi/2 - x}}\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}}\,dx \quad [\because \cos(-x) = \cos x]$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{(1+3^x)}\,dx \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii),

$$2I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\,dx + \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x}\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{(1+3^x)\cos^2 x}{1+3^x}\,dx = \int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{1+\cos 2x}{2}\,dx \quad [\because \cos 2x = 2\cos^2 x - 1]$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_{-\pi/2}^{\pi/2} = \frac{1}{2}[\pi]$$

$$\Rightarrow \quad 2I = \pi/2 \quad \Rightarrow \quad I = \frac{\pi}{4}$$

20. (d) Let $P(a, b)$ and $Q(c, d)$ are any two points.

Given, $OP = OQ$

i.e. $\sqrt{a^2 + b^2} = \sqrt{c^2 + d^2}$

Squaring on both sides,

$$a^2 + b^2 = c^2 + d^2 \quad \text{... (i)}$$

$$R = \{((a, b), (c, d)) : a^2 + b^2 = c^2 + d^2\}$$

$R(x, y)$, $S(1, -1)$, $OR = OS$ (equivalence class)

This gives $OR = \sqrt{x^2 + y^2}$ and $OS = \sqrt{2}$

$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{2}$

$\Rightarrow x^2 + y^2 = 2$ (Squaring on both sides)

$\therefore \quad S = \{(x, y) : x^2 + y^2 = 2\}$

21. (2) Given, $y^2 = a\left[x + \dfrac{\sqrt{a}}{2}\right]$, $a > 0$ ...(i)

Differentiating both sides w.r.t. 'x',

$$2y\frac{dy}{dx} = a[1 + 0] = a \quad \text{...(ii)}$$

Use Eq. (ii) in Eq. (i) to eliminate the constant 'a'.

$$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{2y}\sqrt{\frac{dy}{dx}}\right)$$

$$y^2 - 2xy\frac{dy}{dx} = 2\sqrt{2}\cdot y\sqrt{y}\cdot\frac{dy}{dx}\sqrt{\frac{dy}{dx}}$$

Squaring on both sides,

$$y^4 + 4x^2y^2\left(\frac{dy}{dx}\right)^2 - 4xy^3\frac{dy}{dx} = 8y^3\left(\frac{dy}{dx}\right)^3$$

Thus, degree of above differential equation is 3 and its order is 1.
Difference between degree and order = 3 − 1 = 2

22. (*11*) Given, $3\sin x + 4\cos x = k + 1$ ... (i)

Multiply and divide LHS of Eq. (i) by $\sqrt{3^2 + 4^2} = 5$

i.e. $5\left(\frac{3}{5}\sin x + \frac{4}{5}\cos x\right) = k + 1$

$\Rightarrow 5(\cos\alpha\sin x + \sin\alpha\cos x) = k + 1$

[Let $\cos\alpha = 3/5$ then $\sin\alpha = \sqrt{1-(3/5)^2} = \frac{4}{5}$]

$\Rightarrow 5\sin(x+\alpha) = k + 1$ [Use $\sin(a+b) = \sin a\cos b + \cos a\sin b$]

$\Rightarrow \sin(x+\alpha) = \frac{k+1}{5}$

Let $x + \alpha = \theta$

Then, $\sin\theta = \frac{k+1}{5}$

$\because \quad -1 \le \sin\theta \le 1$

$\Rightarrow \quad -1 \le \frac{k+1}{5} \le 1$

$\Rightarrow \quad -5 \le k + 1 \le 5$

$\Rightarrow \quad -6 \le k \le 4$

$\therefore$ Possible integral values of k are − 6, − 5, − 4, − 3, − 2, − 1, 0, 1, 2, 3 and 4.

i.e. Total 11 integral values of k are possible for which Eq. (i) has solution.

23. (*1*) $\log_4(x-1) = \log_2(x-3)$ (given)

$\Rightarrow \quad \log_{2^2}(x-1) = \log_2(x-3)$

Using property of logarithm, $\log_{b^c} a = \frac{1}{c}\log_b a$

$\Rightarrow \quad \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \quad \log_2(x-1) = 2\log_2(x-3)$

$\Rightarrow \quad \log_2(x-1) = \log_2(x-3)^2$

On comparing, $x - 1 = (x-3)^2$

or $\quad x - 1 = x^2 + 9 - 6x$

$\Rightarrow \quad x^2 - 7x + 10 = 0$

$\Rightarrow x^2 - 5x - 2x + 10 = 0$

$\Rightarrow \quad (x-5)(x-2) = 0$

$\Rightarrow \quad x = 2, 5$

$x = 2$ (rejected) as $x > 3$

$\therefore x = 5$ is only solution i.e. number of solution is 1.

24. (*3*) Given, $x^3 - 2x^2 + 2x - 1 = 0$

i.e. $\quad (x^3 - 1) - (2x^2 - 2x) = 0$

$\Rightarrow (x-1)(x^2 + x + 1) - 2x(x-1) = 0$

$\Rightarrow \quad (x-1)(x^2 + x + 1 - 2x) = 0$

$\Rightarrow \quad (x-1)(x^2 - x + 1) = 0$

$\therefore \quad x = 1$ and $x = \frac{-(-1) \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}i}{2}$

$\therefore$ Roots are 1, $-\omega$, $-\omega^2$.

Then, sum of 162[th] power of the roots

$= (1)^{162} + (-\omega)^{162} + (-\omega^2)^{162}$

$= 1 + \omega^{162} + \omega^{324}$

$= 1 + (\omega^3)^{54} + (\omega^3)^{108}$

$= 1 + (1)^{54} + (1)^{108}$ $\quad [\because \omega^3 = 1]$

$= 1 + 1 + 1 = 3$

25. (*45*) Given, $30\cdot{}^{30}C_0 + 29\cdot{}^{30}C_1 + \ldots + 2\cdot{}^{30}C_{28} + {}^{30}C_{29} = n\cdot 2^m$

This can be written as,

$$\sum_{r=0}^{29}(30-r)\,{}^{30}C_r = n\cdot 2^m$$

or $$\sum_{r=0}^{30}(30-r)\cdot{}^{30}C_r = n\cdot 2^m$$

$$\Rightarrow \sum_{r=0}^{30}30\cdot{}^{30}C_r - \sum_{r=0}^{30}r\cdot{}^{30}C_r = n\cdot 2^m$$

$$\Rightarrow 30\sum_{r=0}^{30}{}^{30}C_r - \sum_{r=0}^{30}r\cdot{}^{30}C_r = n\cdot 2^m$$

Using combination properties,

$30\cdot(2)^{30} - 30\cdot(2)^{29} = n\cdot 2^m$

$\Rightarrow \quad 30\cdot(2)^{29}(2-1) = n\cdot 2^m$

$\Rightarrow \quad 2.15\cdot(2)^{29} = n\cdot 2^m$

$\Rightarrow \quad 15\cdot(2)^{30} = n\cdot 2^m$

Comparing both sides,

$n = 15$ and $m = 30$

$\Rightarrow \quad n + m = 15 + 30 = 45$

26. (*1*) Given $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x$ ... (i)

Let $e^{\sin y} = t$, then $e^{\sin y}\cdot\cos y\cdot\frac{dy}{dx} = \frac{dt}{dx}$

Putting in Eq. (i),

$\frac{dt}{dx} + t\cos x = \cos x$ ... (ii) (Linear form)

Then, $IF = e^{\int \cos x\, dx} = e^{\sin x}$

Solution of differential Eq. (ii) is,

$t\cdot IF = \int \cos x\cdot IF\, dx + C$

$t\cdot e^{\sin x} = \int \cos x\cdot e^{\sin x}dx + C$

Let $e^{\sin x} = e^u$ i.e. let $\sin x = u$ then $\cos x\, dx = du$

$\Rightarrow \quad t\cdot e^{\sin x} = \int e^u du + C = e^u + C$

Put $u = \sin x$ and $t = e^{\sin y}$

$\Rightarrow \quad e^{\sin y}\cdot e^{\sin x} = e^{\sin x} + C$

Given, $y(0) = 0$, this gives $C = 0$

$\Rightarrow \quad e^{\sin y}\cdot e^{\sin x} = e^{\sin x}$

$\Rightarrow \quad e^{\sin y + \sin x} = e^{\sin x}$

$\Rightarrow \quad \sin y + \sin x = \sin x$

$\Rightarrow \quad \sin y = 0$

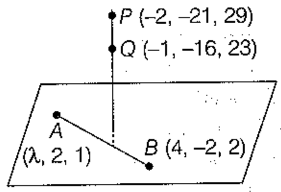
$\Rightarrow \quad y = 0$

$\therefore y(\pi/6) = y(\pi/3) = y(\pi/4) = 0$

Hence, $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$

$= 1 + 0 + 0 + 0 = 1$

27. (*8*) Given $(\lambda, 2, 1)$ be point on the plane which passes through $(4, -2, 2)$ and plane is perpendicular to line joining P and Q.



Given, **AB** is perpendicular to **PQ** i.e., $\mathbf{AB}\cdot\mathbf{PQ} = 0$

Now, $\mathbf{AB} = (4-\lambda)\hat{\mathbf{i}} + (-2-2)\hat{\mathbf{j}} + (2-1)\hat{\mathbf{k}}$

$= (4-\lambda)\,\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\mathbf{PQ} = (-1+2)\hat{\mathbf{i}} + (-16+21)\hat{\mathbf{j}} + (23-29)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

Hence, $\mathbf{AB}\cdot\mathbf{PQ} = 0$

$\Rightarrow \quad (4-\lambda)(1) + (-4)(5) + (1)(-6) = 0$

$\Rightarrow \quad 4 - \lambda - 20 - 6 = 0$

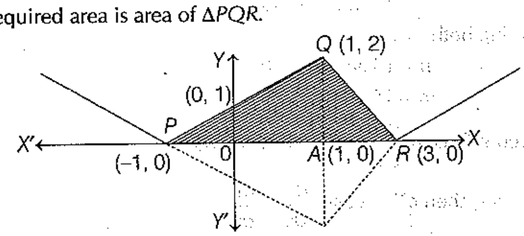
$\Rightarrow \quad \lambda = -22$

Then, $\left(\dfrac{\lambda}{11}\right)^2 - \left(\dfrac{4\lambda}{11}\right) - 4 = \left(\dfrac{-22}{11}\right)^2 - \left(\dfrac{4\times(-22)}{11}\right) - 4$

$= 4 - (-8) - 4 = 8$

28. (*4*) Given, $y = \left|\,|x-1| - 2\,\right|$

Required area is area of ΔPQR.



Area $= \dfrac{1}{2} \times (\text{Base}) \times (\text{Height})$

$= \dfrac{1}{2} \times (PR) \times (AQ)$

$= \dfrac{1}{2} \times 4 \times 2 = 4$

Since, only one curve is given, here assume the area bounded by *X-axis*. Then, the area will be 4 sq unit.

29. (*2*) Let $I = \int_0^{\pi} |\sin 2x|\, dx$

$= 2\int_0^{\pi/2} |\sin 2x|\, dx \quad [\because \sin 2x \text{ is periodic function}]$

$= 2\int_0^{\pi/2} \sin 2x\, dx \; [\sin 2x \text{ is positive in range } (0, \pi/2)]$

$= 2\left[\dfrac{-\cos 2x}{2}\right]_0^{\pi/2}$

$= -[\cos\pi - \cos 0] = -(-1-1) = 2$

$I = 2$

30. (*1*) Given, $\sqrt{3}\cos^2 x = (\sqrt{3}-1)\cos x + 1,\ x \in [0, \pi/2]$

Let $\cos x = t$, then

$\sqrt{3}t^2 = (\sqrt{3}-1)t + 1$

$\Rightarrow \quad \sqrt{3}t^2 - \sqrt{3}t + t - 1 = 0$

$\Rightarrow \quad (\sqrt{3}t^2 - \sqrt{3}t) + (t-1) = 0$

$\Rightarrow \quad \sqrt{3}t(t-1) + 1(t-1) = 0$

$\Rightarrow \quad (t-1)(\sqrt{3}t + 1) = 0$

This gives $t = 1$ and $t = \dfrac{-1}{\sqrt{3}}$

Put, $t = \cos x$, then

$\cos x = 1$ and $\cos x = \dfrac{-1}{\sqrt{3}}$

$\cos x = -1/\sqrt{3}$ is rejected as $x \in [0, \pi/2]$

$\therefore \cos x = 1$

Since, $x \in \left[0, \dfrac{\pi}{2}\right]$, then $\cos x = \cos 0$

This gives $x = 0$ is only solution.

Therefore, number of solution when $x \in [0, \pi/2]$ is 1.